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Effective Mathematical Approach of Balancing Chemical Equations in Schools

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Abstract: This study describes a procedure employing Gaussian elimination method in matrix algebra to balance chemical equations from easy to relatively complex chemical reactions. The result shows that 4 atom of Carbon (C), 14 atoms of Oxygen (O), and 12 atoms of Hydrogen (H) each on both the reactants and products makes the chemical equation balance. This result satisfies the law of conservation of matter and confirms that there is no contradiction to the existing way(s) of balancing chemical equations. The practical superiority of the matrix procedure as the most general tool for balancing chemical equations is demonstrable. In other words, the mathematical method given here is applicable for all possible cases in balancing chemical equations.

Keywords: Balanced equation; Conservation of matters; Reactants; Products; Matrixes.

1. Introduction

We live in a time of extraordinary and accelerating change. New knowledge, tools, and ways of solving and communicating mathematics continue to evolve and emerge. The mathematical principles that students need to learn today is not the same as what their parents and grandparents learnt in the past. The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. The mathematical knowledge necessary to succeed in this changing world is tied to what is taught in schools. In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures and lack of it keeps those doors closed. National Council of Teachers of Mathematics (NCTM) challenges the assumption that mathematics is only for a selected few. On the contrary, everyone needs to understand mathematics. All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence.

Nigerian governments like other developing nations have been placing great emphasis on the study of sciences and mathematics over the years with the principal aim of achieving solid foundation in mathematics and other related science subjects. Mathematics is the science of number, quantities and measurement and it is the backbone of most subjects in all levels of educational systems. Researchers have also described mathematics as the 'soul' of science and technology. Bande [1] and Elegbede [2] stated that mathematics is at the centre of the modern world that is, no sciences without mathematics. Lakatos [3] described mathematics as 'the most perfect of all sciences, the 'mother' [4], the 'queen of all sciences' [5], 'a science of its own right' [4].

The balancing of chemical equations can be made much easier, especially for those who find it difficult, by moving the procedures toward the algorithmic and away from the heuristic. That is, a "step to step" procedure is simpler to master than is the haphazard hopping of inspection, even a highly refined inspection [6].

A popular, casual approach of coping with the challenges of balancing chemical-reaction equations is "by inspection" [7]. The standard balancing-by-inspection approach is to make successive, hopefully intelligent guesses at the coefficients that will balance an equation, continuing until balance is achieved. This can be a straightforward, speedy approach for simple equations. But it rapidly becomes both lengthy and requires more skills for complex reactions that involve many reactants and products that requires balancing.

Indeed, more systematic approaches to implementing the balancing-by-inspection method have been published for use with such challenging equations, and these are slightly faster and more reliable than simple inspection/guesswork. They usually recommend identifying either the "least adjustable" coefficients or the coefficients for a set of sub-reactions and balancing them by inspection first, then balancing the undetermined coefficients, algebraically, in a final step. Balancing by inspection does not produce a systematic evaluation of all of the sets of coefficients that would potentially balance an equation; in other words, the technique encourages the

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notion that there is one, and only one, correct solution for any skeletal equation. In fact, there may be no possible solution, one unique solution, or an infinite number of solutions.

Basically, the substances taking part in a chemical reaction are represented by their molecular formulae, and their symbolic representation is termed as chemical equation [8]. Chemical equation therefore is an expression showing symbolic representation of the reactants and the products usually positioned on the left side and on right side in a particular chemical reaction [9]. Unlike mathematical equations, left side and right side of chemical equations are usually separated using a single arrow pointing to the direction of the products for cases of one way reactions which are most times irreversible whereas a double arrow pointing either direction indicating a reversible reaction [10]. Chemical equations play great role in theoretical as well as industrial chemistry. Mass balance of chemical equations as a century old problem is one of the most highly studied topics in chemical education. It always has the biggest interest for the students on every level. The qualitative and quantitative understanding of the chemical process estimating reactants, predicting the nature and amount of products and determining reaction conditions is necessary to balance the chemical equation. Every student which has general chemistry as a subject is bound to come across balancing chemical equations. Actually, balancing the chemical equations provide an excellent demonstrative and pedagogical example of interconnection between stoichiometry principles and linear algebra. The illustration of Chemical equation is shown by example:

Ethane is a gas similar to methane that burns in oxygen to give carborndioxide gas and steam. The steam condenses to form water droplets. The chemical equation for this reaction is

$$C_2H_6 + O_2 \longrightarrow CO_2 + H_2O \qquad \text{[Not Balanced]}$$

A chemical equation is said to be balanced, provided that the number of atoms of each type on the left is same as the number of atoms of the corresponding type on the right [10]. This leads to the concept of stoichiometry which is defined as the quantitative relationship between reactants and products in a chemical equation [11]. In other words, stoichiometry is the proportional relationship between two or more substances during a chemical reaction [12]. The ratio of moles of reactants and products is given by the coefficients in a balanced chemical equation [11]. It is through this that the amount of reactant needed to produce a given quantity of product, or how much of a product is formed from a given quantity of reactant is determined [12].

Linear algebra at present is of growing importance in engineering research, science, frameworks, electrical networks, traffic flow, economics, statistics, technologies, and many others [13]. It forms a foundation of numeric methods and its main instrument is matrices that can hold enormous amounts of data in a form readily accessible by the computer. This study thus illustrates how to construct a homogenous system of equations whose solution provides appropriate value to balance the atoms in the reactants and with those in the products using matrix algebra.

2. Methodology

In this section the researcher will demonstrate some examples using the linear algebra method to solve many chemical equations for their balancing. All chemical equations balanced here appears in many chemistry textbooks and they are chosen with an intention to balance them using matrix algebra by method of Guassian elimination giving us an upper triangular matrix or Echelon matrix.

Example 2.1 Ethane is a gas similar to methane that burns in oxygen to give carborndioxide gas and steam. The steam condenses to form water droplets. The chemical equation for this reaction is

$$C_2H_6 + O_2 \longrightarrow CO_2 + H_2O$$

Balance the equation.

Solution: To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$wC_2H_6 + xO_2 \longrightarrow yCO_2 + zH_2O$$

Next, we compare the number of carbon (C), oxygen (O) and hydrogen (H) atoms of the reactants with the number of the products. We obtain three linear equations;

C: $2w = y$

H: $6w = 2z$

O: $2x = 2y + z$

Rewriting these equations in standard form, we see that we have a homogenous linear system in four unknowns, that is, w x, y and z.

$$\begin{aligned}
&2w + 0x - y + 0z = 0\\
&6w + 0x + 0y - 2z = 0\\
&0w + 2x - 2y - z = 0\\
&0w + 0x + 0y + 0z = 0
\end{aligned}$$

Alternative

$$\begin{aligned} 2w \quad - y \quad &= 0 \\ 6w \quad - 2z &= 0 \\ 2x - 2y - z &= 0 \end{aligned}$$

Writing this equations or system in matrix form, we have the augmented matrix

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Solving using Echelon form:
Keeping the first row constant and interchange second row for third row and third for the second,

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Keeping the first and second row constant and multiplying the first row by 3 and subtract third row from the first row, we obtain the third row as;

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Then the solution is quickly found from the corresponding equations;

$$2w \quad -y \quad = 0 \qquad (1)$$
$$2x - 2y - z = 0 \qquad (2)$$
$$-3y + 2z = 0 \qquad (3)$$

Using elimination method, to eliminate y we have

$$2w \quad -y \quad = 0 \qquad (1) \times 3$$
$$-3y + 2z = 0 \qquad (3) \times 1$$
$$\therefore z = 3w$$

Using reversible equations we obtain,

$$w = 1, z = 3$$

substituting the value of $w$into equation (1)

$$y = 2$$

substituting the value of y and z into equation (2)

$$x = {}^{7}\!/_{2}$$

that is; $w = 1, x = {}^{7}\!/_{2},\ y = 2,\ z = 3$

since we are dealing with atoms, it is convenient to make the unknowns positive whole numbers, hence multiplying through by 2;

$\Rightarrow w = 2, x = 7, y = 4, z = 6$

Corresponding solution using Matlab

>> $[w\ x\ y\ z]$ = solve $('2 * w - y = 0',\ 6 * w - 2 * z = 0',\ '2 * x - 2 * y - z = 0',\ '0 * w + 0 * x + 0 * y = 0')$

$$w = {}^{z}\!/_{3}$$
$$x = \frac{7*z}{6}$$
$$y = \frac{2*z}{3}$$
$$z = z$$

Since z is an arbitrary constant, we choose $z = 6$ then

$$w = 2, x = 7, y = 4$$

To this end, our balanced equation is

$$2C_2H_6 + 7O_2 \longrightarrow 4CO_2 + 6H_2O$$

Example 2.2 Sodium hydroxide (NaOH) reacts with sulfuric acid (H_2SO_4) yields sodium sulfate (Na_2SO_4) and water.

The chemical equation is;

$$NaOH + H_2SO_4 \longrightarrow Na_2SO_4 + H_2O \qquad \text{[Not Balanced]}$$

To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$w NaOH + x H_2SO_4 \longrightarrow y Na_2SO_4 + z H_2O$$

Next, we compare the number of sodium (Na), oxygen (O), hydrogen (H), and sulfur (S) atoms of the reactants with the number of the products. We obtain four linear equations

Na: $w = 2y$
O: $w + 4x = 4y + z$
H: $w + 2x = 2z$
S: $x = y$
It is important to note that we made use of the subscripts because they count the number of atoms of a particular element. Rewriting these equations in standard form, we see that we have a homogenous linear system in four unknowns, that is, w, x, y and z
$w + 0x - 2y + 0z = 0$
$w + 4x - 4y - z = 0$
$w + 2x + 0y - 2z = 0$
$$0w + x - y + 0z = 0$$
Alternatively;
$$\begin{aligned} w \quad\quad - 2y \quad\quad &= 0 \\ w + 4x - 4y - z &= 0 \\ w + 2x \quad\quad - 2z &= 0 \\ x - y \quad\quad &= 0 \end{aligned}$$
Writing this equations or system in matrix form, we have the augmented matrix
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 1 & 4 & -4 & -1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right)$$
Solving using Echelon form:
Keeping the first row constant and subtracting the second row from that,
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 1 & 4 & -4 & -1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right) \mathrm{R_1{-}R_2} \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right)$$
Now, subtracting the third row from the first row, we obtain
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 1 & 2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right) \mathrm{R_1{-}R_3} \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & -2 & -2 & 2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right)$$
Keeping the first and second row constant and multiplying the third row by (-2) and the fourth row by (4), we obtain
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & -2 & -2 & 2 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 4 & 4 & -4 & 0 \\ 0 & 4 & -4 & 0 & 0 \end{array}\right)$$
Now, adding row two and three, we obtain
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 4 & -4 & 0 & 0 \end{array}\right)$$
Adding second and fourth rows, we get
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{array}\right)$$
Multiplying the fourth row by 3, we have
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 0 & -6 & 3 & 0 \end{array}\right)$$
Adding third and fourth rows, then we obtain
$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & -4 & 2 & 1 & 0 \\ 0 & 0 & 6 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$
Then the solution is quickly found from the corresponding equations;
$$w + 0x - 2y + 0z = 0 \qquad (1)$$
$$0w - 4x + 2y + z = 0 \qquad (2)$$
$$0w + 0x + 6y - 3z = 0 \qquad (3)$$

From equation (3)

$$6y - 3z = 0$$

$$6y = 3z$$

$$y = \frac{1}{2}z \qquad (4)$$

Substituting the value of y into equation (1)

$$w - 2y = 0$$

$$w - 2\left(\frac{1}{2}\right)z = 0$$

$$w = z \qquad (5)$$

Putting equation (4) and (5) into (2)

$$-4x + 2y + z = 0$$

$$-4x + 2\left(\frac{1}{2}\right)z + z = 0$$

$$-4x + 2z = 0$$

Multiply through by (-1)

$$4x - 2z = 0$$

$$4x = 2z$$

$$z = 2x \qquad (6)$$

Hence equation (5) becomes

$$w = 2x$$

Now, putting equation (6) into (4)

$$y = \frac{1}{2}(2x)$$

$$y = x$$

But $y = \frac{1}{2}z$

Using reversible equations we obtain,

$$w = z$$

$$x = \frac{1}{2}z$$

$$y = \frac{1}{2}z$$

$\Rightarrow$

$$w + 0x + 0y - z = 0$$

$$0w + x + 0y - \frac{1}{2}z = 0$$

$$0w + 0x + y - \frac{1}{2}z = 0$$

become the reduced linear system or, the reduced row echelon form,

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Hence, since z can be chosen arbitrary and we are dealing with atoms, it is convenient to choose values so that all the unknowns are positive whole integers. One of such choice is z = 2 which yields

$w = 2$, $x = 1$, and $y = 1$

Corresponding solution using Matlab

>> [w x y z] = solve ('w - 2*y = 0', 'w + 4*x - 4*y - z = 0', 'w + 2*x - 2*z = 0', '$x - y$ = 0')

$$w = z$$

$$x = {}^{z}/_{2}$$

$$y = {}^{z}/_{2}$$

$$z = z$$

Since$z$ is an arbitrary constant, let $z = 2$; then we have

$$w = 2, x = 1, y = 1, z = 2$$

In this case our balance equation is

$$2NaOH + H_2SO_4 \longrightarrow Na_2SO_4 + 2H_2O$$

Example 2.3 Consider the chemical equation below;

$$PhCH_3 + KMnO_4 + H_2SO_4 \longrightarrow PhCOOH + K_2SO_4 + MnSO_4 + H_2O$$

Balance the equation.

Solution: To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$tPhCH_3 + uKMnO_4 + vH_2SO_4 \longrightarrow wPhCOOH + xK_2SO_4 + yMnSO_4 + zH_2O$$

Next, we compare the number of Phenyl (Ph), carbon (C), Potassium (K), Manganese (Mn), oxygen (O), sulfur (S) and hydrogen (H) atoms of the reactants with the number of the products. We obtain seven linear equations;

Ph: $t = w$

C: $t = w$

H: $3t + 2v = w + 2z$

K: $u = 2x$

Mn: $u = y$

O: $4u + 4v = 2w + 4x + 4y + z$

S: $v = x + y$

Rewriting these equations in standard form, we see that we have a homogenous linear system in seven unknowns, that is, t, u, v, w, x, y and z.

$$\begin{gathered}
t + 0u + 0v - w + 0x + 0y + 0z = 0 \\
t + 0u + 0v - w + 0x + 0y + 0z = 0 \\
3t + 0u + 2v - w + 0x + 0y - 2z = 0 \\
0t + u + 0v + 0w - 2x + 0y + 0z = 0 \\
0t + u + 0v + 0w + 0x - y + 0z \ = 0 \\
0t + 4u + 4v - 2w - 4x - 4y - z = 0 \\
0t + 0u + v + 0w - x - y + 0z = 0
\end{gathered}$$

Alternatively;

$$\begin{array}{ccccccccccccccc}
t & & & & & - & w & & & & & & & = & 0 \\
3t & & & + & 2v & - & w & & & & & - & 2z & = & 0 \\
& & u & & & & & - & 2x & & & & & = & 0 \\
& & u & & & & & & & - & y & & & = & 0 \\
& & 4u & + & 4v & - & 2w & - & 4x & - & 4y & - & z & = & 0 \\
& & & & v & & & - & x & - & y & & & = & 0
\end{array}$$

Writing this equations or system in matrix form, we have the augmented matrix

$$\left(\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
3 & 0 & 2 & -1 & 0 & 0 & -2 & 0 \\
0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 & 0 & 0
\end{array}\right)$$

Solving using Echelon form:

Keeping the first row constant, we proceed

$$\left(\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
3 & 0 & 2 & -1 & 0 & 0 & -2 & 0 \\
0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 & 0 & 0
\end{array}\right) R_2 = R_2 - R_1 \sim \left(\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 2 & -1 & 0 & 0 & -2 & 0 \\
0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 & 0 & 0
\end{array}\right)$$

$$\left(\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 2 & -1 & 0 & 0 & -2 & 0 \\
0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 & 0 & 0
\end{array}\right) R_3 = R_3 - 3R_1 \sim \left(\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 2 & 0 & 0 & -2 & 0 \\
0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 & 0 & 0
\end{array}\right)$$

Since the second row is zeros, we replace second row by fifth row and dividing third row by 2, interchanging other row in echelon form

$$\left(\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 4 & 4 & -2 & -4 & -4 & -1 & 0
\end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \end{array}\right) R_5 = R_2 - R_5 \sim \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 4 & 4 & -2 & -4 & -4 & -1 & 0 \end{array}\right) R_6 = R_6 - 4R_2 \sim \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 4 & -2 & -4 & 0 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 4 & -2 & -4 & 0 & -1 & 0 \end{array}\right) R_4 = R_4 - R_3 \sim \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 4 & -2 & -4 & 0 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 4 & -2 & -4 & 0 & -1 & 0 \end{array}\right) R_6 = 4R_3 - R_6 \sim \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -4 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -4 & 1 & 0 \end{array}\right) R_6 = 2R_4 - R_6 \sim \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 6 & -3 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 6 & -3 & 0 \end{array}\right) R_6 = R_6 - R_5 \sim \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 & -3 & 0 \end{array}\right)$$

Then the solution is quickly found from the corresponding equations;

$$\begin{aligned} 7y - 3z &= 0 && (1) \\ 2x - y &= 0 && (2) \\ w + x + y - z &= 0 && (3) \\ v - x - y &= 0 && (4) \\ u - y &= 0 && (5) \\ t - w &= 0 && (6) \end{aligned}$$

From equation (1);

$$7y = 3z$$

Using reversible equation, we have

$$y = 3, z = 7$$

Substituting value of z to equation (2), equation (2) becomes,

$$x = \frac{3}{2}$$

Substituting the value of $x, y\ z$ to equation (3), we get

$$w = \frac{5}{2}$$

Substituting the value of x *and* y to equation (4),

$$v = \frac{9}{2}$$

From equation (5) and (6);

$u = y = 3$ and $t = w = \frac{5}{2}$ respectively

$$\therefore t = \frac{5}{2}, u = 3, v = \frac{9}{2}, w = \frac{5}{2}, x = \frac{3}{2}, y = 3, z = 7$$

Hence, since we are dealing with atoms, it is convenient to choose positive whole values so that all the unknowns are positive integers. As such, multiplying all the obtained values by 2, which yields

$$t = 5, u = 6, v = 9, w = 5, x = 3, y = 6 \text{ and } z = 14$$

Corresponding solution using Matlab

>> $[w\ x\ y\ z]$ = solve ('t − w = 0′,′t − w = 0′,′3 ∗ t + 2 ∗ v − w − 2 ∗ z = 0′,′u − 2 ∗ x = 0′,′u − y = 0′,′4 ∗ u + 4 ∗ v − 2 ∗ w − 4 ∗ x − 4 ∗ y − z = 0′,′v − x − y = 0′)

$$t = (5 * z)/14$$
$$u = (3 * z)/7$$
$$v = (9 * z)/14$$
$$w = (5 * z)/14$$
$$x = (3 * z)/14$$
$$y = (3 * z)/7$$
$$z = z$$

Since z is an arbitrary constant, we let $z = 14$, then

$$t = 5, u = 6, v = 9, w = 5, x = 3, y = 6 \text{ and } z = 14$$

To this end, our balanced equation is

$$5PhCH_3 + 6KMnO_4 + 9H_2SO_4 \longrightarrow 5PhCOOH + 3K_2SO_4 + 6MnSO_4 + 14H_2O$$

Example 2.4 Consider the chemical equation below;

$$K_4Fe(CN)_6 + H_2SO_4 + H_2O \longrightarrow K_2SO_4 + FeSO_4 + (NH_4)_2SO_4 + CO$$

Balance the equation.

Solution: To balance this equation, we insert unknowns, multiplying the reactants and the products to get an equation of the form

$$tK_4Fe(CN)_6 + uH_2SO_4 + vH_2O \longrightarrow wK_2SO_4 + xFeSO_4 + y(NH_4)_2SO_4 + zCO$$

Next, we compare the number of potassium (K), iron (Fe), carbon (C), nitrogen (N), hydrogen (H), sulfur (S) and oxygen (O) atoms of the reactants with the number of the products. We obtain seven linear equations;

K: $4t = 2w$
Fe: $t = x$
C: $6t = z$
N: $6t = 2y$
H: $2u + 2v = 8y$
S: $u = w + x + y$
O: $4u + v = 4w + 4x + 4y + z$

Rewriting these equations in standard form, we see that we have a homogenous linear system in seven unknowns, that is, $t, u, v, w, x, y \text{ and } z$.

$$\begin{aligned}
4t + 0u + 0v - 2w + 0x + 0y + 0z &= 0\\
t + 0u + 0v + 0w - x + 0y + 0z &= 0\\
6t + 0u + 0v + 0w + 0x + 0y - z &= 0\\
6t + 0u + 0v + 0w + 0x - 2y + 0z &= 0\\
0t + 2u + 2v + 0w + 0x - 8y + 0z &= 0\\
0t + u + 0v - w - x - y + 0z &= 0\\
0t + 4u + v - 4w - 4x - 4y - z &= 0
\end{aligned}$$

Alternatively;

$$\begin{array}{lllllllll}
4t & & & -2w & & & & = 0\\
t & & & & -x & & & = 0\\
6t & & & & & & -z & = 0\\
6t & & & & & -2y & & = 0\\
 & 2u & +2v & & & -8y & & = 0\\
 & u & & -w & -x & -y & & = 0\\
 & 4u & +v & -4w & -4x & -4y & -z & = 0
\end{array}$$

Writing this equations or system in matrix form, we have the augmented matrix

$$\left(\begin{array}{ccccccc|c}
4 & 0 & 0 & -2 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\
6 & 0 & 0 & 0 & 0 & 0 & -1 & 0\\
6 & 0 & 0 & 0 & 0 & -2 & 0 & 0\\
0 & 2 & 2 & 0 & 0 & -8 & 0 & 0\\
0 & 1 & 0 & -1 & -1 & -1 & 0 & 0\\
0 & 4 & 1 & -4 & -4 & -4 & -1 & 0
\end{array}\right)$$

Solving using Echelon form:

Dividing the first, fourth and fifth row by 2, we get;

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right) R_2 = 2R_2 - R_1 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right) R_3 = 3R_1 - R_3 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right) R_4 = 3R_1 - 2R_4 \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right)$$

Interchanging rows in echelon form and keeping first and seventh rows constant, we obtain

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right) R_3 = R_2 - R_3 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 4 & 1 & -4 & -4 & -4 & -1 & 0 \end{array}\right) R_7 = 4R_2 - R_7 \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 4 & 4 & 12 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 4 & 4 & 12 & 1 & 0 \end{array}\right) R_7 = R_7 - 3R_3 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 21 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 21 & -1 & 0 \end{array}\right) R_5 = R_5 + 3R_4 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 21 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 21 & -1 & 0 \end{array}\right) R_6 = R_6 + 3R_4 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 21 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 21 & -1 & 0 \end{array}\right) R_7 = R_7 - R_4 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 21 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 21 & -1 & 0 \end{array}\right) R_6 = R_6 - R_5 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 21 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 21 & -1 & 0 \end{array}\right) R_7 = R_5 + 2R_7 \sim \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 42 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 42 & -1 & 0 \end{array}\right) R_7 = R_7 - 21R_6 \left(\begin{array}{ccccccc|c} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 \end{array}\right)$$

Since we have the reduced row echelon matrix, then the solution is quickly found from the corresponding equations;

$$20z = 0$$

But $z \neq 0$, because energy cannot be created or destroyed in a chemical reaction

$2y - z = 0$ (1)

$-6x + z = 0$ (2)

$w - 2x = 0$ (3)

$v + w + x - 3y = 0$ (4)

$u + v - 4y = 0$ (5)

$2t - w = 0$ (6)

From equation (1), we have

$$2y = z$$

Using reversible equation, we have

$$y = 1, z = 2$$

Substituting the value of z into equation (2), we get

$$x = \frac{1}{3}$$

Putting the value of x into equation (3), we obtain,

$$w = \frac{2}{3}$$

Substituting the value of $w, x, and\ y$ to equation (4),

$$v = 2$$

Now, substituting the value of v and y to equation (5);

$$u = 2$$

Putting the value of w to equation (6);

$$t = \frac{1}{3}$$

$\therefore t = \frac{1}{3}, u = 2, v = 2, w = \frac{2}{3}, x = \frac{1}{3}, y = 1$ and $z = 2$

Hence, since we are dealing with atoms, it is convenient to choose positive whole values so that all the unknowns are positive integers. One of such is done by multiplying all the obtained values by 3, which yields

$$t = 1, u = 6, v = 6, w = 2, x = 1, y = 3\ and\ z = 6$$

Corresponding solution using Matlab

>> $[t\ u\ v\ w\ x\ y\ z]$ = Solve $('4*t-2*w=0','t-x=0','6*t-z=0','6*t-2*y=0','2*u+2*v-8*y=0','u-w-x-y=0','4*u+v-4*w-4*x-4*y-z=0')$

$$t = z/6$$
$$u = z$$
$$v = z$$
$$w = z/3$$
$$x = z/6$$
$$y = z/2$$
$$z = z$$

Since z is an arbitrary constant, let $z = 6$; then we have

$t = 1, u = 6, v = 6, w = 2, x = 1, y = 3 \text{ and } z = 6$
In this case, our balance equation is

$$K_4Fe(CN)_6 + 6H_2SO_4 + 6H_2O \longrightarrow 2K_2SO_4 + FeSO_4 + 3(NH_4)_2SO_4 + 6CO$$

3. Discussion

The use of Gaussian elimination in solving these linear equations has been considered as a standard elimination method for solving linear systems that proceeds systematically, irrespective of the particular features of the coefficients. It is a method of great practical importance and is reasonable with respect to computing time and storage demand. The idea of this process is to reduce a system of equations by certain legitimate and reversible algebraic operations (called "elementary operations") to a form in which we can easily see what the solutions to the system are, if there are any. Specifically, we want to get the system in a form where the first equation involves all the variables, the second equation involve all but the first, and so forth. Then it will be simple to solve for each variable one at a time, starting with the last equation, which will involve only the last variable (as shown in example 2.4).

The major goal of Gaussian elimination is to make the upper-left corner element a leading coefficient and use the elementary row operations to get zeros in all positions underneath the leading coefficient. Ultimately, you eliminate all variables in the last row except for one (two as the case may be as shown in example 2.3 below), all variables except for two (three as shown in the last two example 2.3) equation above that one, and so on and so forth to the top equation which has all the variables. Then use reversible equation (backward substitution) to solve for one variable at a time by plugging the values you got into the equations from the bottom to the top.

You can perform three operations on matrices in order to eliminate variables in a system of linear equations;

i. You can multiply any row by a constant (other than zero)
ii. You can switch any row for another
iii. You can add or subtract two rows together.

You can even perform more than one operation. You can multiply a row by a constant and then add it to (subtract it from) another row to change that row. Then it will be simple to solve for each variable one at a time, starting with the last equation, which will involve only the last variable. In a nutshell, this is Gaussian elimination.

Gaussian elimination is probably the best method for solving system of equations if you don't have a computer program or a graphing calculator to help you sole them.

4. Summary

In a balanced equation, coefficients specify the number of molecules (or formula units) of each element involved. The coefficients must satisfy Dalton's (Conservation of mass) requirement that atoms are not created or destroyed in a chemical reaction. There is no fixed procedure for balancing an equation. Although a trial-and-error approach is generally used in classrooms, a systematic algebraic approach is a principle of possibility that often works.

This procedure seems to substantially facilitate the balancing of equations that, traditionally, have been considered difficult for many students. It is interesting that the more difficult the equation, the greater this facilitation appears to be. This procedure allows average students and below average students to experience ready success in balancing, thus avoiding a traditional source of frustration and failure which might contribute to their losing interest in chemistry. One interesting serendipity of this procedure is how quickly it turns able pre-matrix students into extremely fast and accurate balancers.

The immediate importance of the procedure lies in the fact that it can remove the heuristic wall of haphazard inspection, replacing it with a near algorithmic procedure that virtually assures balancing success for average students and below average students. Also, it gives able students an unusual facility. A significant, but less immediate, advantage is the preparation the procedure could offer for future matrix techniques.

5. Conclusion

This allows average, and even low achieving students, a real chance at success. It can remove what is often a source of frustration and failure that turns students away from chemistry. Also, it allows the high achieving to become very fast and very accurate even with relatively difficult equations. A balancing technique based on augmented-matrix protocols was described in this work. Because of its unusual nature, it was best explained through demonstration in the methodology.

The practical superiority of the matrix procedure as the most general tool for balancing chemical equations is demonstrable. In other words, the mathematical method given here is applicable for all possible cases in balancing chemical equations.

References

[1] Bande, T., 2004. "Mathematicians to cooperate with stake holders." *Vanguard Newspaper Thursday, September 2; 11*,

[2] Elegbede, A., 2004. *Students' attitudes to mathematics worries government*. Lagos: The Punch Newspaper, 17(19043), Monday, March 22; 10.

[3] Lakatos, I., 1986. *A Renaissance of Empiricism in the recent philosophy of mathematics? In: T.Tymoczko (Ed.), New directions in the philosophy of mathematics*. Boston: Birkhauser. p. 31.
[4] Mura, R., 1995. "Images of mathematics held by University Teachers of Mathematics education." *Educational Studies in Mathematics,* vol. 28, pp. 33-390.
[5] McGuinnis, J. R., Shama, G., McDuffie, A., Huntley, M. A., King, K., and Watanabe, T., 1996. "Researching the preparation of Specialized mathematics and science upper elementary/middle-level teacher: The 2nd year report." In *Proceedings of the Annual Conference of the National Science Teachers Association.*St. Louis, Missouri. p. 17.
[6] Hutchings, L., Peterson, L., and Almasude, A., 2007. *The Journal of Mathematics and Science, Collaborative Explorations,* vol. 9, pp. 119-133.
[7] Tóth, Z., 1997. "Balancing of chemical equations by inspection." *Journal of Chemical Education,* vol. 74, pp. 1363-11364.
[8] Rao, C. N. R., 2007. *University general chemistry: An introduction to chemistry science*: Rajiv Beri for Macmillan India Ltd. pp. 17-41.
[9] Risteski, I. B., 2009. "A new singular matrix method for balancing chemical equations and their stability." *Journal of the Chinese Chemical Society,* vol. 56, pp. 65-79.
[10] Lay, D. C., 2006. "Linear algebra and its applications." *Pearson Education,* pp. 17-120.
[11] Hill, J. W., Mecreary, T. W., and Kolb, D. K., 2000. *Chemistry for changing times*: Pearson Education Inc. pp. 123-143.
[12] Myers, R. T., Oldham, K. B., and Tocci, S., 2006. "Holt florida chemistry." *Holt, Rinehart and Winston,* vol. 302, pp. 260-287.
[13] Clugston, M. and Flemming, R., 2002. *Advanced chemistry*: Oxford University Press.