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The Study of the Wiener Processes Base on Haar Wavelet

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Abstract: The stochastic system is very importment in many aspacts. Wiener processes is a sort of importment stochastic processes. Wiener square processes is a class of useful stochastic processes in practies,its study is very value.In this paper,we study Wiener square processes using haar wavelet and wavelet transform.we study its some properties and wavelet expansion. Index Wiener Integral processes is a class of useful stochastic processes in practies,its study is very value.In this paper,we study it using haar wavelet and wavelet transform on [0,t].we study its some properties and wavelet expansion.

Keywords: Wiener square processes; Wavelet; Haar; Expansion; Index Wiener Integral processes.

1. Introduction

Wiener processes are a sort of importment stochastic processes. Wiener square processes is a class of useful stochastic processes in practies, its study is very value.

We will take wavelet and use them in a series expansion of signal or function. Wavelet has its energy concentrated in time to give a tool for the analysis of transient, nonstationary, or time-varying phenomena. It still has the oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. We take wavelet and use them in a series expansion of signals or functions much the same way a Fourier series the wave or sinusoid to represent a signal or function. In order to use the idea of multiresolution, we will start by defining the scaling function and then define the wavelet in terms of it.

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing. In fields ranging from Extragalactic Astronomy to Molecular Spectroscopy to Medical Imaging to computer vision, One must recover a signal, curve, image, spectrum, or density from incomplete, indirect, and noisy data .Wavelets have contributed to this already intensely developed and rapidly advancing field .

Wavelet analysis consists of a versatile collection of tools for the analysis and manipulation of signals such as sound and images as well as more general digital data sets ,such as speech, electrocardiograms ,images .Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing .With hindsight the wavelet transform can be viewed as diverse as mathematics ,physics and electrical engineering .The basic idea is always to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks themselves come in different sizes, and are suitable for describing features with a resolution commensurate with their size .

There are two important aspects to wavelets, which we shall call "mathematical" and "algorithmical" .Numerical algorithms using wavelet bases are similar to other transform methods in that vectors and operators are expanded into a basis and the computations take place in the new system of coordinates .As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain, wavelet based algorithms exhibit a number of new and important properties .Recently some persons have studied wavelet problems of stochastic process or stochastic system ([1-18]).

2. Basic Definitions

Definition 1

Let $X(t) = W^2(t), t > 0$ (1)

W(t) is Wiener processes, we call X(t) is Wiener square processes.

We have

$$E(W^2(t) = D(W(t)) + E(W(t))^2 = \sigma^2 t$$

Let s<t, we have

$$
\begin{aligned}
&R(s,t)=EX(t)X(s)=E(W^2(s)W^2(t))\\
&=E(W^2(s)(W(t)-W(s)+W(s))^2\\
&=E(W^2(s)(W(t)-W(s))^2+E(W^2(s))\\
&=\sigma^2 s\sigma^2(t-s)+3\sigma^4 s^2\\
&=\sigma^4 s(t-s)+3\sigma^4 s^2
\end{aligned}
\qquad (2)
$$

Definition 2

Let {x(t) , t∈R} is a stochastic processes on probability space (Ω, g, P) ,we call

$$\mathrm{W(s, x)}=\frac{1}{s}\int_R x(t)(\frac{x-t}{s})dt \qquad (3)$$

is wavelet transform of x(t) . where,ψ is mather wavelet([11]).

Then, we have

$$w(s,x+\tau)=\frac{1}{s}\int_R x(t)\psi(\frac{x+\tau-t}{s})dt \qquad (4)$$

Definition 3

Let mather waveletψ(x) is function:

$$\psi(\mathrm{x})=\begin{cases}1,0\le x<\dfrac{1}{2}\\ -1,\dfrac{1}{2}\le x<1\\ 0,other\end{cases} \qquad (5)$$

we call ψ(x) is the Haar wavelet.

Then, we have

$$\psi(\frac{x-t}{s})=\begin{cases}1, x-\dfrac{s}{2}\le t<x\\ -1, x-s\le t<x-\dfrac{s}{2}\end{cases} \qquad (6)$$

$$\psi(\frac{x+\tau-t}{s})=\begin{cases}1, x+\tau-\dfrac{s}{2}\le t<x+\tau\\ -1, x+\tau-s\le t<x+\tau-\dfrac{s}{2}\end{cases} \qquad (7)$$

3. Some Results about Density Degree

We have

$$
\begin{aligned}
&R(\tau)=E[w(s,y)w(s,y+\tau)]\\
&=E[\frac{1}{s}\int_R x(t)\psi(\frac{y-t}{s})dt][\frac{1}{s}\int_R x(t_1)\psi(\frac{y+\tau-t_1}{s})dt_1]=\frac{1}{s^2}E[\iint_{R^2}x(t)x(t_1)\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]\\
&=\frac{1}{s^2}\iint E[x(t)x(t_1)]\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1\\
&=\frac{1}{s^2}\iint_{R^2}[\sigma^4 t_1(t-t_1)+3\sigma^4 t_1^{\,2}]\psi(\frac{y-t}{s})\\
&\psi(\frac{y+\tau-t_1}{s})dtdt_1
\end{aligned}
$$

$$=\frac{1}{s^2}[\iint_{R^2}\sigma^4 t_1(t-t_1)\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1$$

$$+\iint_{R^2}3\sigma^4 t_1^{\ 2}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$

$$=\frac{1}{s^2}[\iint_{R^2}\sigma^4 t_1 t\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1$$

$$+\iint_{R^2}2\sigma^4 t_1^{\ 2}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$

$$=I_1+I_2$$

Where

$$I_1=\frac{1}{s^2}\iint_{R^2}\sigma^4 t_1 t\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1,$$

$$I_2=\frac{1}{s^2}\iint_{R^2}\ 2\sigma^4 t_1^{\ 2}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1$$

Then,we have

$$I_1=\frac{1}{s^2}\sigma^4[\int_{y-s/2}^{y}tdt\int_{y+\tau-s/2}^{y+\tau}t_1dt_1-\int_{y-s/2}^{y}tdt\int_{y+\tau-s}^{y+\tau-s/2}t_1dt_1$$

The same time, we have

$$-\int_{y-s}^{y-s/2}tdt\int_{y+\tau-s/2}^{y-s/2}t_1dt_1+\int_{y-s}^{y-s/2}tdt\int_{y+\tau-s}^{y+\tau-s/2}t_1dt_1]$$

$$I_2=\frac{1}{s^2}2\sigma^2[\iint_{R^2}\ t_1^{\ 2}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$

$$=\frac{2}{s^2}\sigma^2[\int_{y-s/2}^{y}dt\int_{y+\tau-s/2}^{y+\tau}t_1^{\ 2}dt_1-\int_{y-s/2}^{y}dt\int_{y+\tau-s}^{y+\tau-s/2}t_1^{\ 2}dt_1$$

We let $\sigma=1$, through compute on above

$$-\int_{y-s}^{y-s/2}dt\int_{y+\tau-s/2}^{y-s/2}t_1^{\ 2}dt_1+\int_{y-s}^{y-s/2}dt\int_{y+\tau-s}^{y+\tau-s/2}t_1^{\ 2}dt_1]$$

Then, the zero density degree of W(s , y) is

$$\sqrt{\left|\frac{R''(0)}{\pi^2 R(0)}\right|}$$ can be obtained.

The average density degree of w(s, y) is

$$\sqrt{\left|\frac{R^{(4)}(0)}{\pi^2 R^{(2)}(0)}\right|}$$ *can be obtain all.*

4. Wavelet Expansion of System

In order to use the idea of multiresolution, we will start by defining the scaling function and then define the wavelet in terms of it.

Let real function φ is standard orthogonal element of multiresolution analysis $\{V_j\}\, j\in Z$ (see [7]), then exist $h_k\in l^2$, have

$$\varphi(t)=\sqrt{2}\sum_k\varphi(2t-k)$$

Let $\psi(t)=\sqrt{2}\sum_k(-1)^k h_{1-k}\varphi(2t-k)$

Then wavelet express of $y(t)$ in mean square is

$$y(t)=2^{-\frac{J}{2}}\sum_K C_n^J\varphi(2^{-J}t-n)$$

$$+\sum_{j\le J}2^{-\frac{j}{2}}\sum_{n\in Z}d_n^j\psi(2^{-j}t-n)$$

Where

$$C_n^j = 2^{-\frac{j}{2}} \int_R x(t)\varphi(2^{-j}t-n)dt$$

$$d_n^j = 2^{-\frac{j}{2}} \int_R x(t)\psi(2^{-j}t-n)dt$$

Then have

$$E\left[C_n^j C_m^k\right] = 2^{-\frac{j+k}{2}} \iint_{R^2} E\left[x(t)x(s)\right]$$

$$\varphi(2^{-j}t-n)\varphi(2^{-k}s-m)dsdt$$

$$E\left[d_n^j d_m^k\right] = 2^{-\frac{j+k}{2}} \iint_{R^2} E\left[x(t)x(s)\right]$$

$$\psi(2^{-j}t-n)\psi(2^{-k}s-m)dsdt$$

Where

$$\psi(2^j t-m) = \begin{cases} 1, m2^{-j} \le t < (1/2+m)2^j \\ -1, (1/2+m)2^{-j} \le t < (1+m)2^{-j} \end{cases} \quad (8)$$

$$\psi(2^k s-n) = \begin{cases} 1, n2^{-k} \le s < (1/2+n)2^{-k} \\ -1, (1/2+n)2^{-k} \le s < (1+n)2^{-k} \end{cases} \quad (9)$$

Use (8) and (10) ,we can obtain value of $E\left[d_n^j d_m^k\right]$.

If we let normalized scaling function to have compact support over [0,1],then a solution is a scaling function that is a simple rectangle function

$$\varphi(t) = \begin{cases} 1, 0 \le t \le 1 \\ 0, otherwise \end{cases} \quad (10)$$

Now we consider function $\psi(t)$ that exist compact support set on $[-k_1,k_2],k_1,k_2 \ge 0$, and exist enough large M, have $\int_R t^m \psi(t)dt = 0, 0 \le m \le M-1$, then φ exist compact support set on $[-k_3,k_4]$ satisfy $k_1+k_2 = k_3+k_4, k_3,k_4 \ge 0$.

Let $b(j,k) = \langle y(t), \psi_{jk} \rangle$

$a(j,k) = \langle y(t), \phi_{jk} \rangle$

Let J is a constant, then

$\left\{2^{\frac{J}{2}}\varphi(2^J x-k), k \in Z\right\} \cup \left\{2^{\frac{j}{2}}\psi(2^j t-k), k \in Z\right\}_{j \ge J}$ are a standard orthonormal basis of space $L^2(R)$, then have

$$y(t) = 2^{\frac{J}{2}} \sum_{K \in Z} a(J,K)\varphi(2^J t-K) + \sum_{j \ge J}\sum_{K \in Z} 2^{\frac{j}{2}} b(j,K)\psi(2^j t-K) \quad (11)$$

Therefore, the self-correlation function of $b(j,m)$

$$R_b(j,K;m,n) = E\left[b(j,m)b(k,n)\right]$$

$$= 2^{-\frac{j+K}{2}} \iint_{R^2} E\left[x(t)x(s)\right]\psi(2^j t-m)\psi(2^K s-n)dtds \quad (12)$$

And have also the self-correlation function of $a(j,m)$

$$R_a(j,K;m,n) = E\left[a(j,m)a(k,n)\right]$$

$$= 2^{-\frac{j+K}{2}} \iint_{R^2} E\left[x(t)x(s)\right]\varphi(2^j t-m)\varphi(2^K s-n)dtds \quad (13)$$

Then ,we use (8) and (9) have

$$R_b(j,k;m,n)$$

$$=2^{-\frac{j+k}{2}}\iint_{R^2}E[x(t)x(s)]\psi(2^jt-m)\psi(2^ks-n)dtds$$

$$=2^{-\frac{j+k}{2}}\iint_{R^2}ts\psi(2^js-n)\psi(2^ks-m)dtds$$

$$+2^{-\frac{j+k}{2}}\iint_{R^2}2t^2\psi(2^js-n)\psi(2^ks-m)dtds$$

$$R_a(j,k;m,n)$$

$$=2^{-\frac{j+k}{2}}\iint_{R^2}E[x(t)x(s)]\varphi(2^jt-m)\varphi(2^ks-n)dtds$$

$$=2^{-\frac{j+k}{2}}\iint_{R^2}ts\varphi(2^js-n)\varphi(2^ks-m)dtds+2^{-\frac{j+k}{2}}\iint_{R^2}2t^2\varphi(2^js-n)\varphi(2^ks-m)dtds$$

We have

$$R_b(j,k;m,n)=2^{-\frac{j+k}{2}}[\int_{m2^{-j}}^{(1/2+m)2^{-j}}tdt\int_{n2^{-k}}^{(1/2+n)2^{-k}}sds$$

$$-\int_{m2^{-j}}^{(1/2+m)2^{-j}}tdt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}}sds-\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}}tdt\int_{n2^{-k}}^{(1/2+n)2^{-k}}sds+\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}}tdt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}}sds]$$

$$+2^{-\frac{j+k}{2}}[\int_{m2^{-j}}^{(1/2+m)2^{-j}}2t^2dt\int_{n2^{-k}}^{(1/2+n)2^{-k}}ds\begin{matrix}-\int_{m2^{-j}}^{(1/2+m)2^{-j}}2t^2dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}}ds\\-\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}}2t^2dt\int_{n2^{-k}}^{(1/2+n)2^{-k}}ds\end{matrix}$$

$$+\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}}2t^2dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}}ds]$$

We obtain

$$\varphi(2^jt-n)=\begin{cases}1, n2^j\le t\le(n+1)2^j\\0, other\end{cases}$$

$$\varphi(2^ks-m)=\begin{cases}1, m2^k\le s\le(m+1)2^k\\0, other\end{cases}$$

Then ,we have

$$R_a(j,k;m,n)=2^{-\frac{j+k}{2}}\iint_{R^2}ts\varphi(2^js-n)\varphi(2^ks-m)dtds$$

$$+2^{-\frac{j+k}{2}}\iint_{R^2}2t^2\varphi(2^js-n)\varphi(2^ks-m)dtds$$

5. Basic Definitions of Wiener Processes

Definition 1

Let

$$X(t)=\int_0^t e^{a(t-u)}dw(u), t\ge 0, a\in R-\{0\}$$

W(u) is Wiener processes, we call X(t) is Index Wiener Integral processes.

We have

$$R(s,t)=EX(t)X(s)$$

$$=E\int_0^s e^{a(s-u)}dw(u)\int_0^t e^{a(t-u)}dw(u)$$

$$=\frac{\sigma^2}{2a}(e^{a(s+t)}-e^{a(t-s)})$$

Let

$$\sigma^2=1$$

Definition 2

Let {x(t) , t∈R} is a stochastic processes on probability space (Ω, g, P) ,we call

$$W(s,x)=\frac{1}{s}\int_R x(t)\psi(\frac{x-t}{s})dt$$

is wavelet transform of x(t) . where,ψ is mather wavelet([11]).

Then, we have

$$w(s,x+\tau)=\frac{1}{s}\int_R x(t)\psi(\frac{x+\tau-t}{s})dt$$

Definition 3

Let mather waveletψ(x) is function:

$$\psi(x)=\begin{cases}1, 0\le x<\frac{1}{2}\\ -1, \frac{1}{2}\le x<1\\ 0, other\end{cases} \qquad (5)$$

we call ψ(x) is the Haar wavelet.

Then, we have

$$\psi(\frac{x-t}{s})=\begin{cases}1, x-\frac{s}{2}\le t<x\\ -1, x-s\le t<x-\frac{s}{2}\end{cases}$$

$$\psi(\frac{x+\tau-t}{s})=\begin{cases}1, x+\tau-\frac{s}{2}\le t<x+\tau\\ -1, x+\tau-s\le t<x+\tau-\frac{s}{2}\end{cases}$$

We have

$$R(\tau)=E[w(s,y)w(s,y+\tau)]$$

$$=E[\frac{1}{s}\int_R x(t)\psi(\frac{y-t}{s})dt][\frac{1}{s}\int_R x(t_1)\psi(\frac{y+\tau-t_1}{s})dt_1]=\frac{1}{s^2}E[\iint_{R^2} x(t)x(t_1)\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$

$$=\frac{1}{s^2}\iint E[x(t)x(t_1)]\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1$$

$$=\frac{1}{s^2}\iint_{R^2}\frac{1}{2a}(e^{a(t_1+t)}-e^{a(t-t_1)})\psi(\frac{y-t}{s})$$

$$\psi(\frac{y+\tau-t_1}{s})dtdt_1$$

$$=\frac{1}{s^2}[\iint_{R^2}\frac{1}{2a}e^{a(t_1+t)}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1$$

$$-\iint_{R^2}\frac{1}{2a}e^{a(t-t_1)}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$

$$=I_1-I_2$$

Where

$$I_1=\frac{1}{s^2}[\iint_{R^2}\frac{1}{2a}e^{a(t_1+t)}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1,$$

$$I_2=\iint_{R^2}\frac{1}{2a}e^{a(t-t_1)}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$

Then,we have

$$I_1 = \frac{1}{2as^2}[\int_{y-s/2}^{y} e^{at}dt\int_{y+\tau-s/2}^{y+\tau} e^{at_1}dt_1$$
$$-\int_{y-s/2}^{y} e^{at}dt\int_{y+\tau-s}^{y+\tau-s/2} e^{at_1}dt_1$$
$$-\int_{y-s}^{y-s/2} e^{at}dt\int_{y+\tau-s/2}^{y-s/2} e^{at_1}dt_1$$
$$+\int_{y-s}^{y-s/2} e^{at}dt\int_{y+\tau-s}^{y+\tau-s/2} e^{at_1}dt_1]$$

The same time, we have

$$I_2 = \frac{1}{2as^2}\iint_{R^2} e^{at-at_1}\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$
$$= \frac{1}{2as^2}[\int_{y-s/2}^{y} e^{at}dt\int_{y+\tau-s/2}^{y+\tau} e^{-at_1}dt_1$$
$$-\int_{y-s/2}^{y} e^{at}dt\int_{y+\tau-s}^{y+\tau-s/2} e^{-at_1}dt_1$$
$$-\int_{y-s}^{y-s/2} e^{at}dt\int_{y+\tau-s/2}^{y-s/2} e^{-at_1}dt_1$$
$$+\int_{y-s}^{y-s/2} e^{at}dt\int_{y+\tau-s}^{y+\tau-s/2} e^{-at_1}dt_1]$$

Then, the zero density degree of W(s, y) is

$\sqrt{\left|\frac{R''(0)}{\pi^2 R(0)}\right|}$ can be obtained.

The average density degree of w(s, y) is

$\sqrt{\left|\frac{R^{(4)}(0)}{\pi^2 R^{(2)}(0)}\right|}$ *can be obtain all.*

We consider wavelet expansions of stochastics processes and show that for certain wavelets, the coefficients of the expansion have negligible correlation for different scales. we can introduce a modification of the wavelets. Certain nonstationary processes the wavelets may be chose to give uncorrelated coefficients.

In order to use the idea of multiresolution , we will start by defining the scaling function and then define the wavelet in terms of it.

Let real function φ is standard orthogonal element of multiresolution analysis $\{V_j\} j \in Z$ (see [7]), then exist $h_k \in l^2$, have

$$\varphi(t) = \sqrt{2}\sum_k \varphi(2t-k)$$

Let $\psi(t) = \sqrt{2}\sum_k (-1)^k h_{1-k}\varphi(2t-k)$

Then wavelet express of $y(t)$ in mean square is

$$y(t) = 2^{-\frac{J}{2}}\sum_K C_n^J \varphi(2^{-J}t-n)$$
$$+\sum_{j\le J} 2^{-\frac{j}{2}}\sum_{n\in Z} d_n^j \psi(2^{-j}t-n)$$

Where, $C_n^j = 2^{-\frac{j}{2}}\int_R x(t)\varphi(2^{-j}t-n)dt$

$$d_n^j = 2^{-\frac{j}{2}}\int_R x(t)\psi(2^{-j}t-n)dt$$

Then have

$$E\left[C_n^j C_m^k\right]$$
$$=2^{-\frac{j+k}{2}}\iint_{R^2} E\left[x(t)x(s)\right]\varphi(2^{-j}t-n)\varphi(2^{-k}s-m)dsdt$$
$$E\left[d_n^j d_m^k\right]$$
$$=2^{-\frac{j+k}{2}}\iint_{R^2} E\left[x(t)x(s)\right]\psi(2^{-j}t-n)\psi(2^{-k}s-m)dsdt$$

Where

$$\psi(2^j t-m)=\begin{cases}1, m2^{-j}\le t<(1/2+m)2^j\\ -1,(1/2+m)2^{-j}\le t<(1+m)2^{-j}\end{cases} \quad (8)$$

$$\psi(2^k s-n)=\begin{cases}1, n2^{-k}\le s<(1/2+n)2^{-k}\\ -1,(1/2+n)2^{-k}\le s<(1+n)2^{-k}\end{cases} \quad (9)$$

Use (8) and (10) , we can obtain value of $E\left[d_n^j d_m^k\right]$.

If we let normalized scaling function to have compact support over [0,1],then a solution is a scaling function that is a simple rectangle function

$$\varphi(t)=\begin{cases}1,0\le t\le 1\\ 0, otherwise\end{cases} \quad (10)$$

Now we consider function $\psi(t)$ that exist compact support set on $[-k_1,k_2],k_1,k_2\ge 0$, and exist enough large M, have $\int_R t^m\psi(t)dt=0,0\le m\le M-1$, then φ exist compact support set on $[-k_3,k_4]$ satisfy $k_1+k_2=k_3+k_4,k_3,k_4\ge 0$.

Let $b(j,k)=<y(t),\psi_{jk}>$,

$a(j,k)=<y(t),\phi_{jk}>$

Let J is a constant, then

$\left\{2^{\frac{J}{2}}\varphi(2^J x-k),k\in Z\right\}\cup\left\{2^{\frac{j}{2}}\psi(2^j t-k),k\in Z\right\}_{j\ge J}$ are a standard orthonormal basis of space $L^2(R)$,

then have

$$y(t)=2^{\frac{J}{2}}\sum_{K\in Z}a(J,K)\varphi(2^J t-K)$$
$$+\sum_{j\ge J}\sum_{K\in Z}2^{\frac{j}{2}}b(j,K)\psi(2^j t-K) \quad (11)$$

Therefore, the self-correlation function of $b(j,m)$

$$R_b(j,K;m,n)=E[b(j,m)b(k,n)]$$
$$=2^{-\frac{j+K}{2}}\iint_{R^2} E\left[x(t)x(s)\right]\psi(2^j t-m)\psi(2^K s-n)dtds \quad (12)$$

And have also the self-correlation function of $a(j,m)$

$$R_a(j,K;m,n)=E\left[a(j,m)a(k,n)\right] \qquad =2^{-\frac{j+K}{2}}\iint_{R^2} E\left[x(t)x(s)\right]\varphi(2^j t-m)\varphi(2^K s-n)dtds$$

(13)

Then , we use (8) and (9) have

$$R_b(j,k;m,n)$$
$$=2^{-\frac{j+k}{2}}\iint_{R^2} E[x(t)x(s)]\psi(2^j t-m)\psi(2^k s-n)dtds$$
$$=2^{-\frac{j+k}{2}}\iint_{R^2} e^{a(t+s)}\psi(2^j s-n)\psi(2^k s-m)dtds$$
$$+2^{-\frac{j+k}{2}}\iint_{R^2} e^{a(t-s)}\psi(2^j s-n)\psi(2^k s-m)dtds$$

$$R_a(j,k;m,n)$$
$$=2^{-\frac{j+k}{2}}\iint_{R^2} E[x(t)x(s)]\varphi(2^j t-m)\varphi(2^k s-n)dtds$$
$$=2^{-\frac{j+k}{2}}\iint_{R^2} e^{a(s+t)}\varphi(2^j s-n)\varphi(2^k s-m)dtds+2^{-\frac{j+k}{2}}\iint_{R^2} e^{a(t-s)}\varphi(2^j s-n)\varphi(2^k s-m)dtds$$

We have

$$R_b(j,k;m,n)$$
$$=2^{-\frac{j+k}{2}}[\int_{m2^{-j}}^{(1/2+m)2^{-j}} e^{at}dt\int_{n2^{-k}}^{(1/2+n)2^{-k}} e^{as}ds-\int_{m2^{-j}}^{(1/2+m)2^{-j}} e^{at}dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}} e^{as}ds-\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}} e^{at}dt\int_{n2^{-k}}^{(1/2+n)2^{-k}} e^{as}ds+\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}} e^{at}dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}} e^{as}ds]$$
$$+2^{-\frac{j+k}{2}}[\int_{m2^{-j}}^{(1/2+m)2^{-j}} e^{at}dt\int_{n2^{-k}}^{(1/2+n)2^{-k}} e^{-as}ds-\int_{m2^{-j}}^{(1/2+m)2^{-j}} e^{at}dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}} e^{-as}ds-\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}} e^{at}dt\int_{n2^{-k}}^{(1/2+n)2^{-k}} e^{-as}ds$$
$$+\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}} e^{at}dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}} e^{-as}ds]$$

We obtain

$$\varphi(2^j t-n)=\begin{cases}1, n2^j\le t\le(n+1)2^j\\0, other\end{cases}$$
$$\varphi(2^k s-m)=\begin{cases}1, m2^k\le s\le(m+1)2^k\\0, other\end{cases}$$

Then, we have

$$R_a(j,k;m,n)=2^{-\frac{j+k}{2}}\iint_{R^2} e^{a(t+s)}\varphi(2^j s-n)\varphi(2^k s-m)dtds$$
$$-2^{-\frac{j+k}{2}}\iint_{R^2} e^{a(t-s)}\varphi(2^j s-n)\varphi(2^k s-m)dtds$$

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