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New Way for Solving Naiver - Stokes Equation

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Abstract
This paper gives a new way for solving Navier - Stokes equation (N - S equation for short in following) by using following steps: Step 1, Sets up Wind - Pressure/density equation (1), suits for points in air ($n_a \in A$=∪ n_a) and solves solutions u_a and p_a/ρ_a. (1 - s). Step 2. Sets up Wave equation, described by N - S equation (2), suits for points in sea ($n_s \in S$=∪ n_s), and solutions u_s and p_s/ρ_s. (2 - s) (unknown) Step 3. Sets up Wind - Wave equation (3), by (3) = (1) ∩ (2), suits for points in boundary between air and sea. ($n_{as} \in A \cap$ S) and their solutions: u_a, u_s and p_a/ρ_a , $p_s/p_a/\rho_a$, .(3 - s). Step 4. Sets up boundary conditions. by For differential form, $u_a = u_s$ and $p_a/p_a = p_s/p_a$. (4 - s1). For integral form. The wind dynamic energy = Potential energy of the high of sea. (4 - s2). Now, we have : the solutions of (2- s) is solved by (3 - s) and by (4 - s1), i.e., $u_a = u_s$ and $p_a/\rho_a =$ p_s/ρ_s. Or (1) = (2) . Then (2) is solved.
Keywords: Wind - pressure/density equation; Wave equation; N - S equation; Wind - wave equation.

1. Introduction
Recalled of the Millennium prize problem of N - S equation , offering by Clay Mathematics Institute [1].

"This is the equation which governs the flou of fluids such as water and air. However, there is no proof for the **most basic equations**. One can ask : do **solutions exist,** and are they **unique** ? Why ask for a proof? Because a proof gives not only certitude, but also understanding.".

From this statement, we know that the proof pf existence and unique of solution for **most basic equations (**or **general form)** N - S equation has not been solved yet.

So that, we ask:

1, the range of method for proof.

2, Does the solution of "existence and unique of the general form N - S equation" exist ?

3, The example of proof special (modified) form of N - S equation.

We discus the first asking. The type of method. for proof.

There are two kinds of methods for proof. One belongs to the "**certain type**", which gives the proof by continuous results like differential equation(s), integral equation(s), etc. The other belongs to **"non-certain type**"..Where the problem involves random, statistic, probability, etc., with numerical numerical results.Observably, The proof for existence and unique of N - S equation must be **certain type**.

For the second and third asking, we will discus in **§ 3.**

In this paper, we discuss the following problems:

2. Wind - Pressure/Density Equation
Since the "wind - pressure/density equation"play an important role of our method, therefore, we introduce it here.

To set up and solve [2, 3] the"wind - pressure/density equation", at first, we recalled the"Wind Speed equation" [2, 3] is also called the"Wind - Temperature equation"for convenient. Since the"wind speed equation " is the base of wind description of certain type [4].

$$\partial u/\partial t + g = k(\partial T/\partial s). \qquad \text{(1.2-13) of [2]}$$

$$\partial u/\partial t = k(\partial T/\partial s), \qquad \text{(1.2-13}_a\text{) of [2]}$$

Where u = u(s, t) is the wind speed, T is the temperature, t is time, s = xI + yJ + zK is the displacement, I. J, K are unit vectors in Cartesian co-ordinates. g is gravity acceleration. k is a constant. (1.2-13) of [2] is called approximate "wind speed equation" of a point in air. ($1.2\text{-}13_a$) of [2] is called the exact"wind speed equation".

The"wind speed equation"(1.2-13) of Tian-Quan [2] is also called the"wind - temperature equation" ("w -T equation" in short).

$\dot{\mathbf{u}}$ = (c/m)∇.T (2-1)

(2 -1) is called the "w - T equation" . Where $\dot{u}$ = ∂ u/ ∂ t, u = u(s, t) is the wind speed, t id time, c is a constant, m is mass, k = (c/m). ∇ is called the ("Hamiltan, del, etc.") operator",

$\nabla = \frac{\partial}{\partial x}I + \frac{\partial}{\partial y}J + \frac{\partial}{\partial z}K$, (2-2)

i, j, k are unit vectors in Cartesian co-ordinates. T is the absolute temperature.

(2 - 1) states that the derivatives of wind speed u respective to time t proportions to the derives of temperature T respective to space s (track).

The existence and solution of (2-1) has been proved in Tian-Quan [3].

The solution of (2 - 1) in component form are:

$\dot{u}_x(x, t) = \exp[k(x - t^2)] = k(\partial T_x/\partial x)$, (2-3)

$\dot{u}_y(y, t) = \exp[k(y - t^2)] = k(\partial T_y/\partial y)$, (2-4)

$\dot{u}_z(z, t) = \exp[k(z - t^2)] = k(\partial T_z/\partial z)$, (2-5)

The solution of (2-1) in vector form is:

$\dot{u} = \exp[k(s - t^2)] = k\nabla T$, (2-6)

Now, we turn to "Wind – Pressure/Density Equation"("w– p/ρ Equation" in short)

Using B - C law, we can get "w - p/ρ Eq." from "w - T equation" directly.

By B - C law (combination of Boyle's law and Charles law), we have

pv = cT, (2-7)

Where p = p(s, t) is a (unknown function) stress tensor, with equal compressive normal stress an zero shearing stress.

$$P = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} = p\begin{bmatrix} I \\ J \\ K \end{bmatrix} = p[I + J + K], \quad (2\text{-}8)$$

$p = \sigma_{xx} = \sigma_{yy} = \sigma_{zz}$,

$$\nabla P = \left(\frac{\partial}{\partial x}I + \frac{\partial}{\partial y}J + \frac{\partial}{\partial z}K\right) p[I + J + K] = \frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z},$$

(2-9)

v = volume of a point in air. v = dxdydz. (2-10)

Multiplying both sides of (2-7) by ∇, and use (2-9), we have

∇pv = c ∇T, (2-11)

Dividing both sides of (2-11) by m (point mass), we have

$\nabla p/(m/v) = \nabla p/\rho_{air}$ = (c/m) ∇T = k ∇T, (2-12)

Where $\rho_{air} = m/v$. k = c/m.

Substituting (2-12) into (2-1), we have

$\mathring{u} = k\nabla T = \nabla p/\rho_{air}$, (2-13)

(3-13) is called the "Wind - Pressure/Density Equation" ("w - p/ρ equation." in short). It is a vector PDE. It states that ů is in proportion ro $\nabla(p/\rho_{air})$, or the derivative of wind u speed respect to time proportions to the derivative of p/ρ respect to track (space). Where u and p/ρ_{air} are two parts of unknown variables to be determined.

Solution of "w – p/ρ equation" (2-13).

The solution of scalar form of (2-13) can be obtained from (2-3), (2-4) (2-5)and (2-6) by changing k = c/m to $k = \rho_{air}^{-1} = 1/\rho_{ar}$, i.e.,

$\dot{u}_x(s,t) = \exp[k(x - t^2)] = k(\partial p_x/\partial x)$, $(k = \rho_{air}^{-1})$ (2-14)

$\dot{u}_y(s,t) = \exp[k(y - t^2)] = k(\partial p_y/\partial y)$, $(k = \rho_{air}^{-1})$ (2-15)

$\dot{u}_z(s,t) = \exp[k(z - t^2)] = k(\partial p_z/\partial z)$, $(k = \rho_{air}^{-1})$ (2-16)

The solution of vector form of (2-13) is:

$\dot{u}(s,t) = \exp[k(s - t^2)] = k\nabla p(s,t)/\rho_{air}$, $(k = \rho_{air}^{-1})$ (2-17)

(2-17) shows that two parts of u and $p(s, t)/\rho_{air}$ are solved, i.e., they are known variables.

The calculation of ρ_{air} is discussed in § 7 Appendix. □

3. The Wave Equation describing by the General form of N - S Equation

The "Wave Equation" , described by the general form of N - S equation shown as follows [1]:

$\tau_{sea}\left(\frac{\partial \boldsymbol{w}}{\partial t} + \boldsymbol{w}\nabla\boldsymbol{w}\right) = -\nabla p_{sea} + \nabla.T_s + \boldsymbol{f}$. (3-1)

Where **w** is the flow velocity; τ_{sea} is the fluid density; p_{sea} is the pressure; $.T_s$ is the (viscosity) stress tensor; **f** represents the body force per unit volume.

3.1. Proof of Non-Existence of Solution of General form of N - S *Equation*

To prove the existence of solutions and unique of (3-1) with unknown variables w, p_{sea}, τ_{sea}, T_{sea}, etc. is very difficulty. event impossible.

In fact, we have proved that to prove the existence and unique of solutions of general form is impossible. by paper [5] published in (2020).

Non-Existence of Solution of Rotation Floe in N - S Equs [5]

Here, we simply recalled this paper Using three steps we prove the non-existence of the general form of N - S equation (3-1).

Step1. To prove the non-existence of a special form "Rotation flow in N - S equation, instead of to prove the existence of solutions of the general form of N - S equation. by decomposition method.

The decomposition method is a method for lowering down high order D.E. (Differential Equation) by Eigen value equation. and has been used to solve dimpling and buckling of spherical crust[6]. Where the Eigen value equation with ∇ operator have been solved.

The Eigen value problem related to D.E. and vibration problems have been systemically studied in literature [7].

Step 2. Construct an Eigen value equation.

How to select special case?

The special case should be simple to solve and has not been solved yet.

Here, we chose the Eigen value equation (3-2)

$(\nabla\otimes V)\otimes V = -\lambda^2 V,$ (3-2)

Where V is the velocity of Rotation flow; ∇ is an operator (see (2-2));

$$(\mathbf{\nabla\otimes V}) = \mathrm{rot}\,\mathbf{V} = \mathbf{D} = \begin{vmatrix} I & J & K \\ \partial_x & \partial_y & \partial_z \\ V_x & V_y & V_z \end{vmatrix} = (\partial_y V_z - \partial_z V_y)\mathbf{I} - (\partial_x V_z - \partial_z V_x)\mathbf{J} + (\partial_x V_y - \partial_y V_x)\mathbf{K}, \quad (3\text{-}3)$$

(3-3) represents rot V (called vorticity); λ is the Eigen value; ⊗ is the outer product of vector; $\partial_x = \partial/\partial x, \partial_y = \partial/\partial y, \partial_z = \partial/\partial z$.

D is a determinant with three rows and three columns.

What is the relation between (3-1) and (3-2)?

(3-1) is the general form of N - S equation, (3-2) is a special form of N - S equation.

The right hand side of (3-1) represents the forced terms of a non-homogeneous equation. If we choose

$\frac{\partial \mathbf{w}}{\partial t} + \mathbf{W\nabla W} = 0,$ (3-4)

and make

$\mathbf{w} = \mathbf{rot\,V} = (\mathbf{\nabla \times V}),$ (3-5)

and neglect the first term $\frac{\partial w}{\partial t}$ (w independent to time t), then (3-3), or (3-1) becomes to (3-2) with ($\lambda \neq 0$).

Which means that (3-2) is a special form of general form (3-1).

Step 3. To prove that (3-2) only have zero solution, λ = 0. (V = 0).

Cho0sing a vector **U,** such that

$V\otimes U = \mathbf{1},$ (**1** -- unit vector) (3-6)

i.e.,

$$\mathbf{V\otimes U} = \begin{vmatrix} \mathbf{I} & \mathbf{J} & \mathbf{K} \\ V_x & V_y & V_z \\ U_x & U_y & U_z \end{vmatrix} = (V_yU_z - V_zU_y)\mathbf{I} - (V_xU_z - V_zU_x)\mathbf{J} + (V_xU_y - V_yU_x)\mathbf{K}, \quad (3\text{-}7)$$

Multiplying outer product ⊗ to both sides of (3-2), we have

$$(\mathbf{\nabla\otimes V})\otimes(\mathbf{V\otimes U}) = (\mathbf{\nabla\otimes V})\otimes\mathbf{1} = \mathbf{D}\otimes\mathbf{1} = \begin{vmatrix} \mathbf{I} & \mathbf{J} & \mathbf{K} \\ D_x & D_y & D_z \\ \mathbf{I} & \mathbf{J} & \mathbf{K} \end{vmatrix} = 0 = -\lambda^2(\mathbf{V\otimes U}) = -\lambda^2\mathbf{1}, \quad (3\text{-}8)$$

Where two rows are the same in determinant, therefore,

$\lambda^2 = 0$, and $\mathbf{V} = \mathbf{0}.$ (3-9) ☐

3.2. Example of Existence of Solution of Special form of N - S Equation

"Motion Equation and Solution of Mushroom Cloud"[8]

In Kamke [7], the PDE of mushroom cloud is set up by N - S equation (3-1). In order to solve the PDE, a modified N - S equation is made. Which can be solved and the solution can be checked by visible of mushroom cloud, and compared with similar solution of "wind speed equation of a point in air" (the motion of air is in-visible).

Here, we simply introduce [8].

3.2.1. Derive the PDE

model -- Using model-in-model, to describe the motion of hot air surrounding by cold air.

Based on scientific laws: Boyle's law, Shares law, Einstein's mass-energy equivalence, gravity - buoyancy field, Principle of Minimum Energy Release, Principle of Reciprocal Displacement.

Green formula, Newton second law. Heat and other energy.

Set up PDE by simplifying the general form of N - S equation (with 6 unknown variables to 2 variables, linked with (2-1)).

$\frac{\partial \mathbf{u}}{\partial t} = k\mathbf{\nabla_c}T + \mathbf{g},$ (3-10)

Where **u** is the flow velocity, k is a constant, T is the temperature ($T = T_h/T_c$, T_h - hot air temperature, T_c - cold air temperature),

g is the gravity acceleration. ∇_c is the cylindrical coordinates' del operator, for z-axis-symmetry, ∇_c is:

$\mathbf{\nabla_c} = \mathbf{e_r}\frac{\partial}{\partial r} + \mathbf{e_\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \mathbf{k}\frac{\partial}{\partial z} = \mathbf{e_r}\frac{\partial}{\partial r} + \mathbf{0} + k\frac{\partial}{\partial z},$ (3-11)

3.2.2. Solution of Vector PDE

The solution of vector PDE is complicated. Usually, its component form (scalar function) is easier solved. Here, the component form of vector PDE is solved by method of separating variables.

At first, simplified (3-10), let

$\mathbf{u_1}(s,t) = \mathbf{u}(s,t) + \mathbf{g}t,$ (3-12)

Suppose that the variables $s = s(r,z)$, $\mathbf{u}_1(s,t)$, and $T = T(s,t)$ can be separated. Let

$\mathbf{u_1}(s,t) = a(s)b(t),$ (3-13)

$T(s,t) = c(s)f(t),$ (3-14)

Substituting (3-12) - (3-14) into (3-10), we have

$\frac{db(t)}{dt}a(s) = k\left[\frac{\partial c(s)}{\partial r} + \frac{\partial c(s)}{\mathrm{C}z}\right]f(t),$ (3-15)

Separating variables, we have

$\frac{db(t)}{f(t)dt} = k\frac{\left[\frac{\partial c(s)}{\partial r}+\frac{\partial c(s)}{\partial z}\right]}{a(s)},$ (3-16)

In order to make the solution simple, let

$b(t) = f(t),\ a(s) = c(s),$ (3-17)

then, (3-16) becomes:

$\frac{db(t)}{b(t)dt} = k\frac{\nabla_c a(s)}{a(s)} = \text{const},$ (3-18)

If (3-18) holds, then, each sides must be a constant, i.e.,

$\frac{db(t)}{b(t)} = A_1 dt,\ \ (A_1 = \text{const})$ (3-19)

$k\nabla_c a(s)/a(s) = B, (B = \text{const})$ (3-20)

Or

$\frac{\partial a(s)}{a(s)\partial r} + \frac{\partial a(s)}{a(s)\,\partial z} = \frac{B}{k},$ (3-21)

Separating (3-21) into two parts we have

$\frac{\partial a(s)}{a(s)\partial r} = \frac{B_1}{k},$ (3-22)

$\frac{\partial a(s)}{a(s)\,\partial z} = \frac{B_2}{k},$ (3-23)

Integrating both hands of (3-19), (3-22) and (3-23), we have

$b(t) = A\exp[A_1 t],$ (3-24)

$a(s) = A_2\exp[B_1 r + B_2 z],$ (3-25)

Substituting (3-24), (3-25), (3-17) and (3-12) into (3-19), we jave

$u(s,t) = AA_2\exp\,[B_1 r + B_2 z + A_1 t] - gt,$ (3-26)

Substituting (3-24), (3-25), (3-17) into (3-14), we have

$T(s,t) = AA_2 k\,\exp[B_1 r\, B_2 z + A_1 t],$ (3-27)

The solution $u(s,t)$, and $T(s,t)$ are solved. ☐

3.3. Simplified N - S Equation in Cartesian Coordinates

(3-10) is the simplified M - S equation in cylindrical coordinates, just suited for case of Tian-quan [8]. In order to link the case of (2-1), we need to solve the "wave equation" in Cartesian co-ordinates.

At first, we simplify (3-1) by the assumption that the sea is an uniformly, isotropic fluid. then, $\nabla.T_s = 0$, since the∇ applied to uniformly fluid is zero.

Secondly, we use an equivalent flow velocity É to represent the complex flow velocity, i.e.,

$\frac{\partial \mathbf{E}}{\partial t} = \acute{E} = \acute{E}(\mathbf{s},t) = (\frac{\partial \boldsymbol{w}}{\partial t} + \boldsymbol{w}\nabla\boldsymbol{w}),$ (3-28)

Then, (3-1) becomes

$\acute{E} = -\nabla p_{sea}/\tau_{sea} + \mathbf{f}.$ (3-29)

$\acute{E} = \nabla p_{sea}/\tau_{sea},$ (3-30)

Where the term **f** in (3-1) is neglected for approximately.,

Now, (3-30) is called the simplified "Wave Equation". It is a vector PDE. It states that the derivative of wave speed respect to time is proportional to the gradient of pressure/density respect to track (space), i.e., $\acute{E} = \nabla p_{sea}/\tau_{sea}$.

3.3.1. The Solution of Wave Equation (3-30)

Suppose that the variables of $\tau_{sea}\mathbf{E}$ and p_{sea} can be separated, i.e.,

$\mathbf{E} = \mathbf{E}(\mathbf{s}, t) = a(\mathbf{s})b(t),$ **(3-31)**

$p_{sea} = p_{sea}(\mathbf{s}, t) = c(\mathbf{s})d(t),$ (3-32)

Substituting (3-31), (3-32) into (3-30), we have:

$a(s)\dot{b}(t) = d(t)\nabla c(s),$ (3-33)

Moving variables to the same sides, (3-33) becomes:

$\frac{\dot{b}(t)}{d(t)} = \frac{\nabla c(s)}{a(s)} = R = constant,$ (3-34)

For simple, choose

$a(s) = c(s),\ \ b(t) = -\,d(t),$ (3-35)

Substituting（3-35) into (3-34), we have

$\frac{\dot{b}(t)}{b(t)} = -R,$ (3-36)

$\frac{\nabla c(\mathbf{s})}{c(\mathbf{s})} = R,$ (3-37)

Integrating both sides of (3-36) respect to t, we have

$\int \frac{\dot{b}(t)}{b(t)} dt = \ln b(t) = -\int R\, dt = -R\, t = R\, t^{-1},$ (3-38)

$b(t) = \mathrm{Exp}[Rt^{-1}],$ (3-39)

Integrating both sides of (3-37) respect to **s**, we have

$\int \frac{\nabla c(\mathbf{s})}{c(\mathbf{s})} d\mathbf{s} = \int \frac{\frac{\partial}{\partial x}c(\mathbf{s})\mathbf{I}+\frac{\partial}{\partial y}c(\mathbf{s})\mathbf{J}+\frac{\partial}{\partial z}c(\mathbf{s})\mathbf{K}}{c(\mathbf{s})} d(\mathbf{s}) = \int [\frac{\dot{c}(x)}{c(x)} + \frac{\dot{c}(y)}{c(y)} + \frac{\dot{c}(z)}{c(z)}]\, d(x\mathbf{I} + y\mathbf{J} + z\mathbf{K})) = \ln c(x)\mathbf{I} + \ln c(y)\mathbf{J} + \ln c(z)\mathbf{K} = \ln c(\mathbf{s}) = \int R\, d(\mathbf{s}) = R\mathbf{s},$ (3-40)

$c(\mathbf{s}) = \mathrm{Exp}[R\mathbf{s}],$ (3-41)

Substituting (3-39), (3-41) into (3-31), (3-35), we have

$\acute{E} = \frac{\partial}{\partial t}\mathbf{E} = \mathrm{Exp}[R(\mathbf{s} - t^2)]$, (3-42)

Substituting (3-42) into (3-30), we have

$\acute{E} = \mathrm{Exp}[R(\mathbf{s} - t^2)] = \nabla p_{sea}/\tau_{sea},$ (3-43)

Integrating (3-43) respect to **s**, we have

$p_{sea}//\tau_{sea} = \int \mathrm{Exp}[R(\mathbf{s} - t^2)]d\mathbf{s} = \mathrm{Exp}[R(\mathbf{s}^2/2 - t^2)],$ (3-44)

Note that the variables É and p_{sea}/τ_{sea} are unknown functions yet. □

3.3.1. Dimensional Check

The dimension of each term in a correct result/equation must be the same. Using this to check the correctness of a result is called the dimensional check.

The dimension of left side of (3-43) is:

$\dim \acute{E} = \frac{cm}{sec^2},$ (3-45)

The dimension of middle term of (3-43) gives,

$\dim \frac{Rcm}{t^2} = \dim R \times \dim \frac{cm}{t^2} = \dim R \times \frac{cm}{sec^2},$ (3-46)

By (3-45) - (3-46), we have

$\dim R = cm^0,$ (3-47)

The left term of (3-44), we have

$\dim p_{sea}//\tau_{sea} = \frac{N}{cm^2}\frac{cm^3}{m} = \frac{m}{cm^2}\frac{cm}{sec^2}\frac{cm^3}{m} = \frac{cm^2}{sec^2}$ (3-48)

The right side of (3-44), we have

$\dim \mathrm{Exp}[R(\mathbf{s}^2/2 - t^2)] = cm^0(cm^2/sec^2) = \frac{cm^2}{sec^2},$ (3-40)

Two solutions of "wave equation" (3-30) shown in (3-43), (3-44) have been dimensional checked by (3-45) = (3-46), and (3-48) = (3-49), Which shows that (3-43) and (3-44) are correct. □

4. The Win - Wave Equation

The "wind - pressure/density equation" suits for points in air. The "wave equation" suits for points in sea. Usually the motion of wind and wave are in-depending to each other. Here, the "win - wave equation" is couple the "wind - pressure/density equation" and the "wave equation" with points suited for boundary between air and sea.

Note that we just care for "wind - wave equation" belonging to certain type, while the wind - wave coupling belonging to non-certain type like Haoyu, *et al.* [9] is neglected.

The "Wind – Wave Equation"

We set up the "Wind - Wave Equation" by (2-13), (3-30) and boundary condition.

The boundary condition between air and sea.

The boundary condition between air and sea may be represented by differential and integral forms.

4.1. The Differential form of Boundary Condition

The differential form of boundary condition between air and sea is an equilibrium equation shown as:

$\mathbf{F}_{air} = \mathbf{F}_{sea},$ (4-1)

Where $\mathbf{F}_{air} = \mathbf{F}_{air}(s, t)$ and $\mathbf{F}_{sea} = \mathbf{F}_{sea}(s, t)$ are forces applied on a point (s, t) on boundary, **F** = the applied force.

By Newton's second law, (4-1) becomes:

$\mathbf{F}_{air} = m_{air}\, \grave{u} = m_{sea}\acute{E} = \mathbf{F}_{sea},$ (4-2)

Or $\grave{u} = (m_{air}/m_{sea})\acute{E}$, (4-3)

Where m is the mass of the point.

Substituting (2-13), (3-30), (3-43) and (3-44) into (4-3), we have ù = $ù(\mathbf{s},t) = \text{Exp}[k(\mathbf{s}-t^2)] = \nabla P_{air}/\rho_{air} = (m_{air}/m_{sea})$ É $(\mathbf{s}, t) = \text{Exp}[R(\mathbf{s}-t^2)] = \nabla p_{sea}/\tau_{sea} = \nabla p_{sea}$, $(k = \rho_{air}^{-1})$, $(R = cm^0)$, (4-4)

(4-4) involves many equals. We choice:

$p_{air}(\mathbf{s},t)/\rho_{air} = p_{sea}(\mathbf{s},t)/\tau_{sea}$, (4-5)

(4-3) and (4-5) are two relations between ù and É, p_{air} and p_{sea}. These are differential forms of boundary conditions for a point in
boundary.

4.2. The Integral form of Boundary Condition

Using conservation law of mechanical energy on the boundary between air and sea both sides of a point, we have

$\frac{1}{2} m_{air}\mathbf{u}^2 = m_{sea}\mathbf{g}h$, (4-6)

Where m_{air} and m_{sea} are the masses. **u = u(s**, t) is the velocity of air at boundary, **g** is the gravity acceleration. h = h(**s**, t) is the wave-highness. i.e., the distance of a point at boundary from z = 0 (sea level) to its wave-surface, i.e.,

$h \subset \mathbf{s} \in \Omega = \Omega_{air} \cap \Omega_{sea}$; (4-7)

point **s** belongs to set Ω of intersection of set air Ω_{air} and set sea Ω_{sea}.

where Ω is the set of points of the wave-surface.

Integrating both sides of (4-4) for Ω, we have

$\frac{1}{2}\int_0^{\Omega} \mathbf{u}^2(\mathbf{s})d\mathbf{s} = \mathbf{g}\,\frac{m_{sea}}{m_{air}}\int_0^{\Omega} h(\mathbf{s})d\mathbf{s}$, (4-8)

By theorem of mean value integration, (4-8) gives

$\frac{1}{2}\mathbf{u}_{average}^2 = \frac{m_{sea}}{m_{air}}\mathbf{g}h_{average}$, (4-9)

Or simple shown as "The wind dynamic energy = Potential energy of the high of sea, (4 - s2)".

(4-9) is the integration form of boundary condition, which shows that the square of average air velocity proportions to average wave highness. This relation can be check by weather forecast.

5. Conclusion

5.1. This paper Gives a New Way for Solving N - S Equations by Steps

Step 1, solving "wind - pressure/density equation".

sSep 2, set up "wave equation".

sSep 3, set up "wind - wave equation".

Srep 4, Using boundary condition between air and sea, solves solution of "wave equation" by the above steps.

5.2. Our Previous Paper

"**Non-Existence of Solution of Rotation Floe in N- S Equs**[" 5]**proved that the existence and unique of gene*ral* form of N - S equation is non-existed**

5.3. Our Previous Paper

"Motion Equation and Solution of Mushroom Cloud" [7] solved the solution of cylindrical form of simplified N - S equation'

1. This paper gives the solution in Cartesian co-ordinates of simplified N - S equation.
2. All continuous solution belongs to certain type.
3. Application example of linked our problem is given by paper <Air crash and pressure>.

6. Appendix

A1. Calculation of ρ_{air}

There are methods, tools for calculation of ρ_{air}. All these method treat ρ_{air} as function of p and z, i.e., $\rho_{air} = \rho_{air}(p, z)$ [1, 10], and do not connect with wind speed.

Differ from the about methods, we treat the ρ_{air} connected with the **"**w – p/ρ equation" (2-13). Where the ρ_{air} is calculated as following.

A2. Density of Dry air ρ_{dryair}

In the atmosphere around us, 78% Nitrogen (N_2), 21% Oxygen (O_2), and 1% other gases. The N has a molecular weight of 14, so N_2 has a molecular weight of 28, Oxygen has a molecular weight of 16, so O_2 has molecular weight of 32. Given the mixture of gases of molecules weight is around 29. The total weight of 100% air of $1m^3$ is: w =$[0.78 \times 28 + 0.21 \times 32 + 0.01 \times 29] \times 1m^3 = 31.91$ (N).

$\rho_{\mathbf{dryair}} = 31.91g$, (A-1)

Where g == gravity acceleration. **Assumption:** g is independent with position **s**, $g = 9.81$ (m/sec^2). Then, by (A-1), we have:

$\rho_{\mathbf{dryair}} = 31.91 \times 9.91 = 313.0371(kg/m^3)$. (A-2)

Density of Moist Air

Water H_2O, Hydrogen H is the lightest element and has a molecular weight of 1. So a water molecular weight is $(2 \times 1 + 16 = 18)$. Which shows that the water molecular weight is much lighter than the average weight of the molecular found in air.

The density of moist air can be calculated as the sum of two gases: dry air and water vapor in proportion with their partial proportion c.

$$\rho_{moistair} = c \times \rho_{dryair} + (1 - c)\rho_{vapor}\ (c \leq 1), \quad \text{(A-3)}$$

□

References

[1] Available: https://www.claymeth.org/prize/

[2] Tian-Quan, Y., 2016. "Wind speed equation of a point in air." *Fundamental Journal of Modern Physics,* vol. 9, pp. 57-64.

[3] Tian-Quan, Y., 2016-a. "Existence and solution of wind speed equation of a point in air." *British Journal of Mathematics and Computer Science,* vol. 17, pp. 1-5.

[4] Tian-Quan, Y. and Fey, Y., 2021. *Wind speed equation --- the base of wind description of certain type*. International Book Market Service Ltd. member of OmniScriptum Publshing Group: Scholars Press.

[5] Tian-quan, Y., 2020. "Non-existence of solution of rotation flow in n - s equation." *Global Journal of Pure and Applied Mathematics,* vol. 16, pp. 651-655.

[6] Tiab-quan, Y., 2020. "Static mantle density distribution 3 dimpling and buckling of spherical crust." *Global Journal of Science Frontier Research: A Physics and Space Science,* vol. 20, pp. 1-19. Available: https://doi.org/10.34257/GJSFRAVOL20IS8PG21

[7] Kamke, E., 1977. *Differentialgleighungen lŏsungsmethoden undlŏsungen 1 (translated by znang honling)*. Beijing: Academic Publishers.

[8] Tian-quan, Y., 2021. "Motion equation and solution of mushroom cloud." *Atmospheric and Glimate Sciences,* vol. 11, pp. 86 - 97. Available: https://doi.org/10.4236/acs.2021.111006

[9] Haoyu, J., GeChen, and Fangjie, Y., 2015. *An index of wind-wave coupling and its global climatology*. South China University of Tech Open UrRLSECTIONS.

[10] Tian-Quan, Y., 2022. "Air crash and pressure." *Global Journal of Science Frotier Research: A Physcics and Space Science,* vol. 22.