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The Mechanism of Information Storage in Open Systems

Zayko Yu. N.
Russian Presidential Academy of National Economy and Public Administration, Stolypin Volga Region Institute of Administration , 410031, Saratov, Sobornaya st, 23/25, Russia

Abstract: This article is devoted to the description of the mechanism of the transformation of negative entropy (negentropy) coming into the system from outside, into information by doubling bits of information originally presented in the system. The present investigation uses the model of one-dimensional chain of bi-stable elements which is connected with the nearest neibours. The bifurcative character of this phenomenon is revealed.
Keywords: Landauer principle; Bi-stable cell; Negentropy; Bifurcation; Heredity; Variability.

1. Introduction

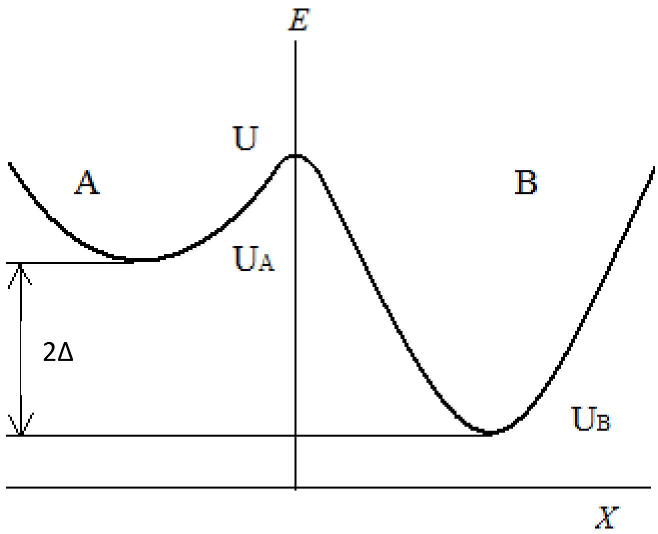
The systems are called as an open systems if they exchange of mass, energy and information with the environment. The open systems' study began in works of I. Prigogine [1-3]. The income of the negative entropy - "negentropy" into the system from outside leads to the lowering of the entropy of the system [4]. Schrödinger [5] showed that this phenomenon is basic in the development of living organisms. There are many sources of negentropy in the environment: sunlight [6], high structural proteins which constitutes the food [5], etc.

However, the mechanism of negentropy transformation into the information is not described until now. This article presents an attempt to improve this situation, based on the model system consisting of the one-dimensional ensemble of bi-stable cells connected with the nearest cell [7]. This approach is based on the works of [8];[9].

2. The Basic Equations of the Model

In Ref. Landauer [9] suggested a model for the description of information processing in arbitrary systems. This model describes the switching cell in the form of an asymmetrical bi-stable potential well for the information degree of freedom (Fig. 1). He considered that process of the information handling goes in the isothermal conditions. This implies that all the other degrees of freedom besides from the informational one play the role of thermostat. In fact, this corresponds to the assumption of an infinite heat capacity of the switching cells [10].

Fig-1. Model of R. Landauer's switching cell. *E* – energy, *X* – information coordinate.



According to Ref. Landauer [9], the statistical ensemble of cells is characterized by the numbers of the ensemble members n_A and n_B in wells A and B, respectively. The behaviors of n_A and n_B are described by the balance equations [9].

$$\frac{dn_A}{dt} = -\nu n_A \cdot \exp\left(-\frac{U-U_A}{kT}\right) + \nu n_B \cdot \exp\left(-\frac{U-U_B}{kT}\right)$$
$$\frac{dn_B}{dt} = \nu n_A \cdot \exp\left(-\frac{U-U_A}{kT}\right) - \nu n_B \cdot \exp\left(-\frac{U-U_B}{kT}\right) \quad (1)$$

Here, t – is the time, T – is the thermostat temperature, ν- is the transition frequency between the ensemble members, U, U_A and U_B – are the energies of the interwell barrier and the lowest energies in different wells, respectively. Difference $\Delta = ½(U_A - U_B)$ represents a half of the energy dissipated during the switching process which is delivered by some controlling force. In the symmetric equilibrium state, which does not carry information $U_A = U_B$ and $n_A = n_B$. When switching which is accompanying the recording of the information is happened then $U_A \neq U_B$ and $n_A \neq n_B$. As a result of switching the system symmetric state relaxes to a new equilibrium distribution

$$n_A = n_B \cdot \exp\left(\frac{U_B - U_A}{kT}\right) \quad (2)$$

as ~ $exp(-\lambda t)$ during time τ, where $\tau^{-1} = \lambda$, with λ being a characteristic value of the equations (1):

$$\lambda = \nu \cdot \exp\left(-\frac{U-U_A}{kT}\right) + \nu \cdot \exp\left(-\frac{U-U_B}{kT}\right) \quad (3)$$

By simple transformations one can show [9], that:

$$\frac{1}{\tau} = \frac{1}{\tau_0} \cdot ch\left(\frac{\Delta}{kT}\right); \frac{1}{\tau_0} = 2\nu \cdot \exp\left(-\frac{U-U_0}{kT}\right); U_0 = \frac{1}{2}\cdot(U_A + U_B) \quad (4)$$

where τ_0 – has a sense of the information lifetime, and τ- is the switching time.

In contrast to Ref. [9], where the consideration was performed in the isothermal approximation, the problem was solved in the adiabatic approximation in Ref. [10], which is closer to the conditions in which real computing devices operate. Therefore, the equations (1) were supplemented by the equation of entropy balance.

Below, we take into account in the equations of the work [10], some adding which describe the information propagation in the z-direction. The initial equations are of the form:

$$\frac{\partial n_A}{\partial t} = -\nu n_A \cdot \exp\left(-\frac{U-U_A}{kT}\right) + \nu n_B \cdot \exp\left(-\frac{U-U_B}{kT}\right) - c\frac{\partial n_A}{\partial z}$$
$$\frac{\partial n_B}{\partial t} = \nu n_A \cdot \exp\left(-\frac{U-U_A}{kT}\right) - \nu n_B \cdot \exp\left(-\frac{U-U_B}{kT}\right) - c\frac{\partial n_B}{\partial z} \quad (5)$$
$$\frac{\partial T}{\partial t} = \frac{2\Delta}{c_H \tau(T)} + \zeta\frac{\partial^2 T}{\partial z^2}$$

Here, T and c_H are the temperature and heat capacity of the cell, respectively, ζ – is the thermal conductivity of the chain [4], c – is the velocity of the propagation of the information in z-direction, and k – is the Boltzmann constant. The last equation (5) describes the transfer of the heat in our system, which we treat as a stationary incompressible fluid. This means that we consider that information transmission rate in the z direction being much larger than the velocity of possible mechanical displacements along z.

The first term on the right-hand side of the last equation (5) describes the local temperature change due to the energy released due to switching of the cell. In essence, it takes into account the principle of Landauer [9], with the modification that we do not assume the isothermal conditions requiring complete removal of the heat from the cell.

Using of the system (5) instead of an explicit description of the connection of adjacent cells, what leads to an infinite system of coupled discrete equations corresponds to the long-wave approximation.

The system (5) is a system of coupled nonlinear equations which can be solved only numerically. Let us introduce new variables:

$$\chi = \frac{T}{T_0}, \alpha = \frac{n_A}{n_A + n_B}, \beta = \frac{n_B}{n_A + n_B}, \tau = \nu t, x = \frac{\nu z}{c};$$
$$A = \frac{U-U_A}{kT_0}, B = \frac{U-U_B}{kT_0}, \mu = \frac{c_H}{k}, \gamma = \frac{\varsigma \nu}{c^2} \quad (6)$$

where T_0 – is the temperature of the chain far from the switching region; and value μ is the number of particles per cell. Equations (5) now looks as follows:

$$\begin{aligned}
&\frac{\partial\alpha}{\partial\tau}+\frac{\partial\alpha}{\partial x}=-\alpha\exp\left(-\frac{A}{\chi}\right)+\beta\exp\left(-\frac{B}{\chi}\right)\\
&\frac{\partial\beta}{\partial\tau}+\frac{\partial\beta}{\partial x}=\alpha\exp\left(-\frac{A}{\chi}\right)-\beta\exp\left(-\frac{B}{\chi}\right)\\
&\frac{\partial\chi}{\partial\tau}=\frac{D}{\mu}\left[\exp\left(-\frac{A}{\chi}\right)+\exp\left(-\frac{B}{\chi}\right)\right]+\gamma\frac{\partial^2\chi}{\partial x^2}
\end{aligned} \tag{7}$$

where D=B-A. Equations (7) have a family of characteristics $x = x_0 - \tau$ (x_0 – is a constant), along which the value $\alpha + \beta$ (or in dimension variables $n_A + n_B$) is conserved. Since through each point of the plane (x, τ) passes exactly one characteristic of that family, the value of $I = \alpha + \beta$ (the Riemann invariant) is a constant throughout said plane provided $I(\tau=0, x)=const=1$.

3. Analysis of the Solutions

For the numeric investigation of the system (7) we have constructed a difference scheme on the grid in the plane (x, τ). Due to the presence of the invariant $I = \alpha + \beta = 1$ the first two equations (7) are reduced to a single equation for α, thus basic equations become as follows

$$\begin{aligned}
&\frac{\partial\alpha}{\partial\tau}+\frac{\partial\alpha}{\partial x}=-\alpha\exp\left(-\frac{A}{\chi}\right)+(1-\alpha)\exp\left(-\frac{B}{\chi}\right)\\
&\frac{\partial\chi}{\partial\tau}=\frac{D}{\mu}\left[\exp\left(-\frac{A}{\chi}\right)+\exp\left(-\frac{B}{\chi}\right)\right]+\gamma\frac{\partial^2\chi}{\partial x^2}
\end{aligned} \tag{8}$$

For the construction of the difference scheme grid the Euler explicit scheme was used [11, 12]. A stability criterion for the problem with the initial data for the heat equation has the form $C_1 < 1$, where $C_1 = 2\gamma\delta_\tau/\delta_x^2$ - is the Courant coefficient [11]; δ_τ and δ_x – are the values of the grid steps in τ and x, respectively. A stability criterion for the wave equation has the form $C_2 = \delta_\tau/\delta_x < 1$[12].

Initial data $\alpha(x, \tau=0)$ was specified in the form of a rectangular pulse of finite length $l = x_2 - x_1$

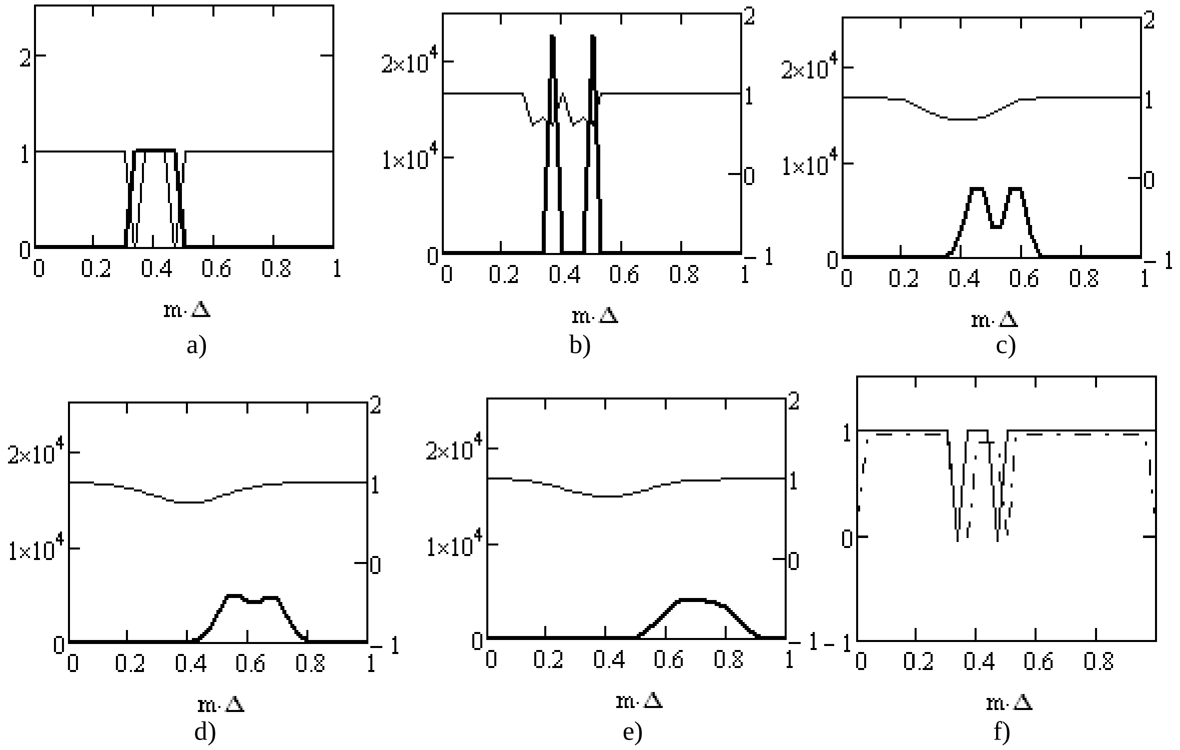
$$\begin{aligned}
&\alpha(x,0)=0, 0<x<x_1, x_2<x<L\\
&\alpha(x,0)=1, x_1<x<x_2
\end{aligned} \tag{9}$$

where L – is the size of the integration in x. Initial data $\chi(x, \tau=0)$ was determined on the basis of equations (8). It follows from the first equation (8) that $\chi(x,0)=-A/\ln(-\alpha_\tau(x,0))$ in the area where $\partial\alpha/\partial x=\beta=0$, while $\chi(x,0)=-B/\ln(\alpha_\tau(x,0))$ in the area where $\partial\alpha/\partial x=\alpha=0$. These relationships help us to imagine the initial temperature distribution χ for a given initial profile α. Let us note that the behavior of $\alpha_\tau(x,0)$ is unknown before a solution is obtained. Therefore it is necessary to assume a plausible behavior of $\alpha_\tau(x,0)$ and compare calculated behavior of $\alpha_\tau(x,0)$ with the assumed one in order to match them.

Note that in the regions where $\alpha(x,0)$ varies quickly (fronts of the pulse, $x = x_{1,2}$) the calculated values of temperature are negative, i.e. $\chi_F=\chi(x=x_{1,2},0) < 0$, which is characteristic for the non-equilibrium systems [4]. This value permits to assess the difference between initial distribution α $(x, 0)$ and the equilibrium one. The solutions of the system (8) for the initial data (9) and the values $\chi_F > -0.01$ were presented in the article [7]. Below are the results of numerical investigation of the system (8) for the smaller values of χ_F. As it turned out the behavior of solutions (8) in this case differs radically from that previously studied in Zayko [7].

Fig. 2 shows typical results of the calculations for the initial values (9) and $\chi_F = -0.07$. They imply that the initial pulse (9) turns in two pulses of shorter duration and greater amplitude, which for some time propagate separately apart from each other and then are merging (Fig. 2, solid line). The temperature profile in accordance with the principle of Landauer and the second rule of thermodynamics seeks from the initial values to the temperature of environment according to the heat conduction equation (thin line in Fig. 2).

Fig-2. The results of the numerical calculations by the Eqs. (8) for the initial values (9). A = 0,1, B = 1,1. Bold solid line – α, thin solid line – χ. The abscissa dimensionless coordinate is $m\Delta$: $\Delta=\delta_x=M^{-1}$ (M=30, $0\le m\le M$), δ_τ=0,02; γ = 0,02, μ = 10^{14} [7]; C_1 = 0,72, C_2 = 0,6; L=1.

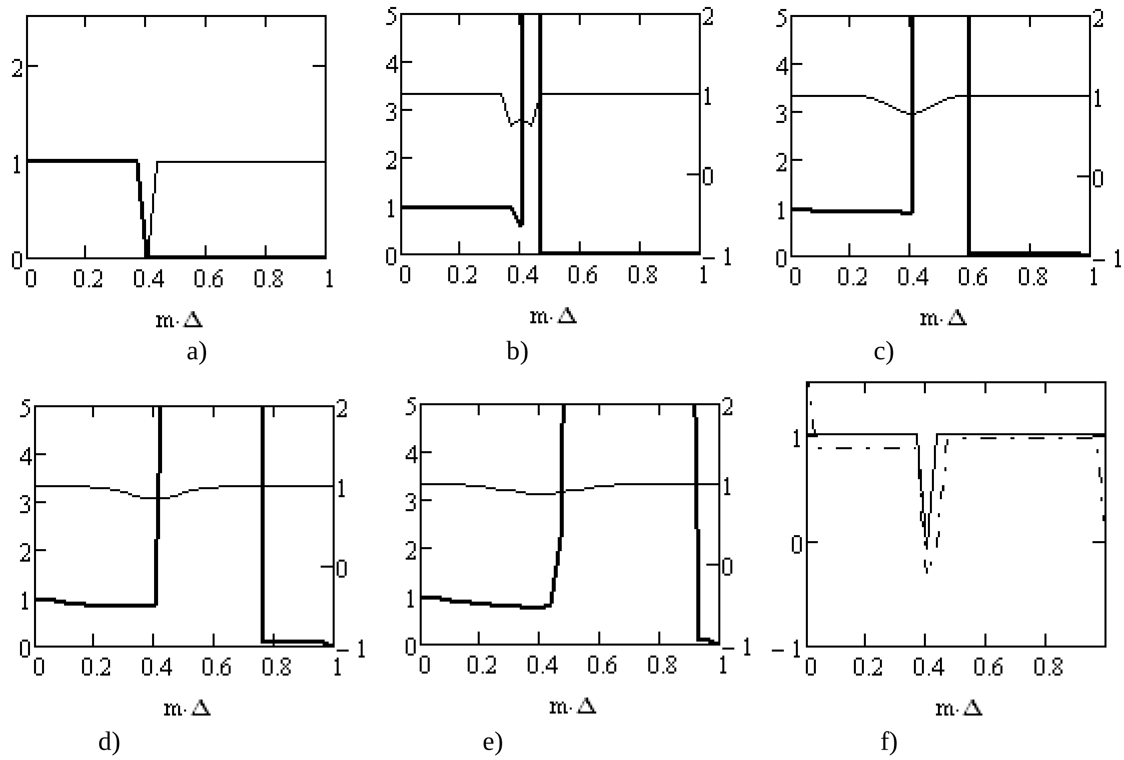


a)-initial profile of α and χ;
b) – e) – behavior of α (left ordinate) and χ (right ordinate) after n steps on τ: b) n=1, c) n=5, d) n=10, e) n=15;
f)- comparison the assumed initial values of χ (solid line) and calculated ones (dot-dash).

Similar results were obtained by numerical analysis of equations (8) for the initial values in the step-like form (10) with χ_F = -0.1(Fig. 3). In this case one pulse occurs near the edge of step, which propagates ahead the edge and becomes broader. The value of the pulse in the maximum is of the same order of magnitude as in the previous case.

$$\alpha(x,0)=1, 0<x<x_1,$$
$$\alpha(x,0)=0, x_1<x<L \qquad (10)$$

Fig-3. The same as in Fig. 3 for the initial values defined by Eqs.(10)



4. Discussion.

The origin of this phenomenon is connected with the bifurcation. To show this we linearize the equation (8). We represent the solution of the Eqs. (8) in the form $\alpha = \alpha_0 + \alpha_1$ and $\chi = \chi_0 + \chi_1$, where $\alpha_1 << \alpha_0$ and $\chi_1 << \chi_0$. Substituting this expansion into the Eqs. (8) and neglecting terms of higher order of smallness, we obtain a system of linear differential equations with respect to α_1 and χ_1. Perform integral Laplace transform [13]

$$\alpha_1(\tau, x) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \alpha_1(\omega) \exp(\omega\tau + ik(\omega)x) d\omega \qquad (11)$$

(k – is a wave number) and similarly for χ_1. The contour of integration in (11) should be traced to the right of all singular points ω_s of $\alpha_1(\omega)$ in the complex plane ω, i.e. $Re(\omega_s) < \sigma$ [13]. Substituting (10) in the system of differential equations for $\alpha_1(\tau,x)$ and $\chi_1(\tau,x)$ we receive a system of linear algebraic equations for $\alpha_1(\omega)$ and $\chi_1(\omega)$. The condition for the solvability of it is the zero value of its determinant. Using this equation, we obtain the dependence $k(\omega. \chi_0)$

$$\gamma k^2 = \frac{D}{\mu \chi_0^2} \left[A \exp\left(-\frac{A}{\chi_0} \right) + B \exp\left(-\frac{B}{\chi_0} \right) \right] - \omega \qquad (12)$$

The points of bifurcation $\chi_0 = \chi_{0B}$ (8) are the branching points of $k(\omega,\chi_0)$, which coincide with zeroes of $k(\omega,\chi_0)$. Value of magnitude of ω in (12) we evaluate from the calculations as $\sim\alpha^{-1}(0,x)\cdot\partial\alpha(0,x)/\partial\tau$. Calculations give the following values: for the pulse (9) $\chi_{0B} \approx$ -0,044÷-0,028 and for the step (10) $\chi_{0B} \approx$ -0,044÷-0,035. The difference stems due to the different places of the assessments.

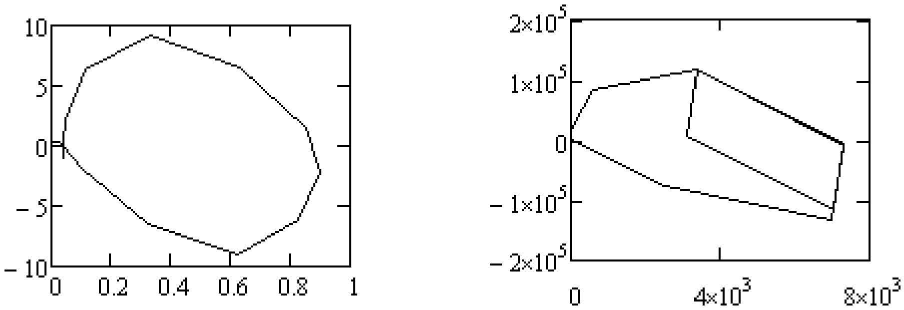
The observed phenomenon looks like the emergence of new bits of information in the system with the income of negentropy. Bifurcation leads to the doubling of the total number of bits in the system.

This mechanism could explain the behavior of different biological systems, both at the cellular and molecular level.

For finding of the solutions of the Eqs. (8) when setting the initial conditions $\chi(x, 0)$ for χ we guided by the Landauer principle, linking a local decrease in the entropy and temperature with the place of localization of bits of information. Note, that in the Ref. [9], these issues are not addressed.

If the income of the negentropy is continued further then doubling of the bits will be continued too as a result of the cascade of doubling bifurcations (Feigenbaum cascade [14], Fig. 4), which under certain conditions leads to chaotic behavior of the system.

Fig-4. Phase portrait of the solution of the Eqs. (8) for initial values of $\alpha(x,0)$ in the pulse form (9).



Abscissa – $\alpha(n\delta_\tau, x)$; ordinate – $\alpha_x(n\delta_\tau, x)$; $n = 5$. Left - $\chi_F = -0.01 > \chi_{B0}$ [7], right - $\chi_F = -0.07 < \chi_{B0}$, χ_{B0} – threshold of bifurcation. The first doubling bifurcation of the period of phase trajectory is seen.

If so, the present model can be used in order to explain the mechanism of the variability so as the explanation of the mechanism of heredity earlier, which is one of the decisive factors of evolution, responsible for the adaptability of organisms to changing of environmental conditions.

5. Conclusions

This work is devoted to the consideration the mechanism of the transformation of negative entropy (negentropy) coming into the system from outside into information with the help of the model of one-dimensional chain of bi-stable elements connected with each other along a certain direction. This phenomenon is connected with the bifurcation of doubling the bits of information in the system and occurs if a level of negative entropy exceeds a certain threshold.

The information in the system is represented in the form of areas (fronts) with a sharp change of $\alpha(x, 0)$, i.e. inverse filling in asymmetric two-well structure. This is known as double data rate (DDR) coding of information [15]. Two types of initial profiles of $\alpha(x, 0)$ are investigated: in the step-like form and in the form of a single rectangular pulse. The first case corresponds to one bit, the second - to two bits which were initially prepared in the system. In both cases, if the negentropy which is measured by negative temperature in fronts' region exceeds in absolute value the certain threshold the character of solutions sharply changes, i.e. the number of fronts, and consequently the number of the information bits is doubling in comparison with the original ones.

Since this treatment does not rely on any specific ways of representing and processing information as well as not associated with a certain type of carriers of information, the results have a wide range of applications. As an example, point out copying of genetic information of DNA. Another example is the process of cell division - mitosis.

In addition to these applications, the present model can be applied to describe the mechanism of variability.

The results obtained in this article impose the doubt in truth of the assumption of the existence a special (biological) form of matter together with gaseous, liquid and solid ones, which is characterized for living organisms [16].

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