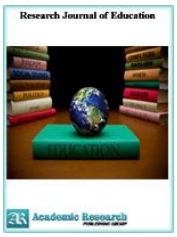

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Comparing Elementary Mathematics Textbooks' Introduction of Symbols for Algebraic Unknowns in Taiwan, Singapore, and Finland

Der-Ching Yang* Graduate Institute of Mathematics and Science Education, National Chiayi University 85, Wen Lung, Ming-Hsiung Chiayi 621, Taiwan

Tseng-Cheng Chiang Graduate Institute of Mathematics and Science Education, National Chiayi University 85, Wen Lung, Ming-Hsiung Chiayi 621, Taiwan

Abstract: This study applied a content analysis method to compare how the algebraic topic of using symbols for unknown quantities is presented in elementary school mathematics textbooks from Taiwan (Nani), Singapore (My Pals Are Here!), and Finland (Laskutaito). Specifically, differences in question types (purely mathematical, verbal, visual, or combined representation), contextual versus noncontextual presentation, and pedagogical content design were compared. The findings showed that (1) fewer visual representations are found in Nani compared with the other textbooks; (2) Taiwan uses more contextual problems than the other countries; and (3) the content design in Taiwan focuses on applying the equivalent axiom to solve for unknown quantities, whereas Singapore and Finland use line segments or divide concepts in geometry graphs. In addition, the Singaporean textbooks teach algebraic simplification, providing this topic earlier than the other countries and enabling students to form connections with junior high school learning. Other implications of this study are discussed, and suggestions for future research are provided.

Keywords: Algebra; Elementary school mathematics textbook; Symbols for unknowns; Taiwan; Finland; Singapore.

1. Introduction

Mathematics textbooks provide students different learning opportunities and directly affect students' learning outcomes (Reys *et al.*, 2010; Yang and Lin, 2015). Therefore, textbooks play an important role in students' mathematics learning (Fan *et al.*, 2013; Huang and Cai, 2011). Many international mathematics educators believe that algebra should be considered basic knowledge for advanced mathematics development (Huang and Cai, 2011; National Council of Teachers of Mathematics, 2000; Smith and Philips, 2000); as Herscovics and Linchevski (1994) pointed out, students who do not build the basic concepts of algebra will hindered in their future study on advanced mathematics. Studies have also showed that students continue to have difficulties in algebra learning, especially regarding the concept of solving equations with unknown variables (Kieran, 2007; Loveless, 2008; National Mathematics Advisory Panel, 2008). Therefore, it is imperative to explore the content arrangement and problem presentation of the unknown variables in algebra textbooks.

The use of symbols such as *a*, *b*, *x*, and *t* to represent unknown quantities is not only a critical element for learning algebra, but also plays a key role in algebraic problem solving (Celik and Gunes, 2013). Related studies have also indicated that the use of such symbols, and how these symbols are explained, are foundational for advanced mathematics learning (Arcavi and Schoenfeld, 1988; Dominguez, 2001; MacGregor and Stacey, 1997). Therefore, understanding the use of symbols for unknowns is crucial.

The design of textbooks affects students' learning opportunities, and researchers believe that they can learn about the advantages and disadvantages of the mathematics textbooks in their own countries by examining other countries' textbooks (Fan *et al.*, 2013; Stigler and Hiebert, 2004). Therefore, this study compared the teaching of symbols for unknowns in the elementary mathematics textbook series *Nani* in Taiwan, *My Pals are Here!* in Singapore, and *Laskutaito* in Finland. The specific research questions were as follows:

1. How is the topic of symbols for unknowns represented (e.g., through symbolic, verbal, visual, or combined forms) in these three sets of elementary mathematics textbooks?
2. What types of problems are presented to teach symbols for unknowns (specifically, problems with and without context) in these three sets of elementary mathematics textbooks?
3. What are the differences in pedagogical design for the topic of symbols for unknowns in these three sets of elementary mathematics textbooks?

*Corresponding Author

2. Theoretical Framework

2.1. Symbols for Unknowns and Variables in Algebra

Within mathematics, algebra is considered a characteristic language (Usiskin, 1997) that enables mathematical problems to be translated into generalized subclasses and formulas (Baki, 2006; Driscoll, 1999; Tall *et al.*, 2000; Usiskin, 1999). It also includes the relationship between numbers and the object of study, such as a group, ring, vector space, or operations and deductions. Algebra contains many connotations, such as unknowns, relationships, equivalence axioms, and regularity, that do not simply focus on computation, but also on understanding relations (Carpenter *et al.*, 2005; Pimm, 1995; Usiskin, 1997). The use of symbols for unknown quantities is a crucial element in algebra, which is central for algebraic problem solving and algebraic concept learning (Celik and Gunes, 2013). In addition, several studies have also indicated that the use and interpretation of symbols representing unknowns is a foundational ability for all higher mathematics subjects (Arcavi and Schoenfeld, 1988; Dominguez, 2001; MacGregor and Stacey, 1997). Thus, the use of symbols for unknowns is a major focus of algebra, and is indispensable for learning advanced algebra content.

Several studies have also indicated that students face difficulties in learning algebra, including the use and interpretation of symbols representing unknowns (Arcavi and Schoenfeld, 1988; Dominguez, 2001; Kinzel, 2000; Li., 2011; Luo, 2004; Philipp, 1999; Rosnick, 1999; Sfard and Linchevski, 1994; Stacey and MacGregor, 1997; Tall and Thomas, 1991). MacGregor and Stacey (1997) suggested that the root of this problem is student confusion about the meanings of different symbols that are representative of each other.

The discussion thus far shows that the use of symbols for unknowns and the abstract nature of algebra make it difficult for students to translate mathematical problems into appropriate symbolic expressions. Therefore, students struggle to understand subsequent algebraic topics, such as variables and equations (MacGregor and Stacey, 1997; Steinberg *et al.*, 1990; Wagner, 1981). This supports the findings of Seeley and Schielack (2007) that understanding and translating variables and algebraic expressions is challenging for many students. All of this underscores the importance of the pedagogical design and presentation of symbols representing unknowns in algebraic learning.

2.2. Classification of Unknown Variables

According to test results, Küchemann (1998) divided student performance into four levels from easy to difficult, with six different interpretations and treatments for the use of unknown symbols. Level 1 included "letter evaluated" and "letter not used," Level 2 included "letter used as an object," Level 3 included "letter used as a specific unknown," and Level 4 included "letter used as a generalized number" and "letter used as a variable." Linsell *et al.* (2013) also discussed symbols for unknowns from three perspectives: generalized numbers, variables, and specific unknowns.

Based on these studies, the present study categorized the use of symbols for unknowns into generalized numbers, specific unknowns, and variables. "Generalized numbers" implies the use of symbols to represent numbers (e.g., a = 3 and b = 7, then a + b = b + a). "Specific unknowns" refers to the symbols used to represent a value that is found by solving a mathematical problem (e.g., if 2n + 1 = 9, n =?). Finally, "Variables" are used when posing or translating mathematical problems; for example, in Y = X + 5, *Y* and *X* are two distinct variables which remain in a particular quantitative relationship with each other although their specific values may change Küchemann (1998) Linsell *et al.* (2013).

3. Method

3.1. Selected Textbook Series

Nani: This is one of the most popular textbook series in Taiwan and has a high proportion of the market share for elementary mathematics textbooks. *Nani* includes 12 student textbooks for grades 1–6, and the topic of symbols for unknowns is covered over two units. The present study reviewed these units specifically.

Laskutaito: Published by WSOY, *Laskutaito* has the highest proportion (about 60%–70%) of the market share for textbooks used in Finnish elementary schools (Saarelainen, 2007). *Laskutaito* has 12 textbooks for grades 1–6, including one unit on symbols for unknowns, which the present study examined.

My Pals are Here!: The textbook series *My Pals are Here! Maths* (MPHM) has the highest market share in Singapore (Singapore Math.com. Inc., 2005), with 12 textbooks for grades 1–6. The MPHM includes one unit on unknown symbols, which was reviewed in the present study.

3.2. Analytical Framework

The current study examined the questions presented in student textbooks used by elementary school students. Based on earlier studies (Yang and Wu, 2010; Zhu and Fan, 2006), the analytical framework included three dimensions: representational forms, contextual features, and presentation styles (Table 1).

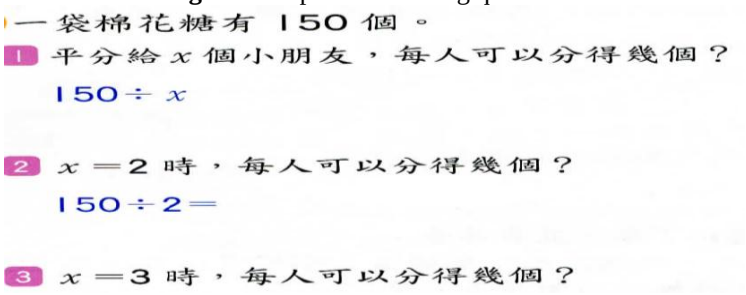
Table-1. Framework for analyzing problems in algebra textbooks.

Dimension	Category
Representational forms	Purely mathematical form
	Verbal form
	Visual form
	Combined form
Contextual features	Contextual questions
	Noncontextual questions
Presentation styles	Chapters
	Subtitles Uniqueness

3.3. Coding and Analysis of Data

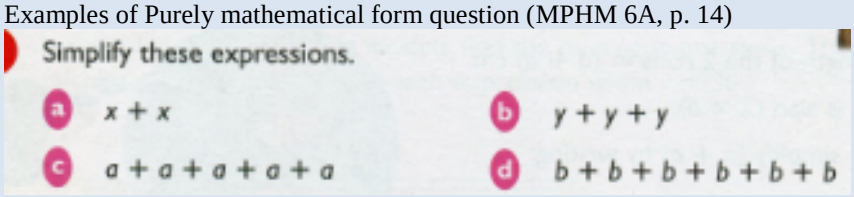
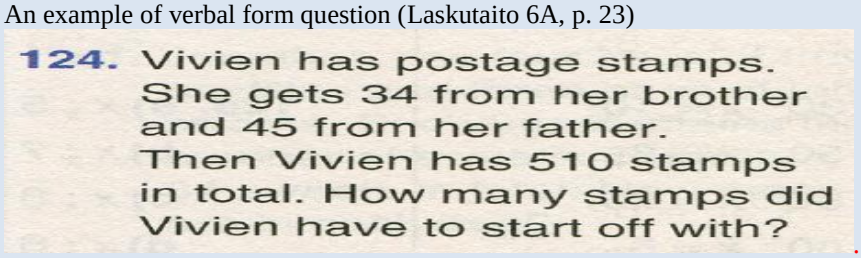
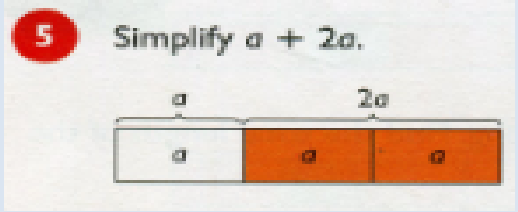
The work examples and exercises found in the student textbooks were counted and the total number of questions was noted. For example, three questions are posed in the Figure 1, indicating the three questions that were counted. Examples of the representational forms defined in this study are given in Table 2.

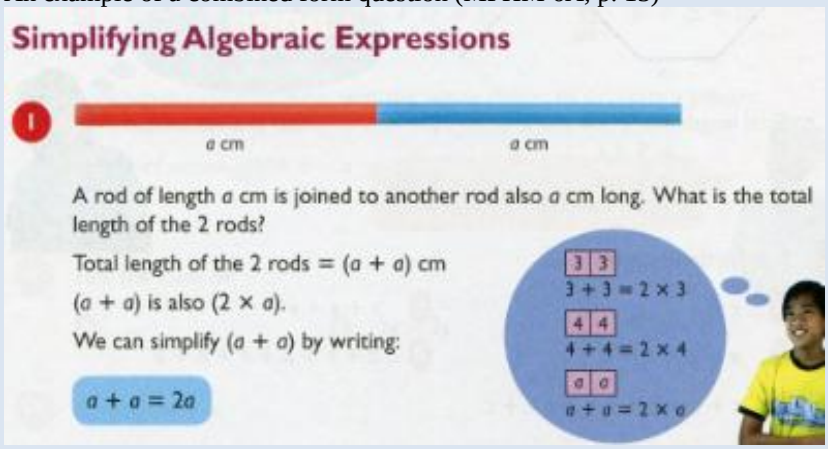
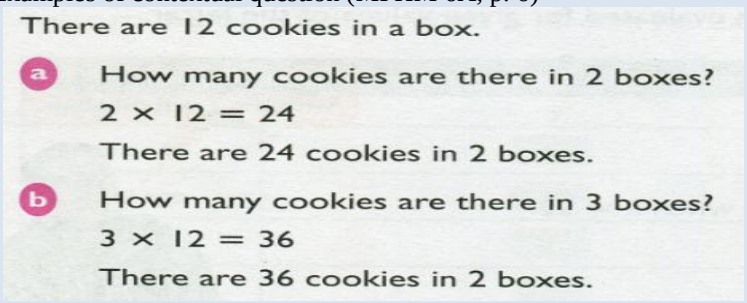
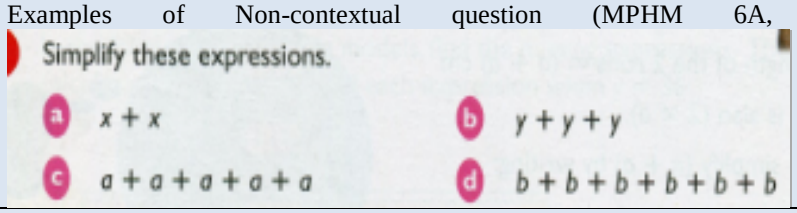
Fig-1. Example for counting questions.



Note: Reproduced from fifth grade *Nani* textbook (*(Nan-I Mathematics Textbook, 2009)*

Table-2. Sample textbook problems and categories.

Example	Category
1. Examples of Purely mathematical form question (MPHM 6A, p. 14) 	Purely mathematical form
2. An example of verbal form question (Laskutaito 6A, p. 23) 	Verbal form
3. An example of a visual form question (MPHM 6A, p. 14) 	Visual form

4. An example of a combined form question (MPHM 6A, p. 13)  	Combined form Contextual question
5. Examples of contextual question (MPHM 6A, p. 6)  	Noncontextual question
6. Examples of Non-contextual question (MPHM 6A, p. 14).  	
7. Chapters were examined in each textbook series. 8. Subtitles were examined in each textbook series. 9. Unique questions presented in each textbook series were examined.	Chapters Subtitles Uniqueness

3.4. Reliability

Three mathematics teachers reviewed the coding reliability by independently sorting the questions in the textbooks into "involving symbols for unknowns" or "not involving symbols for unknowns" categories. This study applied the measures recommended by Wang (1996) and obtained a reliability score of 0.94.

4. Results

Table 3 details the results of the representational forms of questions in the three series of textbooks. More than half of questions in the Finnish textbooks used purely mathematical forms, compared to approximately one-quarter of the questions in the Singaporean and Taiwanese textbooks and approximately one-third of the questions in the Singaporean and Taiwanese textbooks were in verbal form. More than one-quarter of the questions in the Singaporean textbook were in visual form, which is considerably higher than the amount in the Finnish (9.4%) and Taiwanese (2.4%) textbooks. In addition, approximately two-fifths of the Taiwanese questions were in combined form, which is a considerably higher proportion than that found in the Singaporean and Finnish textbooks.

Table-3. Representational types of questions in the three countries' textbooks.

	Purely math form	Verbal form	Visual form	Combined form	Total
Finland	48 (56.5%)	21 (24.7%)	8 (9.4%)	8 (9.4%)	85
Singapore	56 (24.2%)	74 (31.9%)	62 (26.7%)	40 (17.2%)	232
Taiwan	21 (25.3%)	27 (32.5%)	2 (2.4%)	33 (39.8%)	83

Table 4 provides statistics about the contextual versus noncontextual questions among the three countries. The results showed that approximately two-thirds of the questions in the Taiwanese textbooks, and approximately half of

those in the Singaporean textbooks, are contextual. By contrast, only 5.9% of the questions in Finnish textbooks are contextual.

Table-4. Contextual versus noncontextual questions in the three countries' textbooks.

	Contextual question	Noncontextual question	Total
Finland	5 (5.9%)	80 (94.1%)	85
Singapore	97 (41.8%)	135 (58.2%)	232
Taiwan	52 (62.7%)	31 (37.3%)	83

Table 5 reveals the distribution of chapters and explanatory text related to symbols for unknowns presented in the three series of textbooks. Notably, different terms were used by each textbook series. For example, the Finnish textbooks had one chapter called "Revision & Practice," which contained two explanatory sections addressing symbols for unknowns, whereas the Singaporean textbooks had one chapter entitled "Algebra," with three explanatory sections related to the topic. The Taiwanese textbooks included two relevant chapters, "Using Equations" and "Equivalent axiom," with six explanatory sections related to symbols for unknowns. Moreover, the Taiwanese textbooks begin teaching this concept at the grade 5 level, whereas the Finnish and Singaporean textbooks did not include the topic until the grade 6 level.

Table-5. Distribution of relevant chapters and explanatory sections presented in the three textbooks.

	Chapter	explanatory sections	generalized numbers	specific unknowns	Variables	Total (%)
Finland	Revision & practice (6A)	Addition and subtraction equations	0	42	0	42 (49.4%)
		Multiplication and division equations	0	43	0	43 (50.6%)
	Total (%)		0 (0%)	85(100%)	0 (0%)	
Singapore	Algebra (6A)	Using letters as numbers	82	56	0	138 (59.5%)
		Simplifying algebraic expressions	58	4	0	62 (26.7%)
		Word problems	19	13	0	32 (14.8%)
	Total (%)		159(68.5%)	73 (31.5%)	0 (0%)	
Taiwan	Using equation (5B)	Using letters as numbers	9	6	0	15 (18.1%)
		Addition and subtractions equations	0	10	0	10 (12.0%)
		Multiplication and division equations	0	10	0	10 (12.0%)
		Equation	6	4	0	10 (12.0%)
	Equivalent axiom (6A)	Equivalent axiom	8	16	0	24 (28.9%)
		The application of Equivalent axiom	0	14	0	14 (17.0%)
	Total (%)		23(27.7%)	60(72.3%)	0 (0%)	

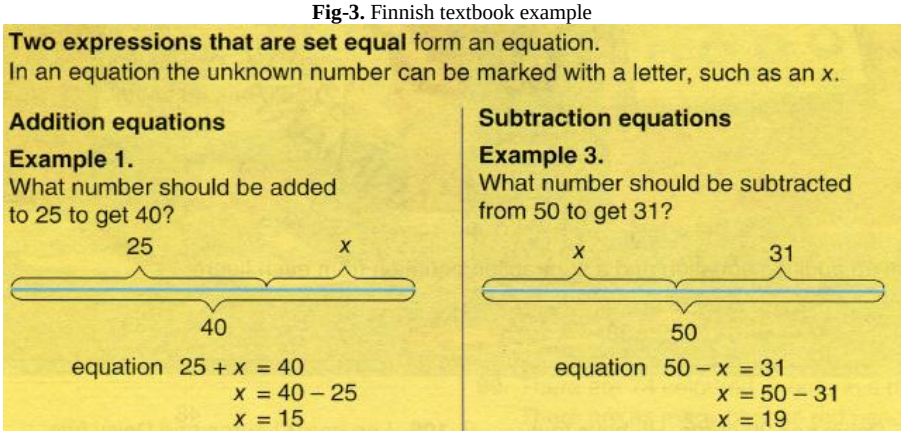
Moreover, the data showed that all of the relevant questions in the Finnish textbooks, and approximately three-quarters of the relevant questions in the Taiwanese textbooks, focus on specific unknowns. This indicates that these textbooks focus on teaching children how to find unknown values within mathematical problems. By contrast, over two-thirds of the relevant questions in the Singaporean textbooks involved generalized numbers, meaning that these textbooks emphasize teaching basic knowledge related to symbols for unknowns. None of the three textbook series addressed variables.

A key difference related to symbols for unknowns was noted among the three textbook series, namely that while Taiwanese textbooks utilized Equivalent axiom to teach students to solve problems (Fig. 1), the Finnish and Singaporean textbooks used part–whole models (i.e., the line-segment method) (Figs. 2 and 3).

Fig-2. Taiwanese textbook example

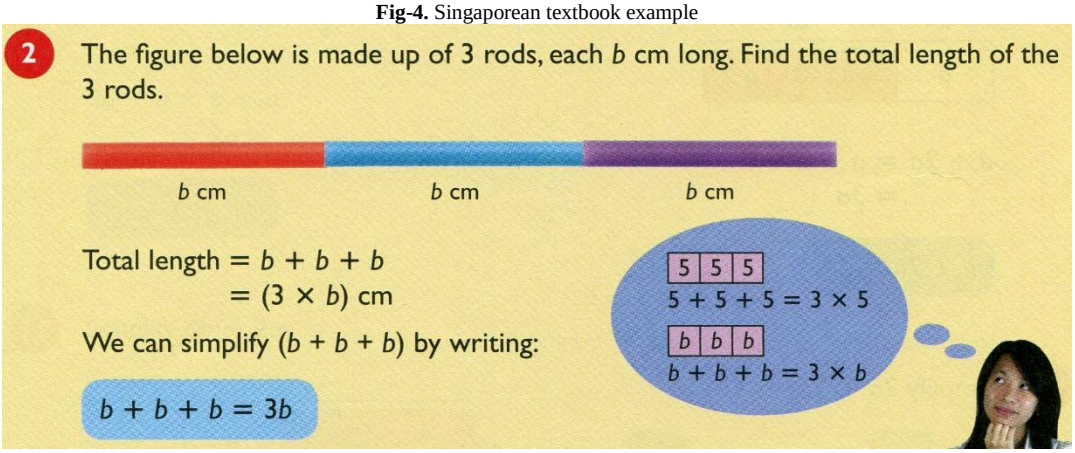
Note: Reproduced from sixth grade *Nani* textbook *(Nan-I Mathematics Textbook, 2009)*

Fig-3. Finnish textbook example



Note: Reproduced from *Laskutaito 6A* (Saarelainen, 2007).

Fig-4. Singaporean textbook example



Note: Reproduced from *MPHM 6A* (Fong *et al.*, 2007).

5. Discussion and Conclusion

This study found that the representational types among the three textbook series vary considerably. Whereas the Finnish textbooks focus on questions in symbolic form, the Singaporean textbooks place more emphasis on verbal and visual forms and the Taiwanese textbooks highlight verbal and combined forms. This finding supports earlier studies that indicated that different countries may take diverse approaches in textbooks perhaps due to cultural differences (Fan *et al.*, 2013; Zhu and Fan, 2006). Moreover, the Singaporean textbooks include many more questions related to unknown symbols, giving Singaporean children more opportunity to practice and become skilled in working with symbols for unknowns. This may be one reason why Singaporean students perform very well on international mathematics assessments (e.g., PISA and TIMSS).

The Taiwanese textbooks include more contextual problems than the other textbooks. This corroborates an earlier study that textbooks in Taiwan emphasize the integration of contextual questions in mathematics activities (Yang and Wu, 2010). According to numerous other studies, the ability of students to apply mathematics knowledge and skills in various contexts in daily life should be a key goal in mathematics education (Graumann, 2011; Muller and Burkhardt, 2007; Niss *et al.*, 2007; Wijaya *et al.*, 2015; Yang and Wu, 2010). Nevertheless, only about 6% of the questions in the Finnish textbooks on the topic of symbols for unknowns are contextual; therefore, the writers of these textbooks should consider designing and integrating more contextual questions.

Overall, considerable variation in pedagogical design was found for teaching this topic. The total number of questions, chapter terminology, and explanations related to symbols for unknown are divergent. Moreover, the age at which these concepts are first presented also varies, with Taiwan introducing the topic to students a year earlier than Finland and Singapore. In addition, Taiwan relies on teaching children through the Equivalent axiom, whereas the Singaporean and Finnish textbooks do not cover this concept. Instead, Singapore and Finland use part–whole models

to teach children to solve problems. The Singaporean textbooks also teach students algebraic simplification, introducing this topic earlier than the other countries and facilitating students' ability to later connect symbols for unknowns with junior high school mathematics. This confirms that different countries often teach the same mathematics topics using diverse approaches perhaps due to cultural differences (Fan *et al.*, 2013; Stigler and Hiebert, 2004; Yang and Wu, 2010; Zhu and Fan, 2006).

Although the present study outlined some of the distinct approaches taken by different countries in their mathematics textbooks, further research is required to understand how these differences affect student learning. Nevertheless, we hope that this study provides mathematics educators with some insight into how the topic of symbols for unknowns is taught.

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