Academic Journal of Applied Mathematical Sciences
ISSN: 2415-2188
Vol. 2, No. 2, pp: 11-18, 2016
URL: http://arpgweb.com/?ic=journal&journal=17&info=aims

Box-Jenkins Method Based Additive Simulating Model for Daily Ugx-Ngn Exchange Rates

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Abstract: A 177-point realization of daily exchange rates of the Uganda shilling (UGX) – Nigerian naira (NGN) from 22nd September, 2015 to 16th March, 2016, is analyzed by Box-Jenkins methods. The original series being non-stationary is differenced seasonally i.e. on a seven-day basis. A further non-seasonal differencing is done to ensure seasonality. These differences of the seasonal differences of the series are modeled by seasonal autoregressive integrated moving average (SARIMA) approach. The first 170 values are used for the modeling process and the remaining 7 are used for out-of-sample forecast goodness-of-fit test. By a new fitting algorithm, it is concluded that the time series follows the additive SARIMA (1,1,0)x(1,1,0)7 model. Forecasts obtained for the daily rates from March 10 to March 16, 2016 agree so closely with the observed values that the calculated goodness-of-fit chi-square test statistic is far from being statistically significant with a p-value of more than 99%. Daily exchange rates between the two currencies may be simulated or forecasted by the model.

Keywords: Ugandan shilling; Nigerian naira; foreign exchange rates; SARIMA models.

1. Introduction

Foreign exchange is a major issue in the discussion of world economy. Any trade relationship between the country Uganda and the country Nigeria is based on the relative value of the Uganda Shilling (UGX) and the Nigerian Naira (NGN). In this write-up the daily exchange rates shall be modelled by Box-Jenkins methods. The particular approach shall be the seasonal autoregressive integrated moving average (SARIMA) approach proposed by Box and Jenkins [1].

In recent times, many authors have adopted the SARIMA modeling approach to model real iife data. Zhang, *et al.* [2] observed that SARIMA modelling outdid standard Poisson regression, autoregressive adjusted Poisson regression and multiple linear regression. Nirmala and Sundaram [3] fitted a SARIMA(0,1,1)x(0,1,1)12 to monthly rainfall in Tamilnadu. Jiang, *et al.* [4] noticed that SARIMA modelling produced better forecasts than dynamic harmonic regression and seasonal –trend decomposition procedure based on Loess. Padhan [5] modelled Indian International tourists footfalls by a SARIMA $(1,1,1)x(2,1,4)_{12}$. Mahsin, *et al.* [6] used a SARIMA$(0,0,1)x(0,1,1)_{12}$ to model raqinfall in Dhaka Division of Bangladesh. Oduro-Gymah, *et al.* [7] fitted a SARIMA$(1,1,1)x(0,1,2)_{12}$ to microwave transmission in Ghana. Liberian inflation rates have been modelled by a SARIMA$(0,1,0)x(2,0,0)_{12}$ [8]. Jianfeng [9] noticed that SARIMA modelling results in closer forecasts to the real data than dynamic linear modelling in forecasting monthly cases of mumps in Hong Kong. He fitted a SARIMA$(2,1,1)x(1,1,1)_{12}$ model to the time series . Li, *et al.* [10] modelled monthly outpatient numbers in China by a SARIMA$(0,1,1)x(0,1,1)_{12}$. Kibunja, *et al.* [11] forecasted monthly precipitation in Mount Kenya region using a SARIMA$(1,0,1)x(1,0,0)_{12}$ model. Valipour [12] observed that SARIMA modelling outdid its non-linear counterpart ARIMA in long-term runoff forecasting. Hassan and Mohamed [13] found that a SARIMA$(0,0,5)x(1,0,1)_{12}$ was the most adequate in the simulation of monthly rainfall drought in the Gadaref region of Sudan. Gikungu, *et al.* [14] fitted a SARIMA$(0,1,0)x(0,0,1)_4$ to quarterly Kenyan inflation rates.

The orthodox and usual approach to SARIMA modelling was proposed by [1]. Suhartono [15] proposed another method based on moving average modelling. In his work he found that according to his definitions a subset SARIMA model outdid both the additive and the multiplicative models for the airline data whereas an additive model best explained variation in the arrival of tourists. Etuk and Ojekudo [16] proposed an alternative modelling algorithm based on duality arguments. This algorithm has been applied to model series with a measure of success [17, 18].

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The seasonal nature of the realization of the exchange rates used for this work makes the application of a SARIMA approach reasonable. The purpose of this write-up is to fit a model to the daily exchange rates of Ugandan shilling (UGX) and Nigerian Naira (NGN) using the algorithm of Etuk and Ojekudo [16]. This work is therefore a further application of the algorithm.

2. Materials and Methods

2.1. Data

The data for this work are 177 values of daily UGX / NGN exchange rates of September 22, 2015 through March 16, 2016. They were obtained from the website www.exchangerates.org/UGX-NGN-exchange-rate-history.html accessed on March 17, 2016. These numbers are interpreted as the quantities of NGN per UGX.

2.2. Seasonal Autoregressive Integrated Moving Average (SARIMA) Models

The definition of a SARIMA model as proposed by Box and Jenkins [1] is as follows. A stationary time series $\{X_t\}$ is said to follow a multiplicative seasonal autoregressive integrated moving average model of order p, d, q, P, D, Q, s designated SARIMA(p,d,q)x(P,D,Q)$_s$ if

$$A(L)\Phi(L^s)\nabla^d\nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \quad (1)$$

where A(L) is a p-order polynomial in L and is called the autoregressive (AR) operator; B(L) is a q-order polynomial in L and is called the moving average (MA) operator; Φ(L) is a P-order polynomial in L called the seasonal AR operator; Θ(L) is a Q-order polynomial in L called the seasonal MA operator. The numbers d and D are the non-seasonal and the seasonal differencing orders respectively. L is the backward shift operator defined by $L^kX_t = X_{t-k}$. the number s is the period of the seasonality of the time series. ∇ and ∇_s are the non-seasonal and the seasonal differencing operators respectively. $\{\varepsilon_t\}$ is a white noise process.

2.3. SARIMA Modelling

Generally the model (1) is estimated beginning with the determination of the orders: p, d, q, P, D, Q and s. The AR orders p and P are estimated by the non-seasonal and seasonal cut-off lags of the partial autocorrelation function, respectively. Similarly the MA orders q and Q are estimated by the non-seasonal and the seasonal cut-off lags of the autocorrelation function respectively. The seasonal period often suggests itself by the known nature of the series. Otherwise it may be suggestive by the correlogram or an analytical inspection of the series. The differencing orders d and D are such that they sum up to 2 at most.

In this work the subset SARIMA modelling algorithm proposed by Etuk and Ojekudo [16] shall be used. It is the autoregressive-moving-average-duality-based version of the algorithm of Suhartono [15].

Suhartono,s algorithm is as follows:

Fit to $\{X_t\}$ the following SARIMA(0,0,1)x(0,0,1)$_s$ model

$$X_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \dots + \beta_s\varepsilon_{t-s} + \beta_{s+1}\varepsilon_{t-s-1} \quad (2)$$

If $\beta_{s+1} = 0$ then the model is said to be *additive*. Otherwise if $\beta_{s+1} = \beta_1\beta_s$, then the model is said to be *multiplicative*. Otherwise it is said to be *subset*.

Etuk and Ojekudo's algorithm which is the dual version of (2) is as follows:

Fit to $\{X_t\}$ the following SARIMA(1,0,0)x(1,0,0)$_s$ model

$$X_t + \alpha_1X_{t-1} + \dots + \alpha_sX_{t-s} + \alpha_{s+1}X_{t-s-1} \quad (3)$$

If $\alpha_{s+1} = 0$ the model is said to be *additive*. If not, if $\alpha_{s+1} = \alpha_1\alpha_s$, the model is said to be *multiplicative*. Otherwise it is said to be *subset*. *Additivity* is ascertained if

$$\widehat{\alpha_{s+1}} < SE(\widehat{\alpha_{s+1}})$$

where SE(.) is the *standard error of and ^ denotes the estimate of.* In other words, the algorithm of Etuk and Ojekudo [16] is the *autoregressive* version of Suhartono [15] method which was worded in *moving average* language. *Multiplicativity* is ascertained if

$$T = (\widehat{\alpha_{s+1}} - \widehat{\alpha_s}\,\widehat{\alpha_1})/SE(\widehat{\alpha_{s+1}})$$

Is not statistically significant where T is t-distributed.

Estimation of the model parameters is done via a non-linear optimization process for the mixed ARMA process.

Often more than one model is entertained based on empirical evidence. Model selection out of the contending ones is done using model identification tools which are information criteria like Akaike Information Criterion (AIC), Schwarz criterion and Hannan-Quinn criterion. Model choice is based onthe minimization of the criteria. The Eviews software which uses the least squares technique is to be used for this work.

3. Results and Discussion

The analysis of the series was restricted to the daily exchange rates from 22nd September, 2015 to 9th March, 2016, that is, 270 values. The remaining 7 values were used to compare with forecasts for the ascertainment of the adequacy or otherwise of the fitted model.

The time-plot of Figure 1 shows a generally positive trend depicting relative depreciation of the

Naira within the time period of interest. The Augmented Dickey Fuller (ADF) Test statistic of the series is of value -1.58. With the 1%, 5% and 10% critical values of -3.47, -2.88 and -2.58 respectiively, the ADF test adjudges the original series as non-stationary. Therefore a 7-day differencing is done. This yields a series with the time-plot of Figure 2 which depicts a generally horizontal trend and a correlogram of figure 3 showing a seasonal nature of period 7 days. The ADF test statistic for these differences is of value -2.21 which on the basis of the same critical values given above makes the null unit-root hypothesis not rejected at the above significant levels. Therefore they are also non-stationary. A further non-seasonal differencing yields a series with the plot of Figure 4 and the correlogram of Figure 5. Evident is a stationary nature which is confirmed by the ADF test with statistics equal to -11.01. Applying the algorithm of Etuk and Ojekudo (3) the SARIMA$(1,1,0)x(1,1,0)_{12}$ estimated in Table 1 is

$$X_t = 0.2037X_{t-1} - 0.6105X_{t-7} + 0.1200X_{t-8} + \varepsilon_t \qquad (4)$$
$$(\pm 0.0809) \quad (\pm 0.0695) \quad (\pm 0.0834)$$

where $\{X_t\}$ is the difference of the seasonal difference of the exchange rates. Clearly the lag 8 coefficient of model (4) is not statistically significant, being less than twice its standard error. That suggests the adoption of the *additive* model, which as given in Table 2 is estimated as

$$X_t = 0.1376X_{t-1} - 0.5900X_{t-7} + \varepsilon_t \qquad (5)$$
$$(\pm 0.0664) \quad (\pm 0.0680)$$

which is clearly better than the model (4) on the basis of the information criteria: AIC, Schwarz criterion and the Hannan-Quinn criterion. Residuals of the model are of zero mean, median and skewness and so it might be said to be fairly normally distributed (see Figure 7). They are also mostly uncorrelated as evident from Figure 6. Moreover the out-of-sample forecasts agree closely with observed values for March 10 to March 16, 2016 (See Table 3).

Figure-1. Time plot of the exchange rates

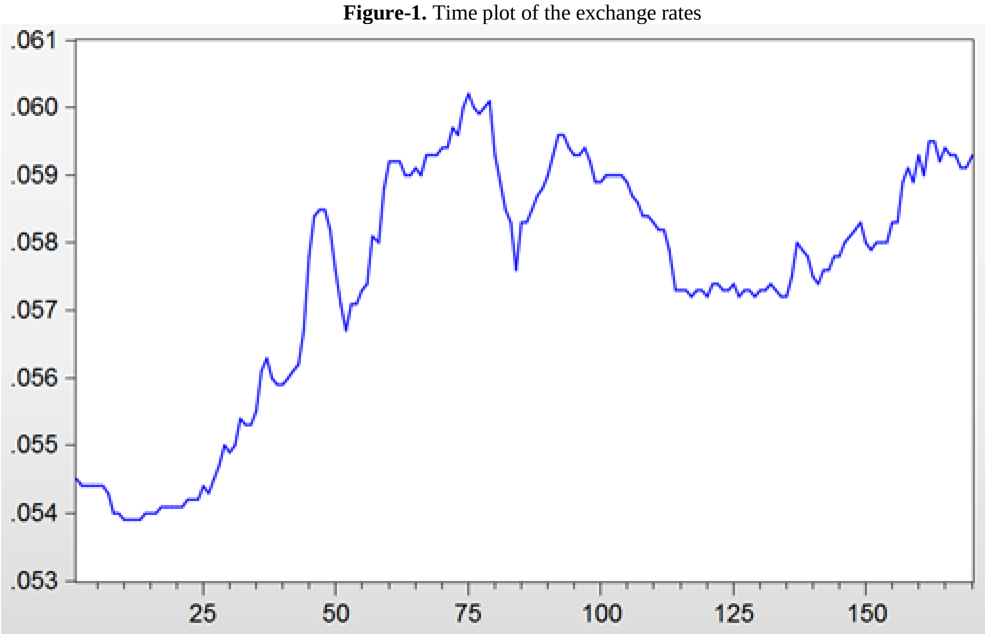


Figure-2. Time plot of the seasonal differences

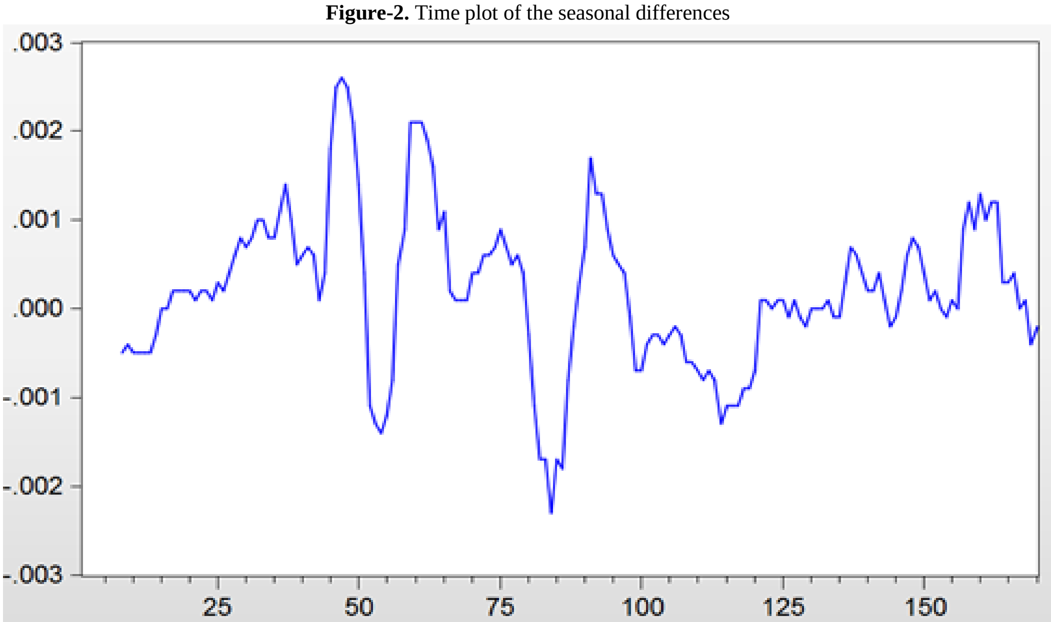
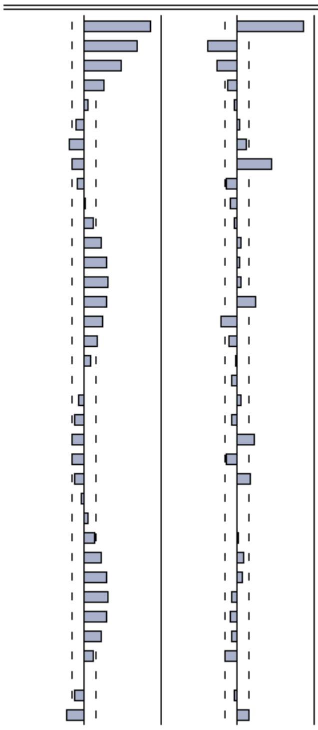


Figure-3. Correlogram of the seasonal differences



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.896	0.896	133.18	0.000
		2	0.723	-0.403	220.38	0.000
		3	0.502	-0.273	262.70	0.000
		4	0.270	-0.118	274.99	0.000
		5	0.057	-0.029	275.55	0.000
		6	-0.108	0.036	277.54	0.000
		7	-0.202	0.116	284.56	0.000
		8	-0.168	0.459	289.45	0.000
		9	-0.089	-0.149	290.84	0.000
		10	0.020	-0.086	290.91	0.000
		11	0.131	-0.035	293.94	0.000
		12	0.231	0.056	303.41	0.000
		13	0.299	0.026	319.43	0.000
		14	0.324	0.049	338.34	0.000
		15	0.308	0.249	355.62	0.000
		16	0.254	-0.220	367.44	0.000
		17	0.176	-0.109	373.16	0.000
		18	0.092	-0.016	374.72	0.000
		19	-0.001	-0.077	374.72	0.000
		20	-0.075	0.042	375.78	0.000
		21	-0.138	-0.075	379.36	0.000
		22	-0.170	0.227	384.85	0.000
		23	-0.169	-0.151	390.30	0.000
		24	-0.121	0.188	393.12	0.000
		25	-0.049	-0.004	393.59	0.000
		26	0.045	0.008	393.99	0.000
		27	0.135	0.017	397.60	0.000
		28	0.233	0.095	408.42	0.000
		29	0.297	0.081	426.12	0.000
		30	0.335	-0.067	448.85	0.000
		31	0.306	-0.090	467.86	0.000
		32	0.235	-0.067	479.23	0.000
		33	0.125	-0.156	482.48	0.000
		34	0.000	0.001	482.48	0.000
		35	-0.135	-0.027	486.31	0.000
		36	-0.236	0.161	498.08	0.000

Figure-4. Time plot of differences of the seasonal difference

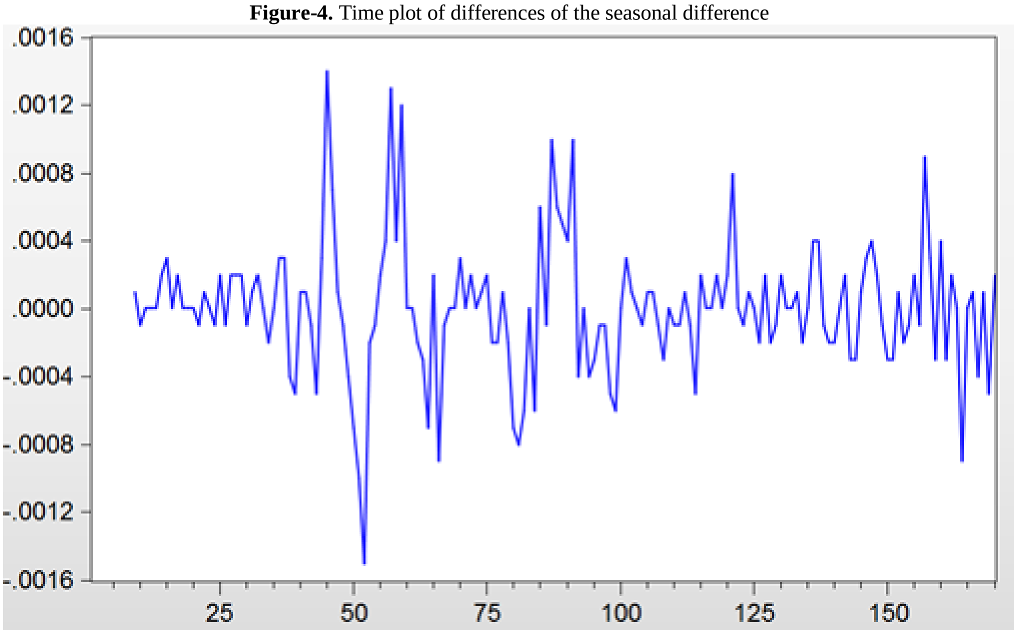
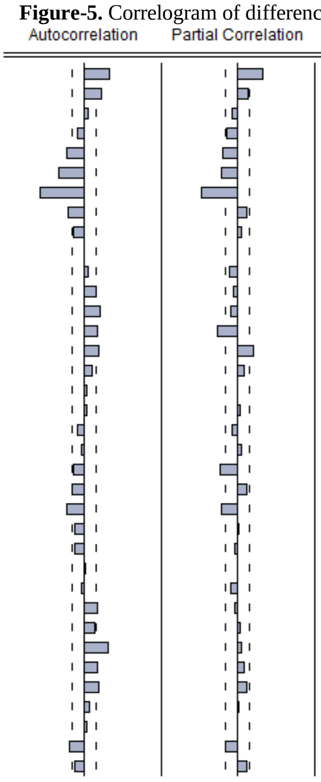


Figure-5. Correlogram of difference of the seasonal difference



	AC	PAC	Q-Stat	Prob
1	0.338	0.338	18.864	0.000
2	0.240	0.142	28.401	0.000
3	0.055	-0.072	28.908	0.000
4	-0.092	-0.143	30.329	0.000
5	-0.234	-0.192	39.566	0.000
6	-0.345	-0.223	59.800	0.000
7	-0.606	-0.493	122.68	0.000
8	-0.222	0.115	131.16	0.000
9	-0.145	0.052	134.82	0.000
10	-0.006	-0.007	134.82	0.000
11	0.050	-0.108	135.27	0.000
12	0.158	-0.057	139.68	0.000
13	0.207	-0.084	147.35	0.000
14	0.185	-0.266	153.47	0.000
15	0.193	0.212	160.21	0.000
16	0.111	0.082	162.46	0.000
17	0.038	-0.002	162.73	0.000
18	0.040	0.028	163.03	0.000
19	-0.085	-0.069	164.38	0.000
20	-0.045	0.044	164.76	0.000
21	-0.141	-0.245	168.53	0.000
22	-0.164	0.116	173.61	0.000
23	-0.230	-0.211	183.75	0.000
24	-0.118	0.018	186.45	0.000
25	-0.116	-0.049	189.05	0.000
26	0.013	-0.006	189.09	0.000
27	-0.033	-0.085	189.31	0.000
28	0.180	-0.036	195.74	0.000
29	0.132	0.040	199.24	0.000
30	0.323	0.045	220.25	0.000
31	0.188	0.082	227.40	0.000
32	0.203	0.119	235.78	0.000
33	0.079	0.020	237.08	0.000
34	0.039	0.005	237.39	0.000
35	-0.189	-0.170	244.90	0.000
36	-0.138	0.119	248.91	0.000

Table-1. Estimation of the sarim(1,1,,0)x(1,1,0) model

Dependent Variable: DSDUXNN1
Method: Least Squares
Date: 03/22/16 Time: 18:11
Sample (adjusted): 17 170
Included observations: 154 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.203658	0.080935	2.516325	0.0129
AR(7)	-0.610471	0.069542	-8.778511	0.0000
AR(8)	0.120007	0.083414	1.438699	0.1523

R-squared	0.414035	Mean dependent var	-1.30E-06
Adjusted R-squared	0.406274	S.D. dependent var	0.000399
S.E. of regression	0.000307	Akaike info criterion	-13.31801
Sum squared resid	1.43E-05	Schwarz criterion	-13.25885
Log likelihood	1028.487	Hannan-Quinn criter.	-13.29398
Durbin-Watson stat	2.035574		

Inverted AR Roots	.84-.40i	.84+.40i	.21-.91i	.21+.91i
	.20	-.58-.73i	-.58+.73i	-.93

Table-2. Estimation of the additive sarima model

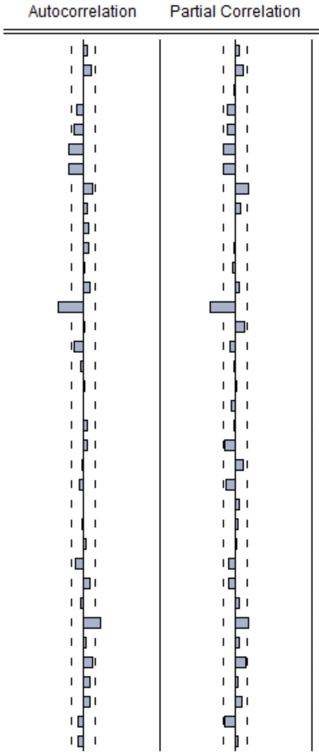
Dependent Variable: DSDUXNN1
Method: Least Squares
Date: 03/22/16 Time: 18:21
Sample (adjusted): 16 170
Included observations: 155 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.137633	0.066373	2.073628	0.0398
AR(7)	-0.589968	0.068039	-8.670989	0.0000

R-squared	0.405990	Mean dependent var	-1.29E-06
Adjusted R-squared	0.402107	S.D. dependent var	0.000398
S.E. of regression	0.000307	Akaike info criterion	-13.32400
Sum squared resid	1.45E-05	Schwarz criterion	-13.28473
Log likelihood	1034.610	Hannan-Quinn criter.	-13.30805
Durbin-Watson stat	1.882951		

Inverted AR Roots	.86-.40i	.86+.40i	.23+.90i	.23-.90i
	-.56+.72i	-.56-.72i	-.91	

Figure-6. Correlogram of the additive sarima model residuals

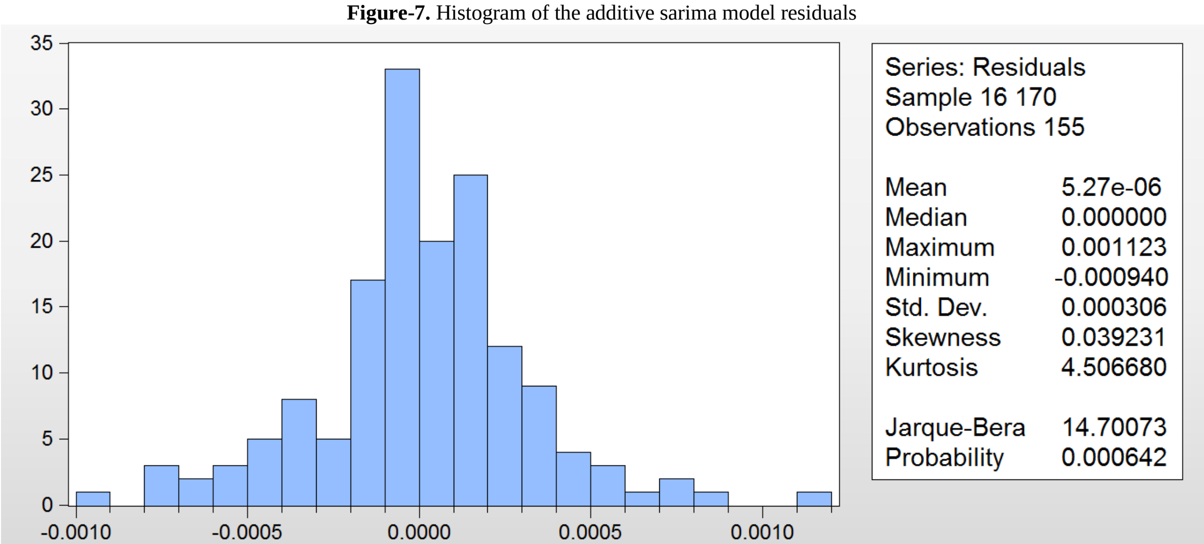


Lag	AC	PAC	Q-Stat	Prob
1	0.056	0.056	0.4927	
2	0.106	0.104	2.2932	
3	-0.002	-0.013	2.2938	0.130
4	-0.096	-0.108	3.7733	0.152
5	-0.117	-0.107	5.9774	0.113
6	-0.195	-0.169	12.214	0.016
7	-0.194	-0.168	18.372	0.003
8	0.126	0.175	21.020	0.002
9	0.051	0.071	21.450	0.003
10	0.072	-0.001	22.321	0.004
11	0.073	-0.009	23.227	0.006
12	0.016	-0.046	23.269	0.010
13	0.090	0.059	24.657	0.010
14	-0.337	-0.340	44.244	0.000
15	0.009	0.121	44.257	0.000
16	-0.129	-0.067	47.189	0.000
17	-0.039	-0.017	47.454	0.000
18	0.024	0.014	47.554	0.000
19	-0.006	-0.055	47.560	0.000
20	0.065	-0.014	48.329	0.000
21	0.044	-0.152	48.675	0.000
22	-0.009	0.105	48.690	0.000
23	-0.058	-0.133	49.307	0.000
24	0.002	0.059	49.307	0.001
25	-0.015	0.030	49.351	0.001
26	0.037	0.014	49.605	0.002
27	-0.113	-0.086	52.049	0.001
28	0.085	-0.095	53.431	0.001
29	-0.048	0.056	53.882	0.002
30	0.235	0.176	64.675	0.000
31	0.025	0.047	64.800	0.000
32	0.116	0.139	67.456	0.000
33	0.094	0.036	69.210	0.000
34	0.089	0.091	70.800	0.000
35	-0.077	-0.145	71.990	0.000
36	-0.070	0.035	72.981	0.000

Table-3. Out-of-sample comparision of forecasted and observed values

Days	Forecasted Rates	Observed Rates
March 10, 2016	0.0596	0.0594
March 11, 2016	0.0599	0.0592
March 12, 2016	0.0597	0.0593
March 13, 2016	0.0600	0.0592
March 14, 2016	0.0597	0.0593
March 15, 2016	0.0600	0.0593
March 16, 2016	0.0601	0.0591

Figure-7. Histogram of the additive sarima model residuals



4. Conclusion

It may be concluded that daily UGX-NGN exchange rates follow an additive SARIMA (1,1,0)x(1,1,0)$_7$ model. Forecasting and simulation of the series may therefore be based on the proposed model (5). The algorithms of Suhartono [15] and Etuk and Ojekudo [16] guarantee parametric parsimony. Precision should however not be sacrificed on the altar of parsimony. The relative benefits and demerits of the novel approach vis-a-vis the traditional ones should be investigated with a view to improving upon the modelling procedure.

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