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Vague Weak Li - Ideals and Normal Vague Li - Ideals of Lattice Implication Algebras

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Abstract: In this paper, we introduce the notions of vague weak LI - ideals and normal vague LI - ideals of lattice implication algebras. We discuss some properties of vague weak LI - ideals and normal vague LI - ideals. We prove that every VLI - ideal extended to normal vague LI - ideal. We study the relations between vague weak LI - ideals and VLI - ideals, vague weak LI - ideals and VILI - ideals, vague weak LI - ideals and vague weak filters, normal vague LI - ideals and vague maximal LI - ideals.

Keywords: Lattice implication algebras; VLI - ideals; VILI - ideals; VWLI - ideals; NVLI - ideals.

1. Introduction

In 1993, Xu [1] first established the lattice implication algebra by combing lattice and implication algebra, and explored many useful structures. The ideal theory serves a vital function for the development of lattice implication algebras. Jun, *et al.* [2] introduced the notion of LI - ideals of a lattice implication algebras and discussed some of their properties. In particularly, Lai, *et al.* [3], Lai and Xu [4], Lai, *et al.* [5] introduced the notion of WLI - ideals in lattice implication algebras and discussed some of their properties.

The concept of fuzzy set was introduced by Zadeh [6]. Xu, *et al.* [7] introduced the notion of Normal fuzzy LI - ideals in lattice implication algebras and discussed some of their properties. The concept of vague set introduced by Gau and Buehrer [8] in 1993. The idea of vague sets is that the membership of every element can be divided into two aspects including supporting and opposing. Rangit Biswas [9] initiated the study of vague algebra by studying vague groups. At first, Ya Qin and Yi Liu [10] applied the concept of vague set theory to lattice implication algebras and introduced the notion of v- filter, and investigated some of their properties.

In this paper, we introduce the notions of VWLI - ideals and normal vague LI - ideals of lattice implication algebras and discuss some of their properties. We prove that every VLI - ideal extended to normal vague LI - ideal. We study the relation between VWLI - ideals and VLI - ideals, between VWLI - ideals and VILI - ideals, NVLI - ideals and vague maximal LI - ideals.

Throughout this article, *L* denotes lattice implication algebra.

2. Preliminaries

In this section we given some definitions and state some results for later use.

Definition 2.1: [11] Let (L , ∨, ∧, ′, 0, I) be a complemented lattice with the universal bounds 0, I. '→ ` is another binary operation of L. Then (L , ∨, ∧, → ′, 0, I) is called a lattice implication algebra, if ∀ x, y, z ϵ L, the following axioms hold:

(I_1) $x \to (y \to z) = y \to (x \to z)$;

(I_2) $x \to x = I$;

(I_3) $x \to y = y' \to x'$;

(I_4) $x \to y = y \to x = I$ implies $x = y$;

(I_5) $(x \to y) \to y = (y \to x) \to x$

(L_1) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$;
(L_2) $(x \wedge y)\ z = (x \to z) \vee (y \to z)$.

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A lattice implication algebra (L, ∨, ∧, →, ′, 0, I) is said to be a lattice H implication algebra if $x \vee y \vee ((x \wedge y) \rightarrow z) = I$, ∀ x, y, z ϵ L.

Theorem 2.2: [11] Let L be a lattice implication algebra, then for any x, y, z ϵ L, the following conclusions hold:

(1) If $I \rightarrow x = I$ then $x = I$.

(2) $I \rightarrow x = x$ and $x \rightarrow 0 = x$.

(3) $0 \rightarrow x = I$ and $x \rightarrow I = I$.

(4) $x \leq y$ if and only if x y = I.

Definition 2.3: [2] Let I be a nonempty subset of a lattice implication algebra L. I is said to be an LI - ideal of L if it satisfies the following conditions:

(1) 0 ϵ I ;
(2) ∀ x, y ϵ L, $(x \rightarrow y)'$ ϵ I and y ϵ I implies x ϵ I.

Definition 2.4: [3] A subset A of L is said to be a weak LI – ideal (briefly, WLI - ideal) of L if $(x \rightarrow y)'$ ϵ A ⇒ $((x \rightarrow y)' \rightarrow y)'$ ϵ A for all x, y ϵ L.

Definition 2.5: [5] A subset A of L is said to be a weak filter of L if $x \rightarrow y$ ϵ A ⇒ $x \rightarrow (x \rightarrow y)$ ϵ A , ∀ x, y ϵ L.

Definition 2.6: [11]. Let $L_1$and L_2 be lattice implication algebras. A map f: $L_1 \rightarrow L_2$ is called an implication homomorphism if $f(x \rightarrow y) = f(x) \rightarrow f(y)$ for all x, y ϵ L_1.

Moreover, if f satisfies the following conditions:

$f(x \wedge y) = f(x) \wedge f(y)$

$f(x \vee y) = f(x) \vee f(y)$

$f(x') = (f(x))'$

for all x, y ϵ L_1 , we say that f is a lattice implication homomorphism.

Definition 2.7: [8] A vague set A in the universal of discourse X is characterized by two membership functions given by:

(1)A truth membership function $t_A : X \rightarrow [0,1]$ and

(2)A false membership function $f_A : X \rightarrow [0,1]$,

Where $t_A(x)$ is a lower bound of the grade of membership of x derived from the "evidence for x", and $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $t_A(x) + f_A(x) \leq 1$. The vague set A is written as $A = \{\langle x, [t_A(x), f_A(x)] \rangle / x \in X\}$. Where the interval $[t_A(x), 1 - f_A(x)]$ is called the value of x in the vague set A and denoted by $V_A(x)$.

Definition 2.8 [8]: The α – cut, A_α of the vague set A is the (α, α) – cut of A and hence given by $A_\alpha = \{x \in X / t_A(x) \geq \alpha\}$, where α ϵ [0, 1].

Definition 2.9[12]: Let A be a vague set of a lattice implication algebra L. A is said to be a vague LI – ideal (briefly VLI – ideal) of L,

If it satisfies the following conditions:

(1) ∀ x ϵ L, $V_A(0) \geq V_A(x)$,
(2) ∀ x, y ϵ L, $V_A(x) \geq \text{imin}\{V_A((x \rightarrow y)'), V_A(y)\}$.

Definition 2.10[13]: Let A be a vague set of a lattice implication algebra L. A is said to be a vague implicative LI – ideal (briefly, VILI – ideal) of L if it satisfies the following conditions:

(1) ∀ x ϵ L, $V_A(0) \geq V_A(x)$,
(2) ∀ x, y, zϵ L, $V_A((x \rightarrow y)') \geq \text{imin}\{V_A(((x \rightarrow y)' \rightarrow y)' \rightarrow z)'), V_A(z)\}$.

Definition 2.11[14]: Let A be a vague set of a lattice implication algebra L. A is said to be a vague implicative LI – ideal (briefly, VPILI – ideal) of L if it satisfies the following conditions:

(1) ∀ x ϵ L, $V_A(0) \geq V_A(x)$,

(2) ∀ x, y, zϵ L, $V_A(y) \geq \text{imin}\{V_A(((y \rightarrow (z \rightarrow y)')' \rightarrow x)'), V_A(x)\}$.

3. Vague Weak LI - ideals

In this section, we introduce the notation of VWLI – ideals and investigate some of their properties.

Definition 3.1: A vague set A of L is said to be a vague weak LI – ideal (briefly, VWLI – ideal) of L, if for any x, y ϵ L, $V_A(((x \rightarrow y)' \rightarrow y)') \geq V_A((x \rightarrow y)')$.

Example 3.2: Let L = {0, a, b, c, d, I} be a set with Cayley table as follows:

→	0	A	B	C	D	I
0	I	I	I	I	I	I
A	C	I	B	C	B	I
B	D	A	I	B	A	I
C	A	A	I	I	A	I
D	B	I	I	B	I	I
I	0	A	B	C	D	I

Define $'$, $\vee$ *and* $\wedge$ operations on L as follows:

$x' = x \to 0$;

$x \vee y = (x \to y) \to y$;

$x \wedge y = ((x \to y)' \to y')'$ for all x, y ϵ L.

Then $(L, \vee, \wedge, \to, ', 0, I)$ is a lattice implication algebra [11]. Let A be a vague set of L defined by $A=\{\langle 0,[0.7,0.2]\rangle\langle a,[0.5,0.3]\rangle,\langle b,[0.5,0.3]\rangle,\langle c,[0.7,0.2]\rangle,\langle d,[0.7,0.2]\rangle,\langle I,[0.5,0.3]\rangle\}$.
Clearly, A is a VWLI – ideal of L.

Corollary 3.3: A vague set A of L is a VWLI – ideal of L if for any x, y ϵ L,

$$V_A((x \to y)' \otimes y') \geq V_A((x \to y)').$$

Theorem 3.4: Every VLI – ideal of L is a VWLI – ideal of L.

Proof: Suppose that A is a VLI – ideal of L. For any x, y ϵ L, we have

$$\begin{aligned}((x \to y)' \to y)' \to (x \to y)' &= (x \to y) \to ((x \to y)' \to y)\\ &= (x \to y) \to (y' \to (x \to y))\\ &= y' \to ((x \to y) \to (x \to y))\\ &= y' \to I\\ &= I.\end{aligned}$$

So $((x \to y)' \to y)' \leq (x \to y)'$ and A is order reversing, we have

$$V_A(((x \to y)' \to y)') \geq V_A((x \to y)').$$

Hence A is a VWLI – ideal of L.

Example 3.5: Let L be a lattice implication algebra defined in the example 3.2. Then, the VLI – ideal $B = \{\langle 0,[0.7,0.2]\rangle,\langle a,[0.5,0.3]\rangle,\langle b,[0.5,0.3]\rangle,\langle c,[0.7,0.2]\rangle,\langle d,[0.5,0.3]\rangle,\langle I,[0.5,0.3]\rangle\}$ is VWLI – ideal of L.

Remark 3.6: The converse of the theorem 3.4 need not to be true. For example the vague set A defined in example 3.2 is VWLI – ideal but it is not a VLI – ideal of L.

Corollary3.7: Let A be a vague set of L,

(1) If A is VILI – ideal of L, then A is VWLI – ideal of L.
(2) If A is VPILI – ideal of L, then A is VWLI – ideal of L.

Corollary 3.8: Every vague lattice ideal of a lattice H implication algebra is a VWLI – ideal.

Theorem 3.9: Let A be a vague set of L. Then A is a VWLI – ideal of L if and only if A_α is a WLI – ideal of L when $A_\alpha \neq \emptyset$, α ϵ [0, 1].

Proof: Let A be a VWLI – ideal of L and α ϵ [0, 1] such that $A_\alpha \neq \emptyset$.
Clearly for any x, y ϵ L, $V_A(((x \to y)' \to y)') \geq V_A((x \to y)')$.
Let x, y ϵ L such that $(x \to y)'$ ϵ A_α

$$\Rightarrow V_A((x \to y)') \geq [\alpha, \alpha]$$
$$\Rightarrow V_A(((x \to y)' \to y)') \geq [\alpha, \alpha]$$
$$\Rightarrow ((x \to y)' \to y)' \in A_\alpha$$

So A_α is VWLI – ideal of L.
Conversely suppose that $A_\alpha \neq \emptyset$ is a WLI – ideal of L, where α ϵ [0, 1].
If A is not a VWLI – ideal of L, then $V_A(((x \to y)' \to y)') < V_A((x \to y)')$ for some x, y ϵ L.
Taking $[\alpha_1, \alpha_1] = \frac{1}{2}[V_A(((x \to y)' \to y)') + V_A((x \to y)')]$.
Then clearly α_1 ϵ [0, 1] and
$V_A(((x \to y)' \to y)') \leq [\alpha 1, \alpha 1] \leq V_A((x \to y)')$ ……………………..(i)
From the right hand side of the inequality (i), we know that $(x \to y)'$ ϵ $A_{\alpha 1}$.
So that $((x \to y)' \to y)'$ ϵ $A_{\alpha 1}$.
It follows that $V_A(((x \to y)' \to y)') \geq [\alpha 1, \alpha 1]$ and this contradicts the left hand side of the inequality (i). So A is a VWLI – ideal of L.

Theorem 3.10: Let A and B are two vague sets of L such that A ⊆ B. If B is a VWLI – ideal of L then A is a VWLI – ideal of L.

Proof: This theorem follows from the theorem3.9 and the theorem 24 in [4].

Definition 3.11: A vague set A of L is said to be a vague weak filter (briefly, VWF) of L, if for any $x, y \in L$, $V_A(x \to (x \to y)) \geq V_A(x \to y)$.
Example 3.12: Let L be a lattice implication algebra defined in the example 3.2. Then, the vague set
$C = \{\langle 0, [0.5, 0.3]\rangle, \langle a, [0.7, 0.2]\rangle, \langle b, [0.7, 0.2]\rangle, \langle c, [0.5, 0.3]\rangle, \langle d, [0.5, 0.3]\rangle, \langle I, [0.7, 0.2]\rangle\}$
is a VWF of L.
Corollary 3.13: A vague set A of L is a VWLI – ideal of L if for any $x, y \in L$,

$$V_A(x' \oplus (x \to y)) \geq V_A(x \to y).$$

Theorem 3.14: Every V – filter of L is a VWF of L.
Proof: Suppose that A is a V- filter of L.
For any $x, y \in L$, we have $(x \to y) \to (x \to (x \to y)) = x \to ((x \to y) \to (x \to y) = x \to I = I$.

So $(x \to y) \leq (x \to (x \to y))$ and A is order preserving, we have

$$V_A(x \to y) \leq V_A(x \to (x \to y)).$$

Hence A is a VWF of L.
Theorem 3.15: Let A be a vague set of L. Then A is a VWF of L if and only if A_α is a weak filter of L when $A_\alpha \neq \emptyset$ $\alpha \in [0, 1]$.
Proof: Let A be a VWF of L and $\alpha \in [0, 1]$ such that $A_\alpha \neq \emptyset$
Clearly for any $x, y \in L$, $V_A(x \to y) \leq V_A(x \to (x \to y))$.
Let $x, y \in L$ such that $(x \to y) \in A_\alpha \Rightarrow V_A(x \to y) \geq [\alpha, \alpha]$

$$\Rightarrow V_A(x \to (x \to y)) \geq [\alpha, \alpha]$$
$$\Rightarrow x \to (x \to y) \in A_\alpha.$$

So, A_α is VWF of L.
Conversely suppose that $A_\alpha \neq \emptyset$ is a weak filter of L, where $\alpha \in [0, 1]$.
If A is not a VWF of L, then $V_A(x \to (x \to y)) < V_A(x \to y)$ for some $x, y \in L$.
Taking $[\alpha_1, \alpha_1] = \frac{1}{2}[V_A(x \to (x \to y)) + V_A(x \to y)]$. Then clearly $\alpha_1 \in [0, 1]$ and
$V_A(x \to (x \to y)) \leq [\alpha 1, \alpha 1] \leq V_A(x \to y)$ ……………………..(i)
From the right hand side of the inequality (i), we know that x y $\in A_{\alpha 1}$.
So that $x \to (x \to y) \in A_{\alpha 1}$.
It follows that $V_A(x \to (x \to y)) \geq [\alpha 1, \alpha 1]$ and this contradicts the left hand side of the inequality (i).
So A is a VWF of L.

Let A be a vague set of L, define a new vague set A′ of L as $A'(x) = A(x')$ for $x \in L$.
Theorem 3.16: The vague set A of L is a VWLI – ideal of L if and only if A′ is a VWF of L.
Proof: It is obvious that if A is a vague set of L, then $(A')_\alpha = (A_\alpha)'$, for all $\alpha \in [0, 1]$.
So, this theorem follows from theorem3.9, theorem 3.3[2] and theorem 3.15.

Let I be a subset of L and $\alpha \in [0, 1]$. Now define vague set A_I by

$$V_{A_I}(x) = [\alpha, \alpha] \text{ if } x \in I$$
$$= [0, 0] \quad \text{otherwise}$$

Theorem 3.17: The subset I of L is a WLI – ideal of L if and only if A_I is a VWLI – ideal of L.
Proof: Suppose that I is WLI – ideal of L.
Then $(x \to y)' \in I$ implies $((x \to y)' \to y)' \in I$ for all $x, y \in L$.
It follows that $V_{A_I}(((x \to y)' \to y)') = V_{A_I}((x \to y)') = [\alpha, \alpha]$.
If $(x \to y)' \notin I$ then $V_{A_I}((x \to y)') = [0, 0]$.
Now two cases are arise, either $(x \to y)' \in I$ or $(x \to y)' \notin I$.
Suppose $(x \to y)' \in I$ then $V_{A_I}(((x \to y)' \to y)') = [\alpha, \alpha] = V_{A_I}((x \to y)')$.
Suppose $(x \to y)' \notin I$ then $V_{A_I}(((x \to y)' \to y)') \geq V_{A_I}((x \to y)')$.
So A_I is a VWLI – ideal of L.
Conversely suppose that A_I is a VWLI – ideal of L.
Let $x, y \in L$ such that $(x \to y)' \in I \Rightarrow V_{A_I}((x \to y)') = [\alpha, \alpha]$

$$\Rightarrow V_{A_I}(((x \to y)' \to y)') = [\alpha, \alpha]$$
$$\Rightarrow ((x \to y)' \to y)' \in I.$$

So I is a WLI – ideal of L.
Theorem 3.18: Let A be a VWLI – ideal of L and the mapping f: $[0, 1] \to [0, 1]$ is a increasing function. Then the vague set $V_A(f)(x) = f(V_A(x))$, for $x \in L$ is a VWLI – ideal of L.
Proof: Let A be a VWLI – ideal of L. Then for any $x, y \in L$, it follows that
$V_A(f)(((x \to y)' \to y)') = f(V_A(((x \to y)' \to y)')) \geq f(V_A(x \to y)') = V_A(f)((x \to y)')$.
Hence $V_A(f)(x)$ is a VWLI – ideal of L.
Theorem 3.19: Let L_1 and L_2 be two lattice implication algebras. The mapping f: $L_1 \to L_2$ is an onto lattice implication homomorphism, then

1. If B is a VWLI – ideal of L_2 then $f^{I}(B)$ is a VWLI – ideal of L_1.
2. If A is a VWLI – ideal of L_1 then f(A) is a VW LI – ideal of L_2.

Proof: Let L_1 and L_2 be two lattice implication algebras. The mapping f: $L_1 \to L_2$ is an onto lattice implication homomorphism.

(1) Let B is a VWLI – ideal of L_2. For any x, y ϵ L_2, it follows that

$f^{-1}(V_B)(((x \to y)' \to y)') = V_B(f((x \to y)' \to y)') \geq V_B(f(x \to y)') = f^{-1}(V_B)((x \to y)')$

So, $f^{-1}(B)$ is a VW LI – ideal of L_1.

(2). Let A is a VWLI – ideal of L_1. Since f: $L_1 \to L_2$ is an onto then for any x, y ϵ L,, there exist u, v ϵ L_1 such that f(u) = x and f(v) = y. Then

$$\begin{aligned} f(V_A)(((x \to y)' \to y)') &= \vee_{f(t) = ((x\ y)'\ y)'} V_A(t) \\ &= V_A(f((u \to v)' \to v)') \\ &\geq V_B(f(u \to v)') \\ &= V_A(f^{-1}(x \to y)') \\ &= f(V_A)((x \to y)'). \end{aligned}$$

So f (A) is a VWLI – ideal of L_2.

Theorem 3.20: Let A and B be two VWLI – ideals of L, then A ∩ B and A ∪ B are also VWLI – ideals.

Proof: Let A and B be two VWLI – ideals of L. For any x, y ϵ L, we have

$VA \cap B(((x \to y)' \to y)') \geq V_A((x \to y)')$ and $V_B(((x \to y)' \to y)') \geq V_B((x \to y)')$.

$$\begin{aligned} VA \cap B(((x \to y)' \to y)') &= \text{imin}\{V_A(((x \to y)' \to y)'), V_B(((x \to y)' \to y)')\} \\ &\geq \text{imin}\{V_A((x \to y)'), V_A((x \to y)')\} \\ &= VA \cap B(x \to y)'). \end{aligned}$$

$$\begin{aligned} \text{And } VA \cup B(((x \to y)' \to y)') &= \text{imax}\{V_A(((x \to y)' \to y)'), V_B(((x \to y)' \to y)')\} \\ &\geq \text{imax}\{V_A((x \to y)'), V_A((x \to y)')\} \\ &= VA \cup B((x \to y)'). \end{aligned}$$

Hence A ∩ B and A ∪ B are also VWLI – ideals of L.

Corollary 3.21: Let$\{A_i / i \in I\}$ is the set of VWLI – ideals of L, then $\bigcup A_i$ and $\bigcap A_i$ are VWLI – ideals of L.

4. Normal Vague LI - Ideals

In this section, we introduce the notion of NVLI - ideals and investigate some of their properties. We study the relation between NVLI - ideals and vague maximal LI - ideals.

Definition 4.1: A VLI – ideal A of L is said to be normal vague LI – ideal (briefly, NVLI – ideal) if there exist x ϵ L such that $V_A(x) = 1$.

Example 4.2: Let L be a lattice implication algebra defined in the example 3.2. Then, the VLI – ideal

$D = \{\langle 0, [1, 0]\rangle, \langle a, [0.5, 0.3]\rangle, \langle b, [0.5, 0.3]\rangle, \langle c, [1, 0]\rangle, \langle d, [0.5, 0.3]\rangle, \langle I, [0.5, 0.3]\rangle\}$ is NVLI – ideal of L.

Remark 4.3: It is clearly if A is NVLI – ideal of L, then $V_A(0) = 1$.

Let A be a vague set of a lattice implication algebra L such that $t_A(x) + f_A(x) \leq t_A(0) + f_A(0)$ for all x ϵ L. Define A^* such that $V_{A^*}(x) = V_A(x) + 1 - V_A(0)$, for all x ϵ L, then A^* is a normal vague set of L.

Theorem 4.4: Let A be a VLI – ideal of L. Then the vague set A^* is a NVLI – ideal of L.

Proof: Let A be a VLI – ideal of L. For any x, y ϵ L, we have

$$\begin{aligned} \text{imin}\{V_{A^*}((x \to y)'), V_{A^*}(y)\} &= \text{imin}\{V_A((x \to y)') + 1 - V_A(0), V_A(y) + 1 - V_A(0)\} \\ &= \text{imin}\{V_A((x \to y)'), V_A(y)\} + 1 - V_A(0) \\ &\leq V_A(x) + 1 - V_A(0) \\ &= V_{A^*}(x) \end{aligned}$$

Thus A^* is a NVLI– ideal of L.

Example 3.5: Let L be a lattice implication algebra defined in the example 3.2. Then, the vague set

$E = \{\langle 0, [0.7, 0.2]\rangle, \langle a, [0.5, 0.3]\rangle, \langle b, [0.5, 0.3]\rangle, \langle c, [0.7, 0.2]\rangle, \langle d, [0.5, 0.3]\rangle, \langle I, [0.5, 0.3]\rangle\}$

is a vague LI – ideal of L. Clearly the vague set

$E^* = \{\langle 0, [1, 0]\rangle, \langle a, [0.8, 0.1]\rangle, \langle b, [0.8, 0.1]\rangle, \langle c, [1, 0]\rangle, \langle d, [0.8, 0.1]\rangle, \langle I, [0.8, 0.1]\rangle\}$

is NVLI – ideal of L.

Theorem 4.6: Let A be a VLI – ideal of L and $V_{A^*}(x) = 0$ for some x ϵ L then $V_A(x) = 0$.

Proof: Let A be VLI – ideal of L and $V_{A^*}(x) = 0$ for some x ϵ L. Then

$$\begin{aligned} & V_{A^*}(x) = 0 \\ & \Rightarrow V_A(x) + 1 - V_A(0) = 0 \\ & \Rightarrow V_A(x) + 1 - 1 = 0 \\ & \Rightarrow V_A(x) = 0. \end{aligned}$$

Theorem 4.7: Any VLI – ideal A of L is normal if and only if $A^* = A$.

Proof: Let A be a N VLI – ideal A of L and x ϵ L.

$V_{A^*}(x) = V_A(x) + 1 - V_A(0) = V_A(x) + 1 - 1 = V_A(x)$.

The converse is obvious.

Corollary 4.8: If A is NVLI – ideal of L then $(A^*)^* = A^*$.

Theorem 4.9: Let A and B are two VLI – ideals of L, then $(A \cup B)^* = A^* \cup B^*$.

Proof: Let A and B are two VLI – ideals of L and x ϵ L, then we have

$$\begin{aligned} V(A \cup B)^*(x) &= VA \cup B(x) + 1 - VA \cup B(x) \\ &= \max\{V_A(x), V_B(x)\} + 1 - \max\{V_A(0), V_B(0)\} \end{aligned}$$

$= \max \{V_{A^*}(x), V_{B^*}(x)\}$
$= V(A^* \cup B^*)(x)$.

Theorem 4.10: Let A be a VLI – ideal of L. If B is a VLI – ideal of L such that $B^* \subseteq A$, A is NVLI – ideal of L.

Definition 4.11: A VLI – ideal is called a maximal if it not L and it is a maximal element of the set of all VLI – ideals of L with respect to vague set inclusion.

Definition 4.12: If A is a maximal vague LI – ideal of L, then

(1) A is NVLI – ideal of L.
(2) A takes only the values 0 and 1.

Proof: Let A be a maximal vague LI – ideal of L.

(1) Suppose that A is not a NVLI – ideal of L. Then for some x ϵ L, we have $V_{A^*}(x) \geq V_A(x)$ and $V_{A^*}(0) \geq V_A(0)$. It follows that A is not a maximal vague LI – ideal of L, Which is contradiction.

(2) Clearly A is NVLI – ideal of L, so $V_A(0) = 1$. Let a ϵL and $V_A(a) \neq 1$.

We claim that $V_A(a) = 0$. If not, then $0 < a < 1$.

Let us consider B be a vague set of L defined by $V_B(x) = \frac{1}{2}\{V_A(x) + V_A(a)\}$ for any x ϵL.

Then $V_B(0) = \frac{1}{2}\{V_A(0) + V_A(a)\} = \frac{1}{2}\{1 + V_A(a)\} \geq \frac{1}{2}\{V_A(x) + V_A(a)\} = VB(x)$.

For any x, y ϵ L, we obtain
$$V_B(x) = \frac{1}{2}\{V_A(x) + V_A(a)\}$$
$$\geq \frac{1}{2}\{\text{imin}\{V_A((x \to y)'), V_A(y)\} + V_A(a)\}$$
$$= \text{imin}\{\frac{1}{2}\{V_A((x \to y)') + V_A(a)\}, \frac{1}{2}\{V_A(y) + V_A(a)\}\}$$
$$= \text{imin}\{V_A((x \to y)'), V_A(y)\}$$

So B is a VLI – ideal of L.

For any x ϵ L**,** we have
$$V_{B^*}(x) = V_B(x) + 1 - V_B(0) = \frac{1}{2}\{V_A(x) + V_A(a)\} + 1 - \frac{1}{2}\{V_A(0) + V_A(a)\}$$
$$= \frac{1}{2}\{V_A(x) + 1\}$$
$$\geq V_A(x)$$

and $V_{B^*}(a) = V_B(a) + 1 - V_B(0) = \frac{1}{2}\{V_A(a) + 1\} < 1 = V_{B^*}(0)$.

Hence B^* is non constant and A is not a maximal vague LI – ideal of L. This is a contradiction.

5. Conclusion

Since W.L. Gai and D.J. Buehrer proposed the notion of vague sets, these ideas have been applied to various fields. In this paper In this paper, we applied these ideas to Lattice implication algebras and introduced the notion of VWLI - ideals and NVLI - ideals. We obtained some properties of VWLI - ideals and NVLI - ideals. we derive the relations between VWLI - ideals and VLI - ideals, between VWLI - ideals and VILI - ideals, between NVLI - ideals and maximal vague LI - ideals. We desperately hope that our work would serve as a foundation for enriching corresponding many - valued logical system.

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