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# The Relationship between GDP and Co2 Emission in Nigeria Using the Least Square Polynomials

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**Abstract:** Nigeria belongs to the group of Next Eleven (N-11), which is a group of 11 countries identified by the investment bank Goldman Sachs in 2007 as having high potential of becoming the world's largest economies in the 21st century. There is a need to determine how it can attain economic growth while conserving energy and reducing emission. This paper looks at the relationship between GDP and $CO_2$ emission in the light of the Environmental Kuznet Curve (EKC). The method of least square polynomials was employed. The results obtained aligned with the EKC hypothesis.

**Keywords:** Environmental kuznet curves; Economic growth; $CO_2$ emission; Least square polynomials.

## 1. Introduction

Empirical researchers have characterized the relationship between economic development and environmental pollution with the environmental Kuznets curve (EKC). Environmental Kuznets Curve states that environmental degradation increases, when countries are in the transition stage of development, but then declines after a threshold level [1, 2]. In the initial stage of development, economies often rely on heavy infrastructure projects, which lead to environmental degradation due to emissions of various pollutants, such as carbon dioxide, sulphur, and nitrogen oxides [3, 4]. Daly [5] argues that increased extraction of natural resources, increased concentration of pollutants and accumulation of waste will therefore results in the environmental degradation and a decline in human welfare, despite rising incomes. Hence, in the initial stage of development, there is some degradation of environment [4]. However, after a threshold level, high sustained economic growth recovers the quality of life and reduces emission of various pollutants. Hence, over the passage of time, the effluence absorption intensity will turn down (see for example Beckerman [6], [7, 8].

Carbon dioxide emissions have grown dramatically in the past century because of human activities, mainly by the use of fossil fuels as well as changes in land use that are directly linked with economic growth and development. The causal relationship between economic development and different indicators of environmental quality has been extensively explored in the recent years by the Environmental Kuznets Curve (EKC) models globally, regionally or country wise by several authors.

Nigeria belong to the group of Next Eleven (N-11), which is a group of 11 countries identified by the investment bank Goldman Sachs in 2007 as having high potential of becoming the world's largest economies in the 21st century [9]. The N-11 consists of Bangladesh, Egypt, Indonesia, Iran, Mexico, Nigeria, Pakistan, Philippines, Turkey, South Korea, and Vietnam. These countries would face issues concerning economic growth and $CO_2$ emission. The N-11 need to determine how they can attain economic growth while conserving energy and reducing emission, [10].

$CO_2$ emissions are directly related to the use of energy, which is an essential factor in the world economy, both for production and consumption. Therefore, the relationship between $CO_2$ emissions and economic growth (see figure 1) has important implications for environmental and economic policies, [11]. Our interest in this paper is the relationship between GDP and $CO_2$ emission in the light of the Environmental Kuznet Curve (EKC). We employed the method of least square polynomials. The results obtained aligned with the EKC hypothesis.

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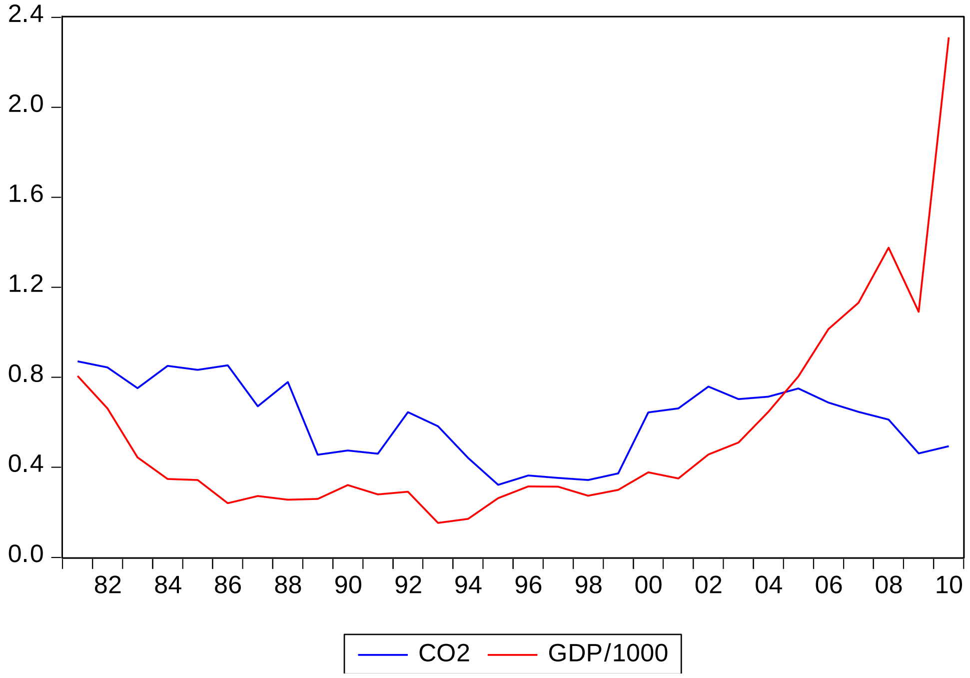
**Figure-1.** The relationship between $CO_2$ emissions and economic growth




## 2. Methodology

We can specify the EKC as

$$y = f(x)$$

Where y is the representation of the environmental quality in this case represented by $CO_2$ emission per capita. $x$ is the representation of economic growth with GDP per Capita as our choice.

We seek to approximate the function $f(x)$ by a polynomial $P_n(x)$ of degree at most n. For $P_n(x)$ to be a good approximation for the function $f(x)$, it must minimise the error

$$|f(x) - P_n(x)|$$

where |.| is a specified measure of deviation. We first show that such a $P_n(x)$ exist and that it is unique and we go ahead to get the polynomials of degree 2 and 3.

**Theorem 1: Weierstrass Theorem**

For any function$f(x)$, continuous on a closed and bounded interval [a,b], there exist an $\varepsilon > 0$ and a polynomial $P_n(x)$ , n$\epsilon N$ such that

$$| f(x) - P_n(x)| < \varepsilon \text{ for all } x \epsilon [a, b]. \qquad (1)$$

By the Weierstrass theorem, we are assured that provided $f(x)$ is continuous, we can get a sequence of polynomial approximating the function in a closed and bounded interval so the existence of $P_n(x)$ is guaranteed.

Taking the $L_2$ norm or mean square error as our measure of deviation we define the mean square error by

$$||f(x) - P_n(x)||_2 = \sum_{I=1}^{n} (P_n(x) - y_i)^2 = \sum_{i=1}^{n}(\sum_{k=0}^{m} (a_k x^k) - y_i)^2, \qquad (2)$$

where

$$P_n(x) = \sum_{k=0}^{m} a_k x^k.$$

In the $L_2$ norm, we can get a unique $P_n(x)$ which minimises the error.

**Theorem 2**

For each approximate function $f(x)$, there is a unique least square polynomial approximation of degree not more than n for which $||f(x) - Pn(x)||_2$ is minimised.

Theorem 2 guarantees the uniqueness of a least square polynomial for the function $(x)$ . Expressing

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

we define

$$\phi(a_0, \ldots, a_n) = \sum_{i=1}^{n} (Pn(x) - y_i)^2.$$

We seek to find the coefficients $(a_0, a_1, \ldots, a_n)$ that minimises $\phi$. $\phi$ is minimum if

$$\frac{\partial}{\partial u} \phi(a_0, \ldots, a_m) = 0, \; j = 0,1, \ldots, m.$$

That mean

$$\frac{\partial \phi}{\partial a_0} = \sum_{i=1}^{n} 2\left[\sum_{k=0}^{m} \left(a_k x_i^k\right) - y_i\right] = 0,$$

$$\frac{\partial \phi}{\partial a_1} = \sum_{i=1}^{n} 2\left[\sum_{k=0}^{m} \left(a_k x_i^k\right) - y_i\right](x_i) = 0$$

and

$$\frac{\partial \phi}{\partial a_m} = \sum_{i=1}^{n} 2\left[\sum_{k=0}^{m}\left(a_k x_i^k\right) - y_i\right]\left(x_i^m\right) = 0.$$

Rearranging we get the m+1 normal equations for the (m+1) unknown coefficient $(a_0, a_1, \ldots, a_n)$ as

$$a_0 + a_1 \sum x_i + \ \ldots + a_m \sum x_i^m = \sum y_i,$$
$$a_0 \sum x_i + a_1 \sum x_i^2 \ldots + a_m \sum x_i^{m+1} = \sum y_i \, x_i.$$
$$a_0 \sum x_i + a_1 \sum x_i^2 \ldots + a_m \sum x_i^{m+1} = \sum y_i \, x_i^2$$

and

$$.a_0 \sum x_i^m + a_1 \sum x_i^2 \ldots + a_m \sum x_i^{2m} = \sum y_i \, x_i^m.$$

These normal equations have a unique solution provided the $x_i$'s are distinct. [12].
Having obtained our $a_i$'s we can specify our EKC as linear, quadratic and cubic polynomials functions.

**i.** The **linear function** is given as

$$y_t = a_o + a_i x_t + \varepsilon_t, \qquad (5)$$

where $y_t$ is $CO_2$ emission, $\boldsymbol{x_t}$ is GDP per capita, $\boldsymbol{a_o}$ is the intercept and $\boldsymbol{\varepsilon_t}$ is an independent and identically distributed (iid) white noise.
If $a_1 > 0$, there is a linearly increasing relationship between $y_t$ and $x_t$. If however $a_1 < 0$, there is a decreasing linear relationship. If $a_1 = 0$, there is no relationship.

**ii.** **Quadratic function** is given as

$$y_t = a_o + a_1 x_t + a_2 x_t^2 + \varepsilon_t \qquad (6)$$

If $\alpha_2 > 0$, then we have the inverted U-curve which satisfies the EKC hypothesis. If $\alpha_2 < 0$, then we have a U-shaped curve.
From equation (6),

$$\frac{dy_t}{dx_t} = a_1 + 2a_2 x_t. \qquad (7)$$

Equating (7) to zero and making $x_t$ the subject we have,

$$x_t = \frac{-a_1}{2a_2}. \qquad (8)$$

Equation (8) is the turning point.

**iii.** The **Cubic function** is given by

$$y_t = a_o + a_1 x_t + a_2 x_t^2 + a_3 x_t^3 + \ \varepsilon_t \qquad (9)$$

Here, when $\alpha_2 < 0$, we have the inverted U-curve, satisfying the EKC hypothesis. If $a_1 > 0$, $\alpha_2 < 0$ and $\alpha_3 > 0$, we have an N-shaped curve. This shows that after an improvement in the environmental quality have been achieved; there is a lapse back to environmental degradation. From (9), we have

$$\frac{dy_t}{dx_t} = a_1 + 2a_2 x_t + 3a_3 x_t^2 \qquad (10)$$

$$\frac{d^2 y_t}{dx_t^2} = 2a_2 + 6a_3 x_t \qquad (11)$$

Equating (11) to zero and making $x_t$ the subject, we have

$$x_t = \frac{-a_2}{3a_3} \qquad (12)$$

$x$ is the inflexion point which could be increasing or decreasing.

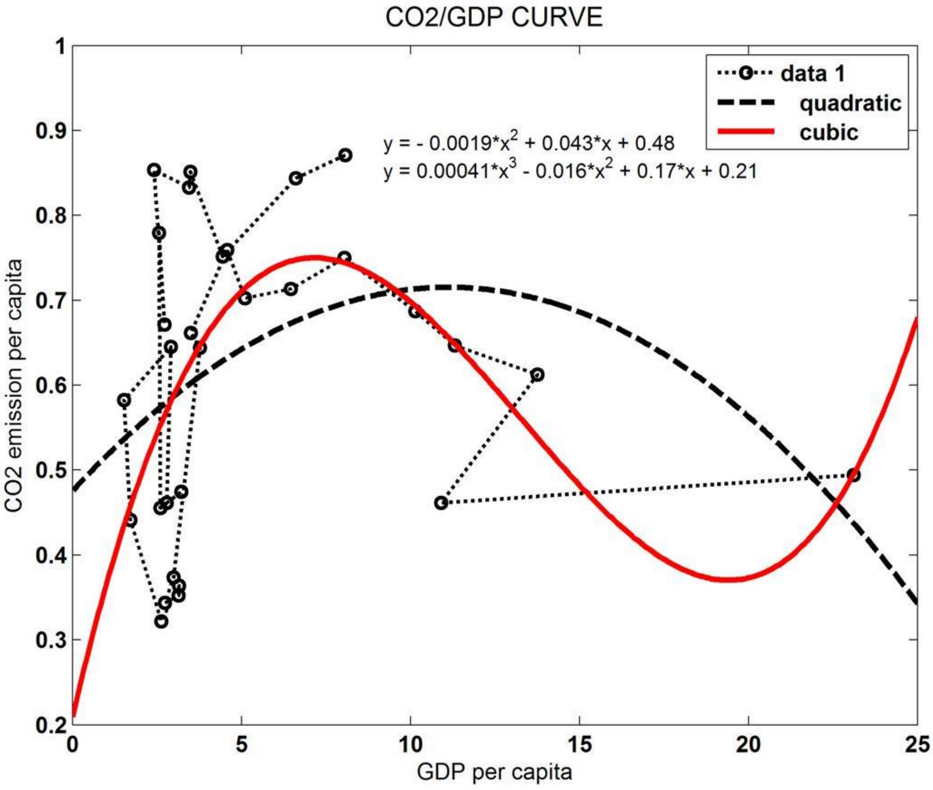
## 2.1. Data Source and Sample

The data used in this paper are the annual time series data for Nigeria from 1981 to 2010 (see table 1). The data were obtained from World Bank's World Development Indicators. The variables employed are CO2 emissions (metric tons per capita), GDP per capita (current US$),

**Table-1.** The annual time series data for Nigeria from 1981 to 2010

| SUMMARY STATISTICS | | |
|---|---|---|
| | CO2 | GDP |
| Mean | 0.613497 | 546.1820 |
| Median | 0.645941 | 346.3337 |
| Maximum | 0.870972 | 2310.861 |
| Minimum | 0.322040 | 153.0762 |
| Std. Dev. | 0.174353 | 459.4922 |
| Skewness | -0.194894 | 2.240809 |
| Kurtosis | 1.763665 | 8.438155 |
| Jarque-Bera | 2.100572 | 62.07303 |
| Probability | 0.349838 | 0.000000 |
| Sum | 18.40492 | 16385.46 |
| Sum Sq. Dev. | 0.881569 | 6122860. |
| Observations | 30 | 30 |

**Figure-2.** The N curve of the CO2 emission and the GDP per capita




## 3. Results and Discussions

The results indicate that the GDP data is normally distributed while the $CO_2$ emission is not normally distributed see (table 1). It indicate also that Nigeria has an inverted U curve as can be seen in the coefficients of the polynomials of degree two and three respectively, that is

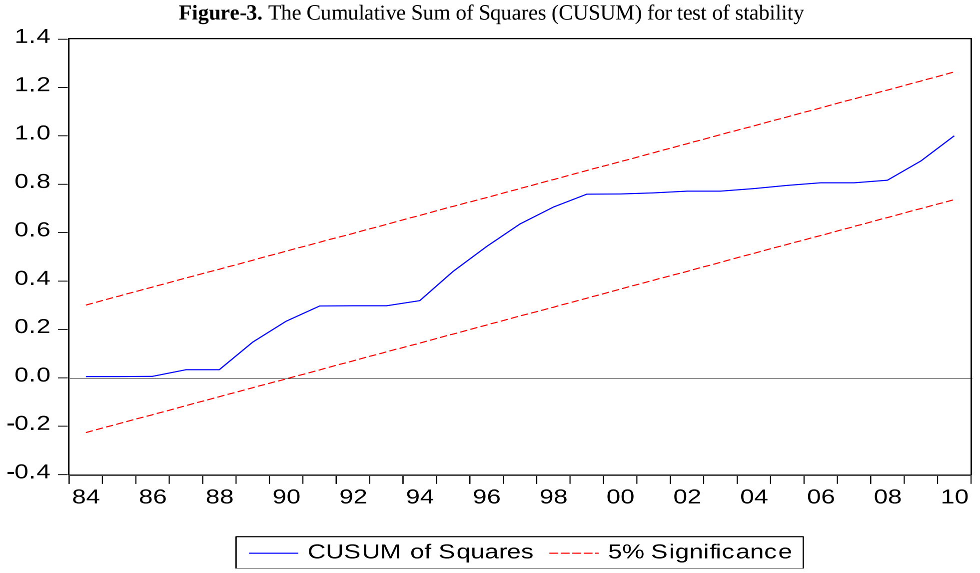
$$-0.0019x^2 + 0.043x + 048$$

$$0.00041x^3 - 0.016x^2 + 0.17x + 0.21$$

The turning point is 11.36 while the point of inflexion is 13.01, these fall in the periods between 2006 and 2008. This corresponds with the period when the GDP was rising steadily

and the carbon emission began to decline. This is in agreement with the EKC hypothesis. The N curve (see figure 2) indicates the tendency of the $CO_2$ emission increasing. The Cumulative Sum of Squares (CUSUM) was used to test the stability of the system and as can be seen in figure 3, it lies within the 5% confidence interval, hence we claim stability.

**Figure-3.** The Cumulative Sum of Squares (CUSUM) for test of stability



## 4. Conclusion

We can conclude that Nigeria's CO2 emission and GDP follow the Environment Kuznet Curve. The emission was increasing and get to the turning point and started decreasing. It however started increasing again, indicating that the battle against environmental degradation is not one to be won once and for all. A concerted effort has to be used to sustain the quality of the environment.

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