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Bayes Estimations for Parameter of the Poisson distribution with Progressive Schemes

Huda Mohammed Alomari
Department of Mathematics, Al-Baha University, Saudi Arabia
Email: haldrawshah@bu.edu.sa

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Abstract

This study introduces maximum likelihood and Bayesian approaches to Poisson parameter estimation using posterior distribution. I discuss three types of loss functions: the asymmetric linear exponential loss function, non-linear exponential loss function, and squared error loss function. Their performance is compared with the maximum likelihood estimator using mean squared error (MSE) as the test criterion. The proposed method with the classical estimator (maximum likelihood estimator) is better than that with the non-classical estimators for point estimation with different sample sizes. Maximum likelihood estimation provides the optimal performance in estimating the Poisson distribution, as evidenced by the asymptotically smallest MSE values. For small true parameter values the results reveal that the Bayesian approaches have good estimation performance.

Keywords: Bayes estimator; Linear exponential (LINXE) loss function; Maximum likelihood estimator; Mean squared error (MSE); Non-linear exponential (NLINEX) loss function; Squared error (SE) loss function.

1. Introduction

Poisson distribution is a discrete distribution that indicates the number of times an event is likely to occur within a given period. It is utilized to determine the probability of an independent event happening in a fixed interval of time with a steady mean rate. There are various applications of Poisson distribution, such as predicting the waiting time between events, the number of typos on the different pages of a book, and the number of births per hour during a given time. Ahmad and Roohi [1], has been explained using a recurrence relation for the Poisson random variables in the first order of negative moment.

Previously, Cohen Jr [2], investigated the maximum likelihood estimation (MLE) of a Poisson distribution that has been modified such that a proportion θ of ones (events) is reported to be zeros (non-events).

Holgate [3], introduced Poisson distribution to estimate bivariate Poisson distribution. Frome, *et al.* [4], introduced MLE to estimate the parameters in a regression model when experimental data are drawn from a Poisson distribution. Hassan, *et al.* [5], considered the Bayes estimator for the parameter of zero-truncated Poisson distribution.

Consul and Jain [6], generalised the Poisson distribution with two parameters. The maximum likelihood, Markov chain Monte Carlo, and Bayes approach to testing hypotheses about the mean of the Poisson parameter have been discussed by Araveeporn [7]. Two Bayesian approaches for Poisson parameter estimation under the squared error loss and quadratic loss functions were studied by Supharakonsakun [8].

This study considers Bayesian estimation with three types of loss functions. The first is the linear-exponential asymmetric linear exponential (LINEX) loss function, which increases roughly linearly on one side of zero and roughly exponentially on the other. The LINEX loss function was introduced by Varian [9] and Zellner [10]. The second is the non-linear exponential (NLINEX) loss function, which is another asymmetric loss function. The NLINEX loss function, proposed by Islam, *et al.* [11] is a modification of the LINEX loss function, where the exponential form of the asymmetric loss function is not always related linearly. The third is the SE loss function, a type of symmetric function also known as the mean of the posterior density function. Note that NLINEX is a mixture of the LINEX and SE loss functions. Thus, it preserves the estimation error attributed to both loss functions. For more details, see Islam, *et al.* [11].

Furthermore, several studies have discussed inferential methods based on the MLE and Bayesian approaches to estimate the unknown parameters of certain distributions. Examples include Exponential distribution [12], Rayleigh distribution [13], Poisson-exponential distribution for complete samples [14] multivariate normal distribution [15] and reliability distribution [16].

This study reports the estimates of the Poisson parameter based on MLE, LINEX, NLINEX, and SE. The remainder of this paper is organized as follows. Section 2 describes the parameter estimation methods and their derivations. A Monte Carlo simulation study and the results are reported comparing the estimation methods in Section 3. The conclusions are presented in Section 4.

2. Estimation Methods for Parameter θ

This section presents methods of estimating the Poisson parameter based on MLE, LINEX, NLINEX, and SE.

2.1. Maximum Likelihood Estimator

MLE is the most popular method for estimating the unknown parameters of a probability distribution based on observations $X_1, X_2, \ldots, X_n$. The maximum likelihood estimate can be computed by maximizing a likelihood function. Suppose $X = (X_1, X_2, \ldots, X_n)$ is a random sample of size n with a probability mass function from independent and identically distributed random variables from a Poisson distribution with parameter θ. A discrete random variable X follows a Poisson distribution with parameter θ if the probability mass function is given by

$$f(X_i|\theta) = \frac{e^{-\theta}\theta^{X_i}}{X_i!}, \tag{1}$$

Where $i = 0, 1, 2, \ldots$ and θ is a constant mean rate of event accrual.

The likelihood, $L(\theta)$, θ based on $X = (X_1, X_2, \ldots, X_n)$ is defined by

$$\begin{aligned} L(X_i|\theta) &= \prod_{i=1}^{n} f(X_i|\theta) \\ &= \frac{e^{-n\theta}\theta^{\sum_{i=1}^{n} X_i}}{\prod_{i=1}^{n} X_i!} \end{aligned} \tag{2}$$

Instead of maximizing the likelihood function itself, one can maximize the natural logarithm of the likelihood function (2) as follows:

$$lnL\,(X_i|\theta) = -n\theta + \sum_{i=1}^{n} X_i\, ln\theta - ln\prod_{i=1}^{n} X_i! \tag{3}$$

To estimate the parameter θ, set the first derivative of (3), with respect to the parameter θ, to zero as follows:

$$\frac{\partial}{\partial\theta}\ln L(X_i|\theta) = -n + \frac{\sum_{i=1}^{n} X_i}{\theta} = 0$$

Thus, the following is obtained

$$\frac{\sum_{i=1}^{n} X_i}{\hat{\theta}} - n = 0$$

By solving the equation for $\hat{\theta}$, one obtains

$$\hat{\theta}_{MLE} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{4}$$

The second derivative of the estimator is less than 0, which means that the estimator is the maximum, where

$$\frac{\partial^2}{\partial\theta^2}\ln L(X_i|\theta) = -\frac{\sum_{i=1}^{n} X_i}{\theta^2}$$

Note that, the value of θ that maximizes the natural logarithm of the likelihood function *log(θ)* is also the value of θ that maximizes the likelihood function $L(\theta)$. Additionally, the MLE of θ is the sample mean.

2.2. Bayes Estimation

Bayesian estimation requires the specification of a prior distribution of the parameter. Here, the informative conjugate prior for θ is a Gamma distribution with parameters α and β, where α is the shape parameter, and β is the scale parameter, and is given as follows:

$$\pi(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} e^{-\beta\theta}\theta^{\alpha-1}, \qquad \alpha,\ \beta,\ \theta > 0 \tag{5}$$

By combining the likelihood function in (2) and the prior distribution in (5), the posterior distribution for the Bayes procedure is obtained, as follows:

$$L(X_i|\theta)\pi(\theta) = \frac{e^{-n\theta}\theta^{\sum_{i=1}^{n} X_i}}{\prod_{i=1}^{n} X_i!}\frac{\beta^\alpha}{\Gamma(\alpha)}e^{-\beta\theta}\theta^{\alpha-1}$$

$$h(X_i|\theta) = \frac{L(X_i|\theta)\pi(\theta)}{\int_0^\infty L(X_i|\theta)\pi(\theta)d\theta}$$

$$= \frac{\frac{e^{-n\theta}\theta^{\sum_{i=1}^{n} X_i}}{\prod_{i=1}^{n} X_i!}\frac{\beta^\alpha}{\Gamma(\alpha)}e^{-\beta\theta}\theta^{\alpha-1}}{\int_0^\infty \frac{e^{-n\theta}\theta^{\sum_{i=1}^{n} X_i}}{\prod_{i=1}^{n} X_i!}\frac{\beta^\alpha}{\Gamma(\alpha)}e^{-\beta\theta}\theta^{\alpha-1}d\theta}$$

$$= \frac{e^{-(n+\beta)\theta}\theta^{\sum_{i=1}^{n}(X_i+\alpha)-1}}{\int_0^\infty e^{-(n+\beta)\theta}\theta^{\sum_{i=1}^{n}(X_i+\alpha)-1}}$$

$$= \frac{(n+\beta)^{\sum_{i=1}^{n} X_i+\alpha}e^{-(n+\beta)\theta}\theta^{\sum_{i=1}^{n}(X_i+\alpha)-1}}{\Gamma(\sum_{i=1}^{n} X_i+\alpha)},$$

which is a Gamma distribution with parameters $\sum_{i=1}^{n} X_i + \alpha$ and $n+\beta$. Then,

$$\theta|X_i \sim Gamma\left(\sum_{i=1}^{n} X_i+\alpha, n+\beta\right).$$

2.2.1. Bayes' Estimator of Parameter θ for LINEX Loss Function

Ahmadi, *et al*. [17], discussed the LINEX loss function for θ. It can be defined as follows:

$$L(D) = b[exp(c(D)) - cD - 1],$$

where $b > 0$, is the scale parameter, $c \neq 0$ denotes the shape parameter, and D represents the estimation error, that is, $D = \hat{\theta} - \theta$.

The Bayes estimator of θ, which depends on the LINEX loss function, is denoted by $\hat{\theta}_{BL}$ and depending on the LINEX loss function, can be expressed as

$$\hat{\theta}_{BL} = \frac{-1}{c}\ln\left[E_\theta\left(exp\left(-c\theta\right)\right)\right], \qquad c \neq 0$$

Provided that $E_\theta(exp(-c\theta))$ exists and is finite, where E_θ denoted the expected value. Here,

$$E_\theta e^{-c\theta} = \int_0^\infty e^{-c\theta}h(X_i|\theta)d\theta$$

$$= \int_0^\infty e^{-c\theta}\frac{(n+\beta)^{\sum_{i=1}^{n} X_i+\alpha}e^{-(n+\beta)\theta}\theta^{\sum_{i=1}^{n}(X_i+\alpha)-1}}{\Gamma(\sum_{i=1}^{n} X_i+\alpha)}d\theta$$

$$= \frac{(n+\beta)^{\sum_{i=1}^{n} X_i+\alpha}}{\Gamma(\sum_{i=1}^{n} X_i+\alpha)}\int_0^\infty e^{-(n+\beta+c)\theta}\theta^{\sum_{i=1}^{n}(X_i+\alpha)-1}d\theta$$

$$= \frac{(n+\beta)^{\sum_{i=1}^{n} X_i+\alpha}}{\Gamma(\sum_{i=1}^{n} X_i+\alpha)}\frac{\Gamma(\sum_{i=1}^{n} X_i+\alpha)}{(n+\beta+c)^{\sum_{i=1}^{n} X_i+\alpha}}$$

$$= \frac{(n+\beta)^{\sum_{i=1}^{n} X_i+\alpha}}{(n+\beta+c)^{\sum_{i=1}^{n} X_i+\alpha}}.$$

Thus,

$$E_\theta e^{-c\theta} = \left(1+\frac{c}{n+\beta}\right)^{-\left(\sum_{i=1}^{n} X_i+\alpha\right)}. \qquad (6)$$

Hence, $\hat{\theta}_{BL}$ is given by

$$\hat{\theta}_{BL} = \frac{-1}{c}\ln\left[\left(1+\frac{c}{n+\beta}\right)^{-\left(\sum_{i=1}^{n} X_i+\alpha\right)}\right]. \qquad (7)$$

2.2.2. Bayes' Estimator of Parameter θ for NLINEX Loss Function

The NLINEX loss function can be defined as

$$L(D) = k\left(exp(cD) + cD^2 - D - 1\right), \qquad k,\ c > 0,$$

where D represents the error in estimating θ by $\hat{\theta}$ then $D = \hat{\theta} - \theta$.

The Bayes estimator of θ based on NLINEX loss function, denoted by $\hat{\theta}_{NBL}$, is discussed in the following theorem.

Theorem 1 For the NLINEX loss function, the Bayes estimator for a parameter θ is written as the following:

$$\hat{\theta}_{BNL} = -[\ln E_\theta(exp(-c\theta)) - 2E_\theta(\theta)]/(c+2), \quad (8)$$

Where E_θ stands for posterior expectation.
[11] provided the proof of this theorem.

From equation (6) we get $E_\theta e^{-c\theta}$. Now, we find E_θ (θ) as follows:

$$E_\theta(\theta) = \int_0^\infty \theta h(X_i|\theta)d\theta$$

$$= \int_0^\infty \theta \frac{(n+\beta)^{\sum_{i=1}^n X_i+\alpha} e^{-(n+\beta)\theta}\theta^{\sum_{i=1}^n (X_i+\alpha)-1}}{\Gamma(\sum_{i=1}^n X_i+\alpha)} d\theta$$

$$= \frac{(n+\beta)^{\sum_{i=1}^n X_i+\alpha}}{\Gamma(\sum_{i=1}^n X_i+\alpha)} \int_0^\infty e^{-(n+\beta)\theta}\theta^{\sum_{i=1}^n (X_i+\alpha+1)-1} d\theta$$

$$= \frac{(n+\beta)^{\sum_{i=1}^n X_i+\alpha}}{\Gamma(\sum_{i=1}^n X_i+\alpha)} \frac{\Gamma(\sum_{i=1}^n X_i+\alpha+1)}{(n+\beta)^{\sum_{i=1}^n X_i+\alpha+1}}.$$

Therefore

$$E_\theta(\theta) = \frac{\sum_{i=1}^n X_i+\alpha}{n+\beta}. \quad (9)$$

By substituting (9) and (6) in (8), the Bayes estimator of θ under NLINEX loss function can be written as follows:

$$\hat{\theta}_{BNL} = \left(\sum_{i=1}^n X_i+\alpha\right)\left(\ln\left(1+\frac{c}{n+\beta}\right)+\frac{2}{n+\beta}\right)/(c+2). \quad (10)$$

The relationship between the Bayes estimator for the parameter θ under LINEX and NLINEX loss functions is given in Theorem 3.2 which is explained by Islam, *et al.* [11].

2.2.3. Bayes Estimator of Parameter θ for Squared Error Loss Function

The Bayesian estimator of θ for the SE loss function can be defined as

$$L(\theta, \hat{\theta}) = (\hat{\theta} - \theta)^2.$$

The Bayes estimate under the squared error loss function is the posterior mean. The BE of θ under SE loss function is given in (9), denoted by $\hat{\theta}_{BS}$. It can be written as

$$\hat{\theta}_{BS} = \frac{\sum_{i=1}^n X_i+\alpha}{n+\beta}. \quad (11)$$

3. Numerical Simulation Results

In this section, the estimation of the Poisson distribution parameter is illustrated from 10, 000 replications with sample sizes (n =10, 20, 30, 50, 75, 100) and true parameters θ = 0.5, 1,5, 10, 15, 20, given arbitrary prior parameters (α, β, c)= (1, 2, 1), (1.5, 2.5, 2), (2, 2, 1), (3, 3, 2) for Bayesian methods with three different loss functions. Maximum likelihood estimator, $\hat{\theta}_{MLE}$ of θ is computed from equation (4). Bayesian estimation, $\hat{\theta}_{BL}$, $\hat{\theta}_{BNL}$, and $\hat{\theta}_{BS}$ are computed from equations (7), (8), and (11), respectively. To compare estimators $\hat{\theta}_{MLE}$, $\hat{\theta}_{BL}$, $\hat{\theta}_{BNL}$, and $\hat{\theta}_{BS}$, the mean square error (MSE) was computed as.

$$MSE = \frac{\sum_{i=1}^s (\theta-\hat{\theta})^2}{s}.$$

MSE was used to identify the best-performing estimation method, with the lowest MSE value meaning that the estimated value of θ was closest to its true value.
The results of the simulation study are listed in Tables 1, 2, 3, 4 and shown in Figure 1.
From the results, the following observations were made:

1- The values of all MSEs decreased as the sample size increased.
2- With increasing θ, the MSE values of all methods increased.
3- It is clear that for θ = 5, 10, 15, 20, the lowest MSE values were obtained by MLE, while the Bayesian approach provided the best estimates for small true parameter values of θ = 0.5 and 1.

4- For θ = 1, MLE exhibited the worst performance for point estimation compared with the other estimation methods.

5- When focusing on the Bayesian estimation approaches, the SE method gave better estimates than those obtained from LINEX and NLINEX loss functions for θ = 5,10,15,20.

For θ = 0.5 and sample size n =10, 20, 30, or 75, the lowest MSE values were obtained by the Bayes estimator under LINE.

Table-1. MSE of different estimators of Poisson distribution, with $\alpha = 1, \beta=2, c = 1$

n	θ	MLE	BNL	BL	BSE
10	0.5	0.050502	0.033776	0.031969	0.034670
	1	0.099874	0.076557	0.078399	0.076586
	5	0.509557	0.996530	1.149211	0.917870
	10	0.998038	3.510970	4.303018	3.178551
	15	1.525568	7.667110	9.470429	6.789033
	20	2.006962	13.32338	16.65893	12.00361
20	0.5	0.024696	0.0205659	0.019952	0.020294
	1	0.049702	0.0433093	0.044628	0.042894
	5	0.244571	0.3954429	0.456993	0.388085
	10	0.508697	1.271059	1.546606	1.154141
	15	0.731793	2.621518	3.176477	2.355563
	20	0.981165	4.503223	5.610954	3.975323
30	0.5	0.0171298	0.0146274	0.014623	0.014200
	1	0.0333097	0.0304442	0.030443	0.030907
	5	0.1700406	0.2399009	0.266538	0.227860
	10	0.3318698	0.7060714	0.247893	0.652572
	15	0.5081242	1.3796080	1.687079	1.258117
	20	0.6664087	2.2669540	2.840443	2.074740
50	0.5	0.0101224	0.0093089	0.0091999	0.009187
	1	0.0198666	0.0186576	0.0191273	0.018722
	5	0.0996972	0.1267040	0.1394557	0.123128
	10	0.1998652	0.3353836	0.3800269	0.320358
	15	0.2997039	0.6439542	0.7542865	0.593225
	20	0.4040265	1.0175340	1.2395180	0.937647
75	0.5	0.0066380	0.0063346	0.006240	0.006336
	1	0.0133812	0.0129072	0.0124868	0.013050
	5	0.0637459	0.0797765	0.0836624	0.076203
	10	0.1320789	0.1909843	0.2212704	0.186443
	15	0.2003226	0.3557853	0.4024462	0.331992
	20	0.2678665	0.5570970	0.6439621	0.495694
100	0.5	0.0047993	0.0048221	0.0049400	0.004898
	1	0.0100234	0.0095552	0.0096270	0.009428
	5	0.0501916	0.0555115	0.0605741	0.055070
	10	0.1028052	0.1350092	0.1448120	0.129490
	15	0.1519033	0.2355401	0.2691264	0.219790
	20	0.2032684	0.3636787	0.4224061	0.335502

Table-2. MSE of different estimators of Poisson distribution, with $\alpha = 1.5$, β=*2,5,* 5, $c = 2$

n	θ	MLE	BNL	BL	BSE
10	0.5	0.493801	0.029284	0.028320	0.031845
	1	0.101804	0.074042	0.076772	0.069969
	5	0.506056	1.359180	1.677169	1.087360
	10	1.005959	5.333362	6.616596	4.160383
	15	1.483913	12.01237	14.94493	9.228407
	20	2.065518	21.11872	26.73860	16.32501
20	0.5	0.024808	0.0190589	0.018414	0.020044
	1	0.050233	0.0425625	0.043759	0.041224
	5	0.246979	0.5395940	0.655547	0.430255
	10	0.494030	1.8979430	2.399565	1.49390
	15	0.729202	4.095880	5.182869	3.153873
	20	0.987438	7.096239	9.117923	5.411159
30	0.5	0.0169205	0.013491	0.013440	0.014170
	1	0.0332975	0.030321	0.030291	0.029284
	5	0.1672686	0.306367	0.361700	0.259028
	10	0.3254338	1.003901	1.273138	0.807252
	15	0.4942541	2.163271	2.689901	1.658168
	20	0.6627450	3.672228	4.678070	2.805890
50	0.5	0.0099141	0.088029	0.008852	0.0092392
	1	0.2006076	0.018638	0.018862	0.0189760
	5	0.0996856	0.157905	0.175632	0.1348723
	10	0.2041021	0.464001	0.566835	0.3812198
	15	0.3011099	0.923898	1.185355	0.7571106
	20	0.3937913	1.571430	2.004889	1.2064580
75	0.5	0.0066537	0.0061112	0.0059782	0.0062003
	1	0.0134114	0.0126253	0.0127258	0.0127560
	5	0.0647493	0.0913944	0.1029666	0.0804847
	10	0.1340716	0.2579344	0.2949821	0.2198021
	15	0.2014200	0.4999491	0.6018946	0.4018282
	20	0.2631775	8.1545310	1.0205901	0.6427475
100	0.5	0.0050446	0.0046239	0.004709	0.0046607
	1	0.0099881	0.0095708	0.0097468	0.0092010
	5	0.0409721	0.0645699	0.0679880	0.0595310
	10	0.0992863	0.1716283	0.2009916	0.1477890
	15	0.1485417	0.3177166	0.3814828	0.2653956
	20	0.1987601	5.2134670	0.6246317	0.4112669

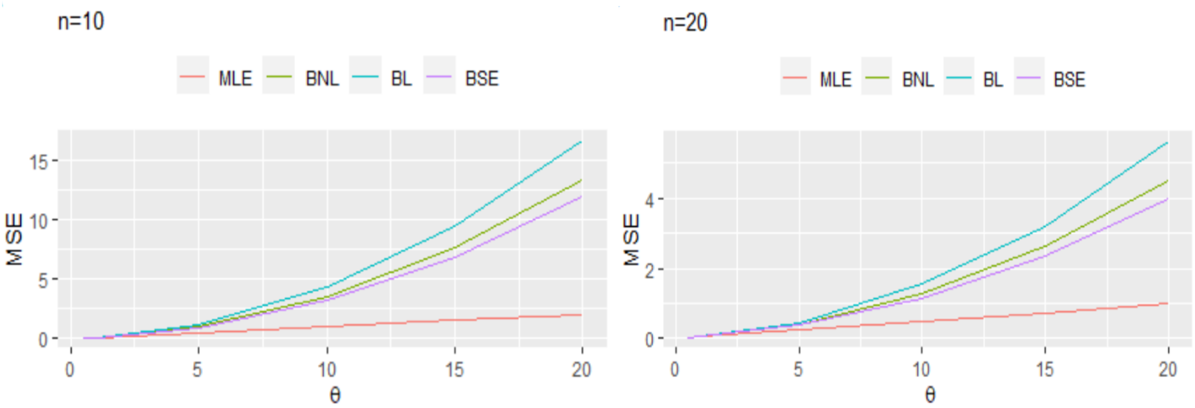
Table-3. MSE of different estimators of Poisson distribution, with $\alpha = 2$, $\beta=2$, $c = 1$.

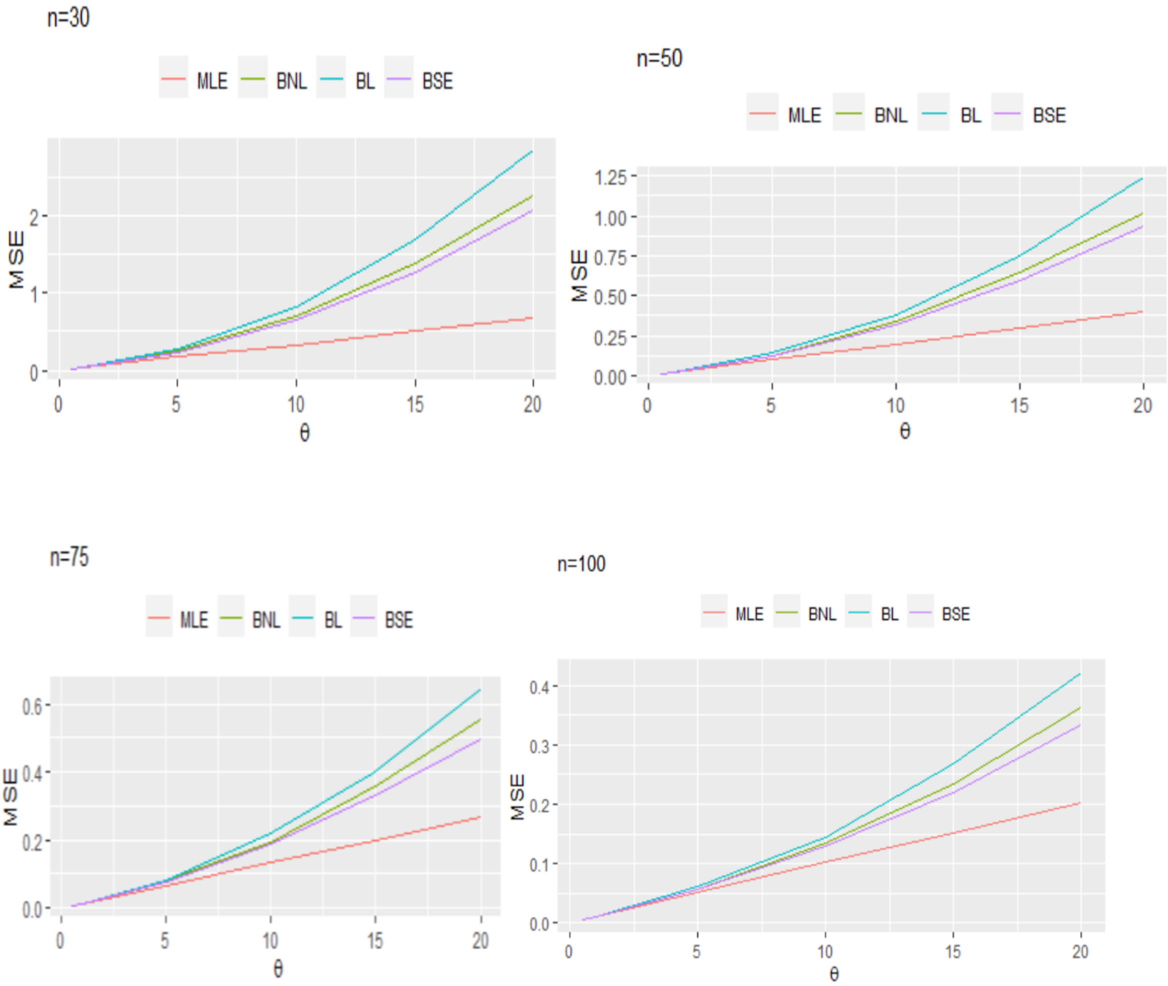
n	θ	MLE	BNL	BL	BSE
10	0.5	0.049613	0.038430	0.035378	0.040813
	1	0.099463	0.067910	0.065282	0.070078
	5	0.498475	0.869419	1.021431	0.783461
	10	1.010366	3.239391	4.020890	2.923388
	15	1.480741	7.347980	8.919138	6.478110
	20	1.956844	12.81537	16.04823	11.34870
20	0.5	0.024888	0.221599	0.020888	0.022288
	1	0.049369	0.041527	0.040017	0.040094
	5	0.247009	0.360695	0.409591	0.341850
	10	0.510284	1.215206	1.441977	1.066989
	15	0.749257	2.471324	3.075869	2.238582
	20	1.020478	4.283689	5.325104	3.849008
30	0.5	0.0164778	0.015562	0.0146705	0.015491
	1	0.0328905	0.029824	0.0287779	0.030022
	5	0.1661426	0.219556	0.2522039	0.208237
	10	0.3351154	0.669059	0.7842565	0.606946
	15	0.4972560	1.303929	1.6189480	1.215151
	20	0.6601329	2.232995	2.7400810	1.983744
50	0.5	0.0100616	0.0096351	0.0094072	0.0097170
	1	0.0200845	0.0181148	0.0183167	0.0186423
	5	0.0980717	0.1202395	0.1320080	0.1160908
	10	0.2027969	0.3167053	0.3756240	0.3007650
	15	0.2970586	0.6102272	0.7260210	0.5703379
	20	0.3929446	0.9840630	1.2059880	0.9037688
75	0.5	0.0067882	0.0063269	0.0062298	0.0064846
	1	0.0134126	0.1262180	0.0125527	0.0127153
	5	0.0672557	0.0779904	0.0800753	0.0728534
	10	0.1321154	0.1925173	0.2104145	0.1817494
	15	0.1952198	0.3494707	0.3984717	0.3186496
	20	0.2633264	0.5399359	0.6361587	0.4962870
100	0.5	0.0048886	0.0050286	0.0048225	0.0044908
	1	0.0102042	0.0096799	0.0096729	0.0095387
	5	0.0487032	0.0559516	0.0564616	0.0550635
	10	0.0972750	0.1342012	0.1465292	0.1304850
	15	0.1502109	0.2314653	0.2594861	0.2202043
	20	0.2000896	0.3504629	0.4134401	0.3273111

Table-4. MSE of different estimators of Poisson distribution, with $\alpha = 3$, $\beta=3$, $c = 2$.

n	θ	MLE	BNL	BL	BSE
10	0.5	0.055441	0.0357918	0.030774	0.042066
	1	0.097564	0.0561682	0.056009	0.058899
	5	0.486016	1,4176350	1.712656	1.141481
	10	1.012824	6.0309502	7.394030	4.876365
	15	1,490865	14. 00411	17.22226	11.35840
	20	2.028845	25.204891	30.97622	20. 40347
20	0.5	0.0255480	0.020593	0.019022	0.023246
	1	0.0497182	0.036842	0.036142	0.038293
	5	0.2491755	0.560198	0.672575	0.466567
	10	0.4828398	2.178532	2.712142	1.756945
	15	0.7391285	4.967351	6.100070	3.900080
	20	0.9945945	8.758717	10.98453	6.892795
30	0.5	0.016932	0.0145839	0.0138779	0.016124
	1	0.0327312	0.2668067	0.0269131	0.027538
	5	0.1659748	0.3184928	0.3699265	0.267965
	10	0.3339096	1.172877	1.4228364	0.944136
	15	0.4967132	2.549100	3.1989610	2.017692
	20	0.674276	4.462129	5.576128	3.508293
50	0.5	0.0101476	0.0091808	0.0090315	0.0095223
	1	0.0196778	0.0174055	0.0173041	0.0176469
	5	0.0996530	0.1588946	0.1846884	0.1380148
	10	0.2001667	0.5260782	0.6489253	0.4439210
	15	0.3013433	1.0913190	1.3605770	0.8861546
	20	0.3966560	1.8874810	2.4182460	1.5060930
75	0.5	0.0066941	0.0062299	0.0061293	0.0063864
	1	0.0128712	0.0121014	0.0122621	0.0120126
	5	0.0670916	0.0968631	0.1052869	0.0856544
	10	0.1352841	0.2895909	0.3397123	0.2403310
	15	0.2014996	0.5778740	0.6948401	0.4749832
	20	0.2635655	0.9865523	1.1890490	0.7881715
100	0.5	0.0049592	0.0047907	0.0046846	0.0049657
	1	0.0100223	0.0094567	0.0094426	0.0094701
	5	0.0509840	0.0652717	0.0723587	0.0595806
	10	0.0963110	0.1905398	0.2222167	0.1630089
	15	0.1480755	0.3678505	0.4340916	0.2988541
	20	0.1972423	0.6077126	0.7287562	0.4910722

Figur-1. The performance of MSE of different estimators θ for Poisson distribution with n = 10, 20, 30, 50, 75, 100.





4. Conclusion

This work studied the MLE and Bayesian methods, including LINEX, NLINEX, and SE loss functions, to estimate the rate parameter of the Poisson distribution. I performed a Monte Carlo simulation to analyze the performance of the methods. I used MSE to compare the performance of these methods. The results of the simulation study demonstrate that MLE performs best in estimating the parameter according to the smallest values of MSE for θ= 5, 10, 15, and 20, while the Bayesian method performs best for θ = 0.5 and 1.

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