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Using the Triangular Fuzzy Numbers for the Verification of a Chosen Decision

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Abstract: Fuzzy numbers play a fundamental role in fuzzy mathematics, analogous to the role played by the ordinary numbers in classical mathematics. In this work we use the Triangular Fuzzy Numbers for the verification of a chosen decision and we give an example illustrating our results in practice. In order to check the creditability of our new fuzzy method, the outcomes of its application are compared in our example with the corresponding outcomes of two traditional methods, the calculation of the mean values and the use of the GPA index.

Keywords: Decision making (DM); Verification of a decision; Grade point average (GPA) index; Fuzzy logic (FL); Fuzzy numbers (FNs); Triangular fuzzy numbers (TFNs); Defuzzification of TFNs; Center of gravity (CoG) defuzzification technique.

1. Introduction

1.1. Decision Making

Decision Making (DM) is the process of choosing a solution between two or more alternatives, aiming to achieve the best possible results for a given problem. Obviously the above process has sense if, and only if, there exist more than one *feasible solutions* and a suitable criterion that helps the decision-maker (d-m) to choose the best among these solutions. We recall that a solution is characterized as feasible, if it satisfies all the restrictions imposed onto the real system by the statement of the problem as well as all the natural restrictions imposed onto the problem by the real system; e.g. if x denotes the quantity of stock of a product, it must be $x \geq 0$. The choice of the suitable criterion, especially when the results of DM are affected by random events, depends upon the d-m's desired goals; e.g. optimistic or conservative criterion, etc.

The rapid technological progress , the impressive development of the transport means, the globalization of the modern society, the enormous changes happened to the local and international economies and other similar reasons led during the last 50-60 years to a continuously increasing complexity of our everyday life. As a result the DM process became in many cases a very difficult task, which is not possible to be based on the d-m's experience, intuition and skills only, as it usually happened in the past. Thus, from the beginning of the 1950's a progressive development started of a systematic methodology for the DM process, which is based on Probability Theory, Statistics, Economics, Psychology, etc and it is known as *Statistical Decision Theory* ([1], etc).

According to the nowadays existing standards the DM process involves the following steps:

- **d_1:** Analysis of the decision-problem, i.e. understanding, simplifying and reformulating the problem in a way permitting the application of the principles of DM on it.
- **d_2:** Collection and interpretation of all the necessary information related to the problem.
- **d_3:** Determination of all the alternative feasible solutions.
- **d_4:** Choice of the best solution in terms of a suitable (according to the d-m's goals and targets) criterion.

One could add one more step to the DM process, the *verification* (checking the creditability) *of the chosen decision* according to the results obtained by applying it in practice. However, this step is extended to areas, which, due to their depth and importance for the administrative rationalism, have become autonomous. Therefore, the step of verification is usually examined separately from the other steps of the DM process.

Notice that the first three steps of the DM process presented above are *continuous* in the sense that the completion of each one of them usually needs some time, during which the d-m's reasoning is characterized by transitions between hierarchically neighbouring steps. The flow-diagram of the DM process is represented in Figure 1 below:

Figure-1. The flow-diagram of the DM process

$$\mathbf{d_1 \leftrightarrow d_2 \leftrightarrow d_3 \rightarrow d_4}$$

1.2. Fuzzy Logic in DM

Situations often appear in everyday life in which definitions have not clear boundaries; e.g. when we speak about the "high mountains" of a country, the "good players" of a football team, etc. The *fuzzy sets* theory was created in response to have a mathematical representation of such kind of situations. For those not familiar to the subject we recall that the notion of a fuzzy set, initiated by Zadeh in 1965 [2], is defined as follows:

Definition 1: Let U denote the universal set of the discourse. Then a fuzzy set A on U (or otherwise a fuzzy subset of U), is a set of ordered pairs of the form.

$A = \{(x, m_A(x)): x \in U\}$, where $m_A : U \to [0,1]$ is the *membership function* assigning to each element of U a real value from the interval [0,1]. -

For reasons of simplicity, many authors identify a fuzzy set with its membership function. Also, a fuzzy set is frequently written in the form of a symbolic sum or, when U has the power of continuous, in the form of a symbolic integral.

The value $m_A(x)$, called the *membership degree (or grade)* of x in A, expresses the degree to which x verifies the characteristic property of A. Thus, the nearer the value $m_A(x)$ to 1, the higher the membership degree of x in A. Obviously each classical (crisp) subset A of U can be considered as a fuzzy subset of U, with $m_A(x)=1$ if $x \in U$ and $m_A(x)=0$ if $x \notin U$. Most of the concepts of classical (crisp) sets can be extended to fuzzy sets. For general facts on fuzzy sets we refer to the book [3].

Fuzzy Logic (FL), the development of which is based on fuzzy sets theory [4], provides a rich and meaningful addition to standard logic. Unlike the standard logic accepting only two states, true or false, FL deals with truth values which range continuously from 0 to 1. Thus something could be *half true* 0.5 or *very likely true* 0.9 or *probably not true* 0.1, etc. In this way FL allows one to express knowledge in a rule format that is close to a natural language expression and therefore it opens the door to construction of mathematical solutions of computational problems which are imprecisely defined.

In particular, a DM problem is frequently expressed in an ambiguous way involving a degree of uncertainty. In such cases the classical Statistical Decision Theory is proved inadequate for helping the d-m to choose the correct decision. On the contrary, FL offers a variety of resources allowing the d-m to frame the goals and constraints of the decision problem in vague, linguistic terms, which may reflect the real situation (e.g. see [5], Section 6.5 of [3], etc).

The steps of the *DM process under fuzzy conditions*, which follow the general lines of the solution of any problem with the methods of FL, involve:

1. *Choice of the universal set of the discourse U*

2. *Fuzzification of the decision problem's data*

In this step the fuzzy goal and the fuzzy constraints of the decision problem are expressed as fuzzy sets in U. For this, one has to define properly the corresponding to each case membership functions[1].

3. *Evaluation of the fuzzy data*

It is logical to define the *fuzzy decision* as the choice that satisfies both the goals and the constraints and, if we interpret this as a logical "and", we can model it with the intersection of all fuzzy goals and constraints of the decision problem. Finally we take the maximum of this set to obtain the best among the existing alternatives (*Bellman-Zadeh's criterion* for DM in a fuzzy environment [5]).

4. *Defuzzification*

In this step the fuzzy outputs of the problem are converted to a crisp number, which allows the expression of the fuzzy decision in the natural language.

1.3. Fuzzy Assessments: A Summary of our Previous Researches

Actually, the verification of a chosen decision (see Section 1.1) constitutes the assessment of the decision problem's solution. The assessment, in general, of a system's effectiveness (i.e. of the degree of attainment of its targets) with respect to an action performed within the system (e.g. problem-solving, DM, learning, etc) is a very important task that enables the correction of the system's weaknesses resulting to the improvement of its general performance.

The assessment methods commonly used in practice are based on the principles of bivalent logic (yes-no). However, assessment cases frequently appear in everyday practice characterized by a degree of uncertainty and/or ambiguity. A teacher, for example, sometimes is not sure about a particular numerical grade characterizing a student's performance. Obviously, in such cases a crisp characterization is not the most appropriate for the assessment. In fact, FL due to its nature of characterizing an ambiguous case with multiple values offers wider and richer resources for such kind of situations.

This gave as several times in past the impulse to apply principles of FL for the assessment of human or machine (in case of CBR systems [6]) skills using as tools the corresponding system's *total uncertainty* (e.g. see [7] and its relevant references, Section 3 of [8], section 2 of [9], etc), the *CoG defuzzification technique* (e.g. see Sections 3 of [8] and [9], etc) as well as two recently developed variations of this technique, the *Triangular* (e.g. see [10], etc) and

[1] We recall that the choice of the membership function, usually based on statistical data of experiments performed with samples of the population under study, is subjective and therefore not unique. However, a necessary condition for the creditability of a fuzzy set in representing a real situation is that the criteria of this choice are compatible to the rules of the common logic.

of the *Trapezoidal* (e.g. see Section 3 of [6], Section 4 of [9], etc) *fuzzy assessment models* that treat better than the CoG the ambiguous assessment cases being at the boundaries between two successive assessment grades (e.g. between A and B for student assessment). The use of the CoG technique for assessment purposes, as well as the above mentioned two variations of it were initiated by Igor Subbotin (e.g. [6, 9-11], etc) Professor of Mathematics at State University in Los Angeles and coauthor of the present author in many publications (e.g. [6, 9, 10], etc).

1.4. Organization of the Paper

Fuzzy numbers (FNs) play a fundamental role in fuzzy mathematics, analogous to the role played by the ordinary numbers in classical mathematics. Our target in the present paper is to extend our older researches mentioned in Section 1.3 by using the simplest form of FNs, i.e. the *Triangular Fuzzy Numbers (TFNs)*, for the verification (assessment) of a chosen decision. Notice that, there exist strong logical pro arguments for employing this approach. In fact, roughly speaking a TFN (a, b, c), with a, b and c real numbers such that $a< b < c$, says that *"b lies in the interval [a, c]"* which means *"approximately equal to b",* an expressions that constitutes the basis for a fuzzy assessments.

The rest of the paper is organized as follows: In Sections 2 and 3 we present the FNs and the TFNs respectively, the basic arithmetic operations on them and *a* defuzzification method of TFNs with the help of the CoG technique. In Section 4 we consider an example from everyday life and we apply the TFNs, as well as two other traditional methods (calculation of the mean values and GPA index), for the verification of the chosen in our example decision. The last Section 5 is devoted to our final conclusions and to a brief exposition of our future plans for further research on the subject.

2. Fuzzy Numbers

2.1. Definitions

A FN is a special form of fuzzy set on the set ***R*** of real numbers. For introducing the notion of a FN, it becomes necessary to give first the following three introductory definitions:

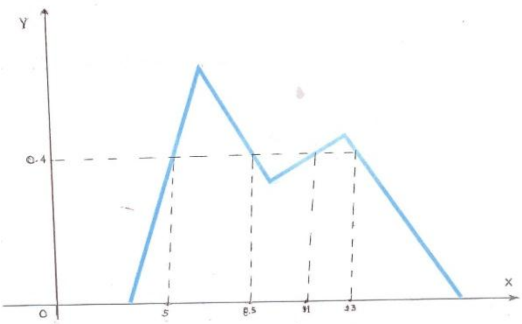
Definition 2: A fuzzy set A on U with membership function $y = m(x)$ is said to be *normal*, if there exists x in U, such that $m(x) = 1$.

Definition 3: Let A be a fuzzy set on U, and let x be a real number of the interval [0, 1]. Then the x-*cut* of A, denoted by A^x, is defined to be the set $A^x = \{y \in U: m(y) \geq x\}$.

Definition 4: A fuzzy set A on ***R*** is said to be *convex,* if its x-cuts A^x are ordinary closed real intervals, for all x in [0, 1].

For example, for the fuzzy set A whose membership function's graph is represented in Figure 2, we observe that $A^{0.4} = [5, 8.5] \cup [6, 7]$ and therefore A is not a convex fuzzy set.

Figure-2. Graph of a non convex fuzzy set

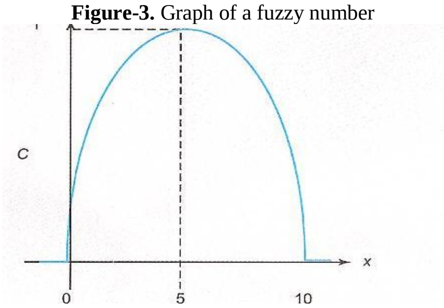


We are ready now to give the definition of a FN:

Definition 5: A FN is a normal and convex fuzzy set A on ***R*** with a piecewise continuous membership function.

Figure 3 represents the graph of a FN expressing the fuzzy concept: *"The real number x is approximately equal to 5".* We observe that the membership function of this FN takes constantly the value 0 outside the interval [0, 1], while its graph in [0, 1] is a parabola.

Figure-3. Graph of a fuzzy number



Since the x-cuts A^x of a FN A are closed real intervals, we can write $A^x = [A_l^x, A_r^x]$ for each x in [0, 1], where A_l^x, A_r^x are real numbers depending on x. The following statement defines a *partial order* in the set of all FNs:

Definition 6: Given the FNs A and B we write $A \leq B$ (or $\geq$) if, and only if, $A_l^x \leq B_l^x$ and $A_r^x \leq B_r^x$ (or $\geq$) for all x in [0, 1]. Two FNs for which the above relations hold are called *comparable*, otherwise they are called *non comparable*.

For general facts on FNs we refer to Chapter 3 of the book of Theodorou [12], which is written in Greek language, and also to the classical on the subject book of Kaufmann and Gupta [13].

2.2. Arithmetic Operations on FNs

The basic arithmetic operations on FNs are defined in general in two alternative ways, which are *equivalent* to each other:

(i) *With the help of their x-cuts and the Representation-Decomposition Theorem* for fuzzy sets[2]: In fact, if *A* and *B* are given FNs, and "*" denotes an arithmetic operation (addition, subtraction, multiplication or division) between them, then applying the above theorem for the fuzzy set *A* * *B* we find that $A * B = \sum_{x\in[0,1]} x(A*B)^x$. But the x-cuts of the FNs are ordinary closed real intervals, therefore, if we define that $(A * B)^x = A^x * B^x$ (where, for reasons of simplicity, "*" in the second term of the above equation denotes the corresponding operation between closed real intervals), the *fuzzy arithmetic is turned to the well known arithmetic of the closed real intervals* [3].

(ii) By applying the *Zadeh's extension principle* ([3], Section 1.4, p.20), which provides the means for any function f mapping the crisp set X to the crisp set Y to be generalized so that to map fuzzy subsets of X to fuzzy subsets of Y.

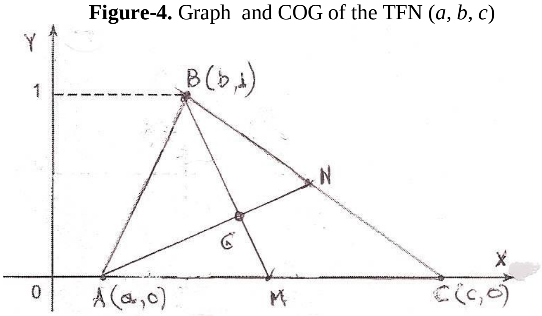
In practice the above two general methods of the fuzzy arithmetic, requiring laborious calculations, are rarely used in applications, where the utilization of simpler forms of FNs is preferred.

3. Triangular Fuzzy Numbers

3.1. Definition and Basic Properties of TFNs

The membership function's graph of a TFN (*a, b, c*) is presented in Figure 4. We observe that its value is equal to 0, if x is outside the interval [*a, c*], while in the interval [*a, c*] its graph is the union of two straight line segments forming a triangle with the X-axis.

Figure-4. Graph and COG of the TFN (*a, b, c*)



Therefore, the analytic definition of a TFN is given as follows:

Definition 7: Let *a, b* and *c* be real numbers with *a* < *b* < *c*. Then the *TFN* (*a, b, c*) is a FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \text{ or } x > c \end{cases}$$

---

[2] The Representation-Decomposition Theorem of Ralesscou-Negoita ([14], Theorem 2.1, p.16) states that a fuzzy set A can be completely and uniquely expressed by the family of its x-cuts in the form $A = \sum_{x\in[0,1]} xA^x$.

[3] We recall that an arithmetic operation "*" between closed real intervals is defined by the general rule $[a, b] * [a_1, b_1] = \{x * y: x, y \in \boldsymbol{R}, a \leq x \leq a_1, b \leq y \leq b_1\}$ [13].

In the above definition we obviously have that $m(b)=1$, while b need not be in the "middle" of a and c.
The following two Propositions refer to basic properties of TFNs that we are going to use later in this paper:

Proposition 1: The x-cuts A^x of a TFN A = (a, b, c), $x \in [0, 1]$, are calculated by the formula $A^x = [A_l^x, A_r^x] = [a + x(b - a), c - x(c - b)]$.

Proof: Since $A^x = \{y \in \boldsymbol{R}: m(y) \geq x\}$, Definition 6 gives for the case of A_l^x that

$\frac{y-a}{b-a} = x \Leftrightarrow y = a + x(b-a)$. Similarly for the case of A_r^x we have that $\frac{c-y}{c-b} = x$
$\Leftrightarrow y = c - x(c - b)$.

Proposition 2: *(Defuzzification of a TFN)* The coordinates (X, Y) of the COG of the graph of the TFN (a, b, c) are calculated by the formulas $X = \frac{a+b+c}{3}$, $Y = \frac{1}{3}$.

Proof: The graph of the TFN (a, b, c) is the triangle ABC of Figure 4, with A $(a, 0)$,
B $(b, 1)$ and C $(c, 0)$. Then, the COG, say G, of ABC is the intersection point of its medians AN and BM, where N $\left(\frac{b+c}{2}, \frac{b}{2}\right)$ and M $\left(\frac{a+c}{2}, 0\right)$. Therefore the equation of the straight line on which AN lies is $\frac{x-a}{\frac{b+c}{2}-a} = \frac{y}{\frac{1}{2}}$, or $x + (2a - b - c)y = a$ (1). In the same way one finds that the equation of the straight line on which BM lies is $2x + (a + c + 2b)y = a + c$ (2).

Since $D = \begin{vmatrix} 2 & a+c-2b \\ 1 & 2a-b-c \end{vmatrix} = 3(a-c) \neq 0$, the linear system of (1) and (2) has a unique solution with the respect to the variables x and y determining the coordinates of the triangle's COG.
The proof of the Proposition is completed by observing that

$$D_x = \begin{vmatrix} a+c & a+c-2b \\ a & 2a-b-c \end{vmatrix} = a^2 - c^2 + ba - bc = (a+c)(a-c) + b(a-c)$$

$$= (a-c)(a+c+b) \text{ and } D_y = \begin{vmatrix} 1 & a+c \\ 2 & a \end{vmatrix} = c - a.$$

3.2. Arithmetic Operations on TFNs

It can be shown [13] that the two general methods for defining arithmetic operations on FNs presented in Section 2.2 lead to the following simple rules for the *addition* and *subtraction* of TFNs:

Let A = (a, b, c) and B = (a_1, b_1, c_1) be two TFNs. Then

- The sum A + B = $(a+a_1, b+b_1, c+c_1)$.
- The difference A - B = A + (-B) = $(a-c_1, b-b_1, c-a_1)$, where –B = $(-c_1, -b_1, -a_1)$ is defined to be the *opposite* of B[4].

In other words, the opposite of a TFN, as well as the sum and the difference of two TFNs are also TFNs. On the contrary, the product and the quotient of two TFNs, although they are FNs, *they are not always TFNs*. However, in the special case where a, b, c, a_1, b_1, c_1 are in $\boldsymbol{R}^+$, it can be shown [13] that the fuzzy operations of *multiplication* and *division* of TFNs can be *approximately performed* by the rules:

- The product A. B = (aa_1, bb_1, cc_1).
- The quotient A : B = A . B^{-1} = $\left(\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}\right)$, where $B^{-1} = \left(\frac{1}{a_1}, \frac{1}{b_1}, \frac{1}{c_1}\right)$ is defined to be the *inverse* of B.

In other words, in $\boldsymbol{R}^+$ the inverse of a TFN, as well as the product and the division of two TFNs can be approximately considered to be TFNs too.

Further, one can define the following two *scalar operations*:

- k + A= (k+a, k+b, k+c), $k \in \boldsymbol{R}$
- kA = (ka, kb, kc), if k>0 and kA = (kc, kb, ka), if k<0.

We close this section with the following definition, which will be used later in this paper for verifying a chosen decision with the help of TFNs:

Definition 8: Let A_i, i = 1, 2, …, n be TFNs, where n is a non negative integer, $n \geq 2$. Then we define the *mean value* of the above TFNs to be the TFN

$$A = \frac{1}{n}(A_1 + A_2 + \ldots + A_n).$$

[4] Obviously A + (-A) = (a-c, 0, c-a) $\neq$ O = (0, 0, 0), where the TFN O is defined by O(x) = 1, if x = 0 and O(x)=0, if x $\neq$ 0

4. Verification of a Chosen Decision

In this section we use the TFNs for verifying, in an example taken from everyday life, a chosen decision. Further, and in order to check the creditability of this approach, the outcomes of its application in our example are compared with the corresponding outcomes of two traditional assessment methods, the calculation of the *mean values* and of the *Grade Point Average (GPA) index*. Our example is the following:

EXAMPLE: A car industry circulates a new model in the market in two different types, the Luxury (L) Class and the Regular (R) Class. Six months after the purchase of their cars the customers were asked to complete a written questionnaire (based on a climax from 0 to 100) concerning the degree of satisfaction for their new cars. The scores corresponding to the customers' answers were the following:

L-Class: 100(2 times), 99(3), 98(5), 95(8), 94(7), 93(1), 92 (6), 90(5), 89(3), 88(7), 85(13), 82(6), 80(14), 79(8), 78(6), 76(3), 75(3), 74(3), 73(1), 72(5), 70(4), 68(2), 63(2), 60(3), 59(5), 58(1), 57(2), 56(3), 55(4), 54(2), 53(1), 52(2), 51(2), 50(8), 48(7), 45(8), 42(1), 40(3), 35(1).

R-Class: 100(1), 99(2), 98(3), 97(4), 95(9), 92(4), 91(2), 90(3), 88(6), 85(26), 82(18), 80(29), 78(11), 75(32), 70(17), 64(12), 60(16), 58(19), 56(3), 55(6), 50(17), 45(9), 40(6).

The above answers were classified by the staff of the industry's marketing department to the following five categories: A (85-100) = fully satisfied customers, B (75-84) = very much satisfied customers, C (60-74) = satisfied customers, D (50-59) = rather satisfied customers and F (0-49) = unsatisfied customers.

The questionnaire's data is summarized below in Table 1:

Table-1. Questionnaire's data

Customers' Categories	L Class	R Class
A	60	60
B	40	90
C	20	45
D	30	45
F	20	15
Total	170	255

We apply now the above mentioned assessment methods for the evaluation of this data as follows:

4.1. Mean Values

Calculating the mean values m_L and m_R of the scores of the customers' answers for the L-Class and the R-Class respectively one finds that $m_L \approx 76.006$ and $m_R \approx 75.09$. This shows that all the customers were very much satisfied with their new cars, with the customers who purchased the L-Class being slightly more satisfied than those who purchased the R-Class.

4.2. GPA Index

We recall that the GPA index is a weighted mean, where more importance is given to the higher scores by attaching greater coefficients (weights) to them. In other words the GPA index focuses on the *quality performance* of a group of individuals.

The GPA index, a very popular in USA and some other Western countries assessment method, is calculated by the formula GPA = $\frac{n_D+2n_C+3n_B+4n_A}{n}$ (3), where n_A, n_B, n_C, n_D and n_E denote the numbers of the corresponding group's members who belong to the categories A, B, C, D and F respectively and n is the total number of the group's members (e.g. see [15]). In the worst case ($n_F = n$) formula (3) gives that GPA = 0, while in the ideal case ($n_A = n$) it gives that GPA = 4. Therefore, we have that $0 \leq$ GPA ≤ 4. Consequently, any value of GPA greater than the half of its maximal value (4:2 = 2) may be considered as corresponding to a satisfactory quality performance of the corresponding group.

In our case, replacing the data of Table 1 to formula (3), one finds that both values of the GPA for the customers of the L-Class and the R-Class are equal to $\frac{43}{17} \approx 2.529$. This means that the industry's customers were equally satisfied with their new cars.

4.3. Use of the TFNs

We assign to each category of the customers' answers a TFN (denoted by the same letter) as follows: A= (85, 92.5, 100), B = (75, 79.5, 84), C = (60, 67, 74), D= (50, 54.5, 59) and F = (0, 24.5, 49). The middle entry of each of the above TFNs is equal to the mean value of the scores attached to the corresponding category, while the left and right entries are equal to the minimal and maximal score respectively of the corresponding category. In this way a TFN corresponds to each customer representing his (her) answers to the questionnaire.

We observe now that in Table 1 we actually have 170 TFNs representing the answers of the customers who purchased the L-class and 255 TFNs representing the answers of the customers who purchased the R-class.

Therefore, it is logical to accept that the overall evaluation of the answers of each of the two groups of customers is given by the mean value of the corresponding TFNs (see Definition 8).

Let us denote the above means by the letters L and R respectively. Then, performing straightforward calculations, one finds that

$$L = \frac{1}{170}.(60A+40B+20C+30D+20F) \approx (63.53, 71.74, 83.47) \text{ and}$$

$$R = \frac{1}{255}.(60A+90B+45C+45D+15F) \approx (65.88, 72.63, 79.53).$$

Observing the left and right entries of the TFNs L and R one understands that the answers of the industry's customers show that both groups of them were from satisfied (63.53 and 65.88) to very much satisfied (83.47 and 79.53) from their new cars[5]; however, this is a rough approximation only of their overall opinion. Further, applying the formula of Proposition 1 one finds that the x-cuts of the two TFNs are L^x = [63.53+8.21x, 83.47-11.73x] and R^x = [65.88+6.75x, 79.53-6.9x] respectively. But $63.53+8.21x \leq 65.88+6.75x \Leftrightarrow 1.46x \leq 2.35 \Leftrightarrow x \leq 1.61$, which is true, since x is in [0, 1]. On the contrary, $83.47-11.73x \leq 79.53-6.9x \Leftrightarrow 3.94 \leq 4.83x \Leftrightarrow 0.82 \leq x$, which is not true for all values of x. Therefore, according to Definition 6, the *TFNs L and R are not comparable*, which means that at this stage one can not decide which of the two groups of customers is more satisfied by the new cars.

In order to overcome this difficulty we shall *defuzzify* the TFNs R and L. In fact, by Proposition 2, the COGs of the triangles forming the graphs of the TFNs L and R have x-coordinates equal to $X = \frac{63.53+71.74+83.47}{3} \approx 72.91$ and

$$X' = \frac{65.88+72.63+79.53}{3} \approx 72.68 \text{ respectively.}$$

Observe now that the GOGs of the graphs of L and R lie in a rectangle with sides of length 100 units on the X-axis (customer scores from 0 to 100) and one unit on the Y-axis (normal fuzzy sets). Therefore, *the nearer the x-coordinate of the COG to 100, the better the corresponding group's opinion about the new cars*, Thus, since $X > X'$, the customers who purchased the L-Class were more satisfied from their new cars, than those who purchased the R-Class.

4.4. Comparison of the Applied Verification Methods

The calculation of the mean values of the customer scores and the use of TFNs has shown that the customers who purchased the L-Class were (slightly) more satisfied from their new cars than those who purchased the R-Class. On the contrary, the calculation of the GPA index has shown that the customers of both groups were equally satisfied. This small difference of the evaluation results can be easily explained by the fact that GPA is a weighted index giving more importance to the higher scores (to which greater coefficients are assigned).

But, the most important for the industry thing is that in all cases it was proved that its customers were more or less satisfied from their new cars, which means that the decision to circulate in the market the two Classes of its new model was a successful decision.

5. Conclusion

In this paper we utilized the TFNs for the verification of a chosen decision (circulation of two types of a new model in the market by a car industry). The comparison of the verification results obtained by this approach with the corresponding results obtained by two traditional assessment methods (mean values and GPA index) gave a strong indication of the creditability of our new fuzzy assessment method. The general character of this method enables its use in future for the assessment of a variety of other human or machine skills, in order to get safer statistical results about its applicability in practice and its validity. Further, other forms of FNs could be used for the same purpose; e.g. the *Trapezoidal Fuzzy Numbers (TpFNs)*, which are straightforward generalization of the TFNs (e.g. see [10, 13, 16] etc).

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