Academic Journal of Applied Mathematical Sciences

ISSN(e): 2415-2188, ISSN(p): 2415-5225

Vol. 2, No. 7, pp: 70-76, 2016

URL: http://arpgweb.com/?ic=journal&journal=17&info=aims

Pure Moving Average Vector Bilinear Time Series Model and Its Application

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Abstract: Most time series assume both linear and non linear components because of their random nature. Thus, the classical linear models are not appropriate for modeling series with such behaviour. This work was motivated by the need to propose a vector moving average (MA) bilinear concept that caters for the linear and non linear components of a series on the basis of the 'orders' of the linear MA process. To achieve this, a matrix that preserved the 'orders' of the linear processes was formulated with given conditions. With the introduction of diagonal matrix of lagged white noise processes, some special bilinear models emerged and the 'orders' of the pure linear MA processes were maintained in both the linear and non linear parts. The derived vector bilinear models were applied to revenue series, and the result showed that the models gave a good fit which depicted its validity.

Keywords: Vector representation; Matrix representation; Bilinear moving average process; Autocorrelation function; Partial autocorrelation function.

1. Introduction

Moving average linear models have formed an important class that has provided useful descriptions for most random processes arising in a large variety of problems. Perhaps its popularity has been as a result of its technical convenience.

However, most time series have been found with evidence of non stationarity thus rendering the classical linear models inappropriate for modeling. It was in this view that the bilinear models were developed.

The interesting feature of a bilinear system is that though it is non-linear, its structural theory is analogous to that of linear systems [1].

Iwueze [2] studied the existence and computation of all second order moments of the vector valued time series of the form

$$X_t = Ce_t + AX_{t-1} + \sum_{j=1}^{q} b_j e_{t-j} + \sum_{j=1}^{q} B_j X_{t-j} e_{t-j}$$

where $X_t = \left(X_t, X_{t-1}, X_{t-2}, \ldots, X_{t-p+1}\right)^T$, C and bj are given $p \times 1$ matrices with real entries, A and B_j are given matrices with real entries; and $p = \max(r,m), g = \min(m,s), q = \max(h,g)$. He found that the vectorial representation leads to an important result on matrix algebra with respect to the spectral radius of Kronecker product of matrices.

According to Rao [1]; a bilinear time series model $BL(p,r,m,k)$ is given by the difference equation:

$$X(t) + \sum_{j=1}^{p} a_j X(t-j) = \sum_{j=0}^{r} c_j e(t-j) + \sum_{l=1}^{m} \sum_{l^1=1}^{k} b_{ll^1} X(t-l) e(t-l^1) \qquad (1)$$

where $\{e(t)\}$ is an independent white noise process and $c_0 = 1$. $\{X(t)\}$ is termed the bilinear process. The autoregressive moving average model $ARMA(p,r)$ is obtained from (1) by setting $b_{ll^1} = 0 \ \forall \ l$ and l^1.

Parameter estimation of bilinear processes has been studied for particular cases by Bouzaachane, *et al.* [3].

Boonchai and Eivind [4] gave the general form of multivariate bilinear time series models as :

$$X(t) = \sum A_i . X(t-i) + \sum M_j . e(t-j) + \sum\sum\sum B_{dij} . X(t-i) . e_d(t-j) + e(t)$$

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Here the state $X(t)$ and noise $e(t)$ are n-vectors and the coefficients A_i, M_j and B_{dij} are n by n matrices. If all $B_{dij} = 0$, we have the class of well-known vector $ARMA$-models.

In bilinear autoregressive process, lagged values of X_t forms the basic building block in the linear part. Thus, we expect the purely random process $\{\varepsilon_t\}$ to occupy the linear parts of a bilinear moving average process, while the non linear component comprises the product of X_t and ε_t.

This research work seeks to address a vector form of bilinear moving average models with respect to the 'orders' of the pure linear moving average (MA) models.

The data used for estimation is a three source monthly generated revenue (for a period of ten years) of Ik. L.G.A. in Nigeria.

2. Theoretical Framework and Matrix Formulation

Let us consider an $n-$dimensional vector $\boldsymbol{X}_t$ of time series :

$$\boldsymbol{X}_t^{\prime} = [X_{1t}, X_{2t}, ..., X_{nt}].$$

Define Q to be a set of all linear MA orders of $\boldsymbol{X}_t$; so that $q_i \in Q$ and

$$Q = \{q_1, q_2, ..., q_n\}; i = 1,2,...,n.$$

2.1. Linear and Non Linear Separation

While maintaining the MA orders, we distinguish the linear vector component matrix as:

$$\boldsymbol{X}_t = \sum_{l=1}^{max\, q_i} \lambda_l \varepsilon_{t-l} + U_t \tag{2}$$

By introducing a diagonal matrix $\varepsilon_{t-l}^{(D)}$ of lagged values of white noise process, the non linear vector part emerges:

$$\boldsymbol{X}_t = \sum_{l=1}^{max q_i} \varepsilon_{t-l}^{(D)} \{b_l X_{t-k^*}\} + U_t \tag{3}$$

where U_t is a vector of white noise.

Now combining the two equations above gives a bilinear vector moving average (BIVMA) matrix as shown:

$$\boldsymbol{X}_t = \sum_{l=1}^{max\, q_i} \lambda_l \varepsilon_{t-l} + \sum_{l=1}^{max q_i} \varepsilon_{t-l}^{(D)} \{b_l X_{t-k^*}\} + U_t \tag{4}$$

with a special condition that $b_{k^*l.ij} = 0 \quad \forall \quad i = j$ which avoids the emergence of simultaneous equation models.

Any element of $\boldsymbol{X}_t$ could be obtained from the above expression.

For a pure BIVMA processs, $k^* = 0$

Thus,

$$\boldsymbol{X}_t = \sum_{l=1}^{max q_i} \lambda_l \varepsilon_{t-l} + \sum_{l=1}^{max q_i} \varepsilon_{t-l}^{(D)} \{b_l X_{t-0}\} + \boldsymbol{U}_t . \tag{5}$$

To preserve the orders of MA in our various bilinear expressions; the following definitions and conditions are applied:

i.) $\lambda_l = \{\lambda_{l.ij}\}_{n\times n}$ such that

$\lambda_{l.ij} \neq 0$ if $\exists \; \varepsilon_{jt-l}$

$\lambda_{l.ij} = 0$ if ε_{jt-l} does not exist.

ii.) $b_l = \{b_{k^*l.ij}\}_{n\times n}$

iii.) $b_{k^*l.ij} = 0 \quad \forall \quad i = j$.

Also for pure BIVMA, we have,

$b_l = \{b_{0l.ij}\}_{n\times n}$ such that

$b_{0l.ij} \neq 0$ if $\exists \;\; \varepsilon_{jt-l}$

$b_{0l.ij} = 0$ if ε_{jt-l} does not exist.

iv.) $\boldsymbol{\varepsilon_{t\text{-}l}} = \{\varepsilon_{it-l}\}_{n\times 1}$

$\boldsymbol{\varepsilon_{t\text{-}l}}$ is a white noise process

v.) $\varepsilon_{t-l}^{(D)} = Diag\{\varepsilon_{it-l}\}_{n\times n}$

vi.) $X_{t-0} = \{X_{it-0}\}_{n\times 1}$

vii.) $\boldsymbol{U_t} = \{u_{it}\}_{n\times 1}$; $\boldsymbol{u_t}$ is an independent white noise process

Hence we can write (5) as :

$$\{X_{it}\}_{n\times 1} = \sum_{l=1}^{\max q_i}\{\lambda_{l.ij}\}_{n\times n}\{\varepsilon_{it-l}\}_{n\times 1} + \sum_{l=1}^{\max q_i}\{\varepsilon_{it-l}\}_{n\times n}^{(D)}\left[\{b_{0l.ij}\}_{n\times n}\{X_{it-0}\}_{n\times n}\right] + \{u_{it}\}_{n\times 1} \qquad (6)$$

which can be simplified and expanded as follows :

Let $q^* = maxq_i$.

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ \cdot \\ \cdot \\ \cdot \\ X_{nt} \end{bmatrix} = \begin{bmatrix} \lambda_{1.11} & \lambda_{1.12} & \cdot & \cdot & \cdot & \lambda_{1.1n} \\ \lambda_{1.21} & \lambda_{1.22} & \cdot & \cdot & \cdot & \lambda_{1.2n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \lambda_{1.n1} & \lambda_{1.n2} & \cdot & \cdot & \cdot & \lambda_{1.nn} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{nt-1} \end{bmatrix} + \begin{bmatrix} \lambda_{2.11} & \lambda_{2.12} & \cdot & \cdot & \cdot & \lambda_{2.1n} \\ \lambda_{2.21} & \lambda_{2.22} & \cdot & \cdot & \cdot & \lambda_{2.2n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \lambda_{2.n1} & \lambda_{2.n2} & \cdot & \cdot & \cdot & \lambda_{2.nn} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-2} \\ \varepsilon_{2t-2} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{nt-2} \end{bmatrix}$$

$$+ \ldots + \begin{bmatrix} \lambda_{q^*.11} & \lambda_{q^*.12} & \cdot & \cdot & \cdot & \lambda_{q^*.1n} \\ \lambda_{q^*.21} & \lambda_{q^*.22} & \cdot & \cdot & \cdot & \lambda_{q^*.2n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \lambda_{q^*.n1} & \lambda_{q^*.n2} & \cdot & \cdot & \cdot & \lambda_{q^*.nn} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-q^*} \\ \varepsilon_{2t-q^*} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{nt-q^*} \end{bmatrix}$$

$$+ \begin{bmatrix} \varepsilon_{1t-1} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \varepsilon_{2t-1} & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \varepsilon_{nt-1} \end{bmatrix} \left\{ \begin{bmatrix} 0 & b_{01.12} & \cdot & \cdot & \cdot & b_{01.1n} \\ b_{01.21} & 0 & \cdot & \cdot & \cdot & b_{01.2n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ b_{01.n1} & b_{01.n2} & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{bmatrix} X_{1t-0} \\ X_{2t-0} \\ \cdot \\ \cdot \\ \cdot \\ X_{nt-0} \end{bmatrix} \right\}$$

$$+\begin{bmatrix} \varepsilon_{1t-2} & 0 & . & . & . & 0 \\ 0 & \varepsilon_{2t-2} & . & . & . & 0 \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & . & . & . & \varepsilon_{nt-2} \end{bmatrix} \left\{ \begin{bmatrix} 0 & b_{01.12} & . & . & . & b_{01.1n} \\ b_{01.21} & 0 & . & . & . & b_{01.2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ b_{01.n1} & b_{01.n2} & . & . & . & 0 \end{bmatrix} \begin{bmatrix} X_{1t-0} \\ X_{2t-0} \\ . \\ . \\ . \\ X_{nt-0} \end{bmatrix} \right\}$$

$$+\ldots+\begin{bmatrix} \varepsilon_{1t-q*} & 0 & . & . & . & 0 \\ 0 & \varepsilon_{1t-q*} & . & . & . & 0 \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & . & . & . & \varepsilon_{nt-q*} \end{bmatrix} \left\{ \begin{bmatrix} 0 & b_{0q^*.12} & . & . & . & b_{0q^*.1n} \\ b_{0q^*.21} & 0 & . & . & . & b_{0q^*.2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ b_{0q^*.n1} & b_{0q^*.n2} & . & . & . & 0 \end{bmatrix} . \begin{bmatrix} X_{1t-0} \\ X_{2t-0} \\ . \\ . \\ . \\ X_{nt-0} \end{bmatrix} \right\} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ . \\ . \\ . \\ u_{nt} \end{bmatrix}$$

3. Illustration

Let us consider the data in appendix A below as a special case where:

$$\boldsymbol{X}_t' = [X_{1t}, X_{2t}, X_{3t}]$$

The distribution of autocorrelation and partial autocorrelation functions of the series shows that:

X_{1t} follows a moving average process of order 1

X_{2t} follows a moving average process of order 1

X_{3t} follows a moving average process of order 2

Thus $Q = \{q_1, q_2, q_3\} = \{1,1,2\}$

So that $max.q = q^* = 2$

Then the BIVMA matrix representation is:

$$\boldsymbol{X}_t = \sum_{l=1}^{2} \lambda_l \boldsymbol{\varepsilon}_{t-l} + \sum_{l=1}^{2} \boldsymbol{\varepsilon}_{t-k}^{(D)} \{\boldsymbol{b}_l \boldsymbol{X}_{t-0}\} + \boldsymbol{U}_t$$

With the given conditions, we expand and simplify the matrices as follows:

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix} = \begin{bmatrix} \lambda_{1.11} & \lambda_{1.12} & \lambda_{1.13} \\ \lambda_{1.21} & \lambda_{1.22} & \lambda_{1.23} \\ \lambda_{1.31} & \lambda_{1.32} & \lambda_{1.33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \\ \varepsilon_{3t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \lambda_{2.13} \\ 0 & 0 & \lambda_{2.23} \\ 0 & 0 & \lambda_{2.33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-2} \\ \varepsilon_{2t-2} \\ \varepsilon_{3t-2} \end{bmatrix}$$

$$+\begin{bmatrix} \varepsilon_{1t-1} & 0 & 0 \\ 0 & \varepsilon_{2t-1} & 0 \\ 0 & 0 & \varepsilon_{3t-1} \end{bmatrix} \left\{ \begin{bmatrix} 0 & b_{01.12} & b_{01.13} \\ b_{01.21} & 0 & b_{01.23} \\ b_{01.31} & b_{01.32} & 0 \end{bmatrix} \begin{bmatrix} X_{1t-0} \\ X_{2t-0} \\ X_{3t-0} \end{bmatrix} \right\}$$

$$+\begin{bmatrix} \varepsilon_{1t-2} & 0 & 0 \\ 0 & \varepsilon_{2t-2} & 0 \\ 0 & 0 & \varepsilon_{3t-2} \end{bmatrix} \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ b_{02.31} & b_{02.32} & 0 \end{bmatrix} \begin{bmatrix} X_{1t-0} \\ X_{2t-0} \\ X_{3t-0} \end{bmatrix} \right\}$$

Which produces :

$$X_{1t} = \lambda_{1.11}\varepsilon_{1t-1} + \lambda_{1.12}\varepsilon_{2t-1} + \lambda_{1.13}\varepsilon_{3t-1} + \lambda_{2.13}\varepsilon_{3t-2}$$
$$+ b_{01.12}\varepsilon_{1t-1}X_{2t-0} + b_{01.13}\varepsilon_{1t-1}X_{3t-0}$$

$$X_{2t} = \lambda_{1.21}\varepsilon_{1t-1} + \lambda_{1.22}\varepsilon_{2t-1} + \lambda_{1.23}\varepsilon_{3t-1} + \lambda_{2.23}\varepsilon_{3t-2}$$

$$+b_{01.21}\varepsilon_{2t-1}X_{1t-0}+b_{01.23}\varepsilon_{2t-1}X_{3t-0}$$

$$X_{3t}=\lambda_{1.31}\varepsilon_{1t-1}+\lambda_{1.32}\varepsilon_{2t-1}+\lambda_{1.33}\varepsilon_{3t-1}+\gamma_{2.31}X_{1t-2}+\lambda_{2.33}\varepsilon_{3t-2}$$
$$+b_{01.31}\varepsilon_{3t-1}X_{1t-0}+b_{01.32}\varepsilon_{3t-1}X_{2t-0}+b_{02.31}\varepsilon_{3t-2}X_{1t-0}$$
$$+b_{02.32}\varepsilon_{3t-2}X_{2t-0}.$$

4. Results

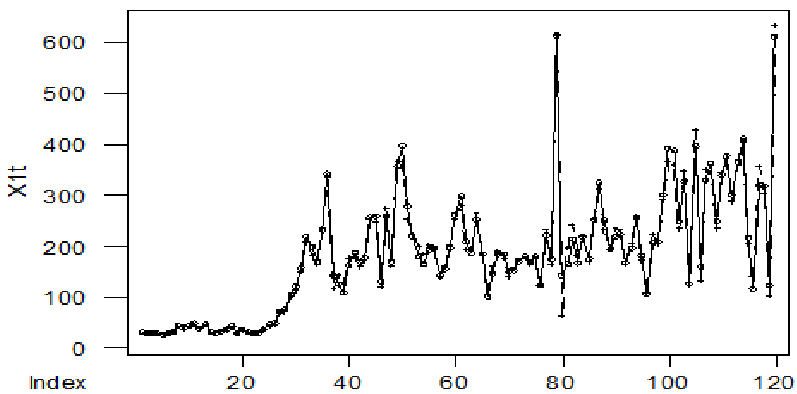
4.1. Estimates for the BIVMA Models

As could be seen above, the orders of pure linear MA is maintained in both linear and non-linear parts of our derived bilinear vector models. These models are linear in states X_{it-k} and ε_{it-l} separately but non-linear jointly as the name 'bilinear' implies. The regression estimates obtained provide the following models for the three vector components (X_{1t}, X_{1t}, X_{1t}).

$$X_{1t}=-0.247\varepsilon_{1t-1}+0.774\varepsilon_{2t-1}+0.252\varepsilon_{3t-1}+0.021\varepsilon_{3t-2}-0.036\varepsilon_{1t-1}X_{2t-0}+0.004\varepsilon_{1t-1}X_{3t-0}$$
$$X_{2t}=0.175\varepsilon_{1t-1}+0.054\varepsilon_{2t-1}-0.361\varepsilon_{3t-1}-0.421\varepsilon_{3t-2}+0.013\varepsilon_{2t-1}X_{1t-0}+0.074\varepsilon_{2t-1}X_{3t-0}$$
$$X_{3t}=0.589\varepsilon_{1t-1}+0.025\varepsilon_{2t-1}+0.432\varepsilon_{3t-1}-0.211\varepsilon_{3t-2}+0.012\varepsilon_{3t-1}X_{1t-0}-0.002\varepsilon_{3t-1}X_{2t-0}$$
$$-0.031\varepsilon_{3t-2}X_{1t-0}+0.012\varepsilon_{3t-2}X_{2t-0}$$

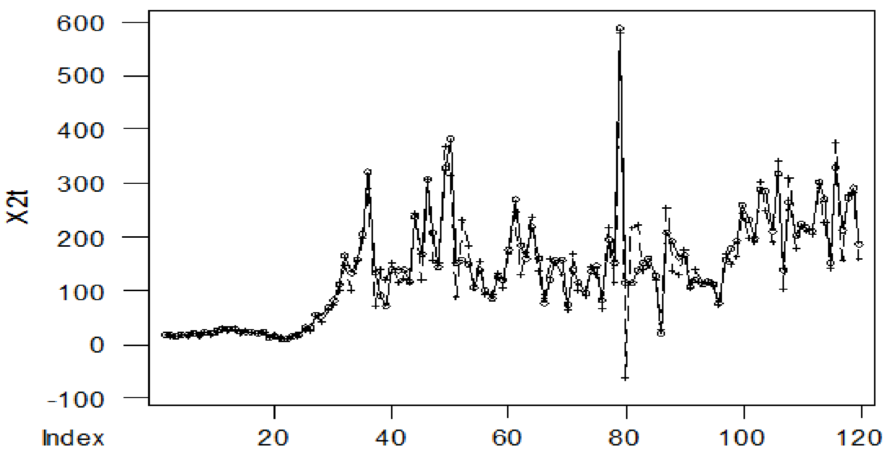
The actual and estimated values for each component vector are plotted below:

Figure-1. Plots of actual and estimates of X_{1t} .



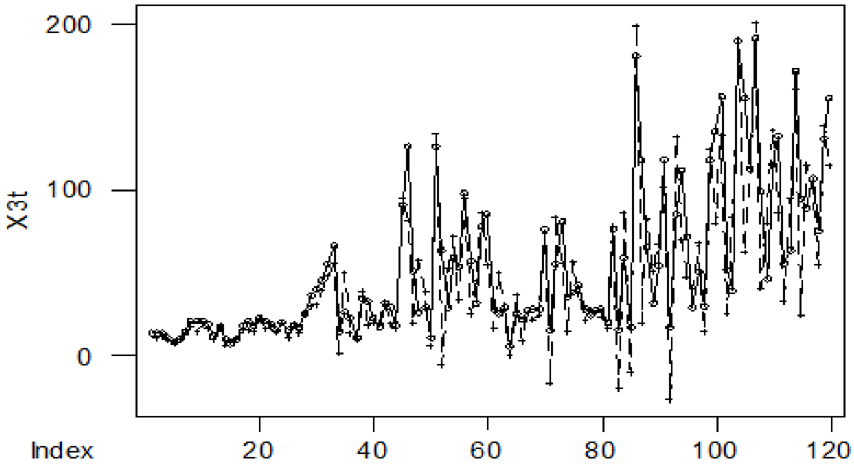
Key: o (actual plot) and + (estimates plot)

Figure-2. Plots of actual and estimates of X_{2t}



Key: o (actual plot) and + (estimates plot)

Figure-3. Plots of actual and estimates of X_{3t} .



Key: o (actual plot) and + (estimate plot)

Each overlay graph above contains two plots. One for the actual values in Appendix A, and the other for the estimated values provided by the above models. As seen in the overlay plots for each component, the reulting models provide good estimates for the data. This is why for each overlay plot, the actual and estimate plots rise and fall together. In other words, there is no significant difference between the actual and the estimated values.

5. Conclusion

There is no gainsaying the fact that some series assume not only linear part, but both linear and non linear component as expressed in our bilinear concept. This is so because of the random nature of observations assume by certain processes. The 'orders' of the proposed bilinear concept are founded on the basis of the 'orders' of the pure linear MA processes. These orders are preserved in both the linear and non linear parts of the bilinear expressions. The models are linear in states X_{it-k} and ε_{it-l} separately but non-linear jointly, as the name 'bilinear' implies. Since there is strong correlation between the actual and the estimated values as evidenced in the different plots, we can unreservedly state that this concept has offered another exciting potential in the modeling of non stationary series.

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Appendix-A. ***Three sources of internal generated revenue*** (X_{1t}, X_{2t}, X_{3t})

S/N	X_{1t}	X_{2t}	X_{3t}	S/N	X_{1t}	X_{2t}	X_{3t}	S/N	X_{1t}	X_{2t}	X_{3t}
1.	30.87	17.01	13.86	41.	186.82	139.41	47.41	81.	164.91	145.21	19.70
2.	31.26	17.31	13.95	42.	169.89	137.98	31.91	82.	215.65	139.52	76.13
3.	29.35	16.10	13.25	43.	176.91	147.73	29.18	83.	167.03	151.33	15.70
4.	30.05	18.68	11.37	44.	256.21	238.38	17.83	84.	219.36	160.19	59.17
5.	25.96	17.46	8.50	45.	260.00	169.12	90.88	85.	176.06	129.01	47.05
6.	30.31	20.55	9.76	46.	434.75	308.15	126.60	86.	251.51	70.66	180.85
7.	31.54	17.04	14.50	47.	258.23	207.11	51.12	87.	325.11	207.01	118.10
8.	45.20	23.85	21.35	48.	169.79	143.58	26.21	88.	257.86	192.54	65.32
9.	41.07	20.57	20.50	49.	358.15	328.97	29.18	89.	195.03	162.92	32.11
10.	45.46	24.86	20.60	50.	397.26	383.01	14.25	90.	220.52	165.52	55.00
11.	48.17	29.65	19.03	51.	279.01	152.71	126.30	91.	225.77	107.42	118.35
12.	40.17	28.67	11.50	52.	220.75	157.39	63.36	92.	167.89	120.52	47.37
13.	45.79	29.76	16.03	53.	178.99	149.68	29.31	93.	198.30	112.85	85.45
14.	32.76	22.89	9.87	54.	164.50	105.69	58.81	94.	257.08	115.70	141.38
15.	30.77	23.25	7.52	55.	192.33	138.53	53.80	95.	183.01	110.86	72.15
16.	32.07	21.97	10.10	56.	198.54	100.29	98.25	96.	106.12	76.75	29.37
17.	37.83	19.64	18.19	57.	143.54	86.21	57.33	97.	207.17	156.60	50.57
18.	43.85	22.60	21.25	58.	155.90	124.20	31.70	98.	209.36	179.21	30.15
19.	30.77	12.60	18.17	59.	198.51	120.68	77.83	99.	309.66	191.79	117.87
20.	37.06	14.53	22.53	60.	260.93	175.79	85.14	100.	391.27	258.99	135.28
21.	31.96	10.61	21.35	61.	299.44	270.84	28.60	101.	388.93	232.97	155.96
22.	29.00	10.30	18.70	62.	211.02	185.08	25.94	102.	250.32	198.14	52.18
23.	30.36	15.04	15.32	63.	188.06	158.68	29.38	103.	328.70	289.35	39.15
24.	36.63	16.90	19.73	64.	252.71	247.66	5.05	104.	475.41	285.73	189.68
25.	45.77.	30.45	15.32	65.	185.72	160.47	25.25	105.	396.98	241.31	155.67
26.	50.00	31.50	18.50	66.	101.75	77.25	24.50	106.	461.13	317.68	143.45
27.	72.50	55.20	17.30	67.	145.56	118.56	27.00	107.	331.10	138.69	192.41
28.	77.18	51.73	25.45	68.	184.41	156.59	27.83	108.	363.17	263.85	99.32
29.	104.08	67.58	36.50	69.	184.41	156.59	27.82	109.	248.5	202.20	46.30
30.	120.70	80.90	39.80	70.	149.33	73.20	76.13	110.	339.98	224.38	115.60
31.	157.31	111.47	45.87	71.	153.39	138.19	15.70	111.	377.75	245.45	132.30
32.	220.45	164.79	55.66	72.	171.38	115.46	55.92	112.	300.42	244.67	55.75
33.	198.76	132.35	66.41	73.	180.48	96.33	84.15	113.	366.28	303.08	63.20
34.	171.03	156.70	14..33	74.	170.13	135.03	35.10	114.	441.37	270.02	171.35
35.	231.97	205.76	26.21	75.	184.16	145.96	38.20	115.	246.69	151.69	95.00
36.	343.58	321.12	22.46	76.	124.36	81.71	42.65	116.	416.48	327.73	88.75
37.	143.73	132.88	10.85	77.	222.96	194.45	28.51	117.	320.97	213.59	107.35
38.	126.16	91.21	34.95	78.	175.75	151.25	24.50	118.	347.35	272.14	75.21
39.	107.93	74.75	33.18	79.	614.93	587.93	27.00	119.	422.91	291.66	131.25
40.	162.04	139.41	22.63	80.	142.32	114.50	27.82	120.	641.23	485.56	155.67