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Interval-Valued Intuitionistic L-Fuzzy Strong β -Filters On β - Algebras

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Abstract: In 2002, J. Neggers and H. S. Kim introduced the notion of β-algebra. The theory of fuzzy sets proposed by L. A. Zadeh in 1965 is generalized in 1986 by K. T. Atanassov an intuitionistic fuzzy sets. The notion of β -filters was introduced by Henri Cartan in 1937. In 1991, C.S. Hoo introduced the concept of the β -filters in BCI-algebras and Satyanarayana, *et al.* [2], introduced interval-valued intuitionistic fuzzy ideals in BF-algebras. In this paper, we define the notion of an Interval-valued intuitionistic L-fuzzy strong β –filters on β-algebra and investigate some of their properties.

Keywords: β -algebras; β -filters; L-fuzzy strong β -filters; interval-valued intuitionistic fuzzy sets; Interval-valued intuitionistic L-fuzzy strong β -filters on β -algebras.

1. Introduction

In 2002, Neggers and Kim [1], introduced the notion of β-algebra. The theory of fuzzy sets proposed by Zadeh [3] in 1965 is generalized in 1986 by Atanassov [4] an Intuitionistic fuzzy sets. Then many researchers have been engaged in extending the concepts and results of abstract algebra. The notion of β-filters was introduced by Henri Cartan in 1937. In Hoo [5] introduced the concept of the β-filters in BCI-algebras. The notion of interval-valued fuzzy set was first introduced by Zadeh [6] as an extension of fuzzy sets. In K. Sujatha and others [7], [8], [9] introduced the notions of Fuzzy β-filters and Intuitionistic Fuzzy β-filters in β-algebras. In this paper, we discuss the concept of interval-valued intuitionistic L-fuzzy strong β-Filter on β–algebras.

2. Preliminares

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1 A β-algebra is a non-empty set X with constant 0 and two binary operations + and - satisfying the following axioms:

(1) $x-0=x$

(2) $(0-x)+x=0$

(3) $(x-y)-z=x-(z+y)\ \forall\ \mathrm{x, y, z} \in \mathrm{X}$

Definition 2.2 Let X and Y be two β-algebras. A mapping $f: X \to Y$ is said to be a β-homomorphism, if $f(x+y)=f(x)+f(y)$ and $f(x-y)=f(x)-f(y)$ for all $x, y \in X$.

Definition 2.3 Let X be a β-algebra and A a β-subalgebra. A is said to be β-filter on X, if for all $x, y \in X$ $\mathrm{x}\Delta\mathrm{y}=\mathrm{x}+(\mathrm{x}+\mathrm{y})$ and $\mathrm{x}\nabla\mathrm{y}=\mathrm{x}-(\mathrm{x}-\mathrm{y})\in \mathrm{A}$

Definition 2.4 Let X be a β-algebra and A be an intuitionistic fuzzy β-subalgebra. A is said to be intuitionistic fuzzy β-filter on X , if it satisfies for all $x, y \in X$.

(1) $\mu_A(x\Delta y) \geq \min\{\mu_A(x), \mu_A(x+y)\}$ and

$\lambda_A(x\Delta y) \leq \max\{\lambda_A(x), \lambda_A(x+y)\}$

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(2) $\mu_A(x\nabla y) \geq \min\{\mu_A(x), \mu_A(x-y)\}$ and
$\lambda_A(x\nabla y) \leq \max\{\lambda_A(x), \lambda_A(x-y)\}$

(3) $\mu_A(y) \geq \mu_A(x)$ and $\lambda_A(y) \leq \lambda_A(x)$ if $x \leq y$

Definition 2.5 Let X be a β-algebra and A be an intuitionistic fuzzy β-subalgebra. A is said to be intuitionistic L-fuzzy β-filter on X, if it satisfies for all $x, y \in X$.

(1) $\mu_A(x\Delta y) \geq \{\mu_A(x) \wedge \mu_A(x+y)\}$ and
$\lambda_A(x\Delta y) \leq \{\lambda_A(x) \vee \lambda_A(x+y)\}$

(2) $\mu_A(x\nabla y) \geq \{\mu_A(x) \wedge \mu_A(x-y)\}$ and
$\lambda_A(x\nabla y) \leq \max\{\lambda_A(x) \vee \lambda_A(x-y)\}$

(3) $\mu_A(y) \geq \mu_A(x)$ and $\lambda_A(y) \leq \lambda_A(x)$ if $x \leq y$.

Definition 2.6 Let X be a β-algebra and A be an Intuitionistic L-fuzzy β-subalgebra. A is said to be an Intuitionistic L-fuzzy strong β-filter on X, if it is satisfies for all $x, y \in X$.

(1) $\mu_A(x\Delta y) = \mu_A(x\nabla y)$

(2) $\lambda_A(x\Delta y) = \lambda_A(x\nabla y)$

(3) $\mu_A(y) \geq \mu_A(x)$ and $\lambda_A(y) \leq \lambda_A(x)$ if $x \leq y$.

By interval number D we mean an interval $[a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$. For interval numbers $D_1 = [a_1^-, b_1^+]$, $D_2 = [a_2^-, b_2^+]$.

We define

- $\min(D_1, D_2) = D_1 \cap D_2 = \min([a_1^-, b_1^+], [a_2^-, b_2^+])$
$= [\min\{a_1^-, a_2^-\}, \min\{b_1^+, b_2^+\}]$
- $\max(D_1, D_2) = D_1 \cup D_2 = \max([a_1^-, b_1^+], [a_2^-, b_2^+])$
$= [\max\{a_1^-, a_2^-\}, \max\{b_1^+, b_2^+\}]$

and put

- $D_1 \leq D_2 \Leftrightarrow a_1^- \leq a_2^-$ and $b_1^+ \leq b_2^+$
- $D_1 = D_2 \Leftrightarrow a_1^- = a_2^-$ and $b_1^+ = b_2^+$,
- $D_1 < D_2 \Leftrightarrow D_1 \leq D_2$ and $D_1 \neq D_2$
- $mD = m[a_1^-, b_1^+] = [ma_1^-, mb_1^+]$, where $0 \leq m \leq 1$.

It is obvious that $(D[0,1], \leq, \vee, \wedge)$ is a complete lattice with $[0, 0]$ as its least element and $[1,1]$ as its greatest element. We now use $D[0,1]$ to denote the set of all closed sub intervals of the interval $[0,1]$.

For interval numbers $D_1 = [a_1^-, b_1^+], D_2 = [a_2^-, b_2^+] \in D[0,1]$ we define

- $D_1 + D_2 = [a_1^- + a_2^- - a_1^-.a_2^-, b_1^+ + b_2^+ - b_1^+.b_2^+]$.

Let L be a given nonempty set. An interval-valued fuzzy set B on L is defined by $B = \{(x, [\mu_B^-(x), \mu_B^+(x)] : x \in L\}$, Where $\mu_B^-(x)$ and $\mu_B^+(x)$ are fuzzy sets of L such that $\mu_B^-(x) \leq \mu_B^+(x)$ for all $x \in L$. Let $\tilde{\mu}_B(x) = [\mu_b^-(x), \mu_B^+(x)]$, then $B = \{(x, \tilde{\mu}_B(x)) : x \in L\}$ Where $\tilde{\mu}_B : L \to D[0,1]$

3. Interval-Valued Intuitionistic L-Fuzzy Strong β-Filter

In this section, we introduce the notion of interval-valued intuitionistic L-fuzzy strong β-filter on a β-algebra. But we begin with the definition and example of Interval-valued intuitionistic L-fuzzy β-filter on a β-algebra.

Definition 3.1 Let X be a β-algebra and A be an interval-valued intuitionistic fuzzy β-subalgebra. A is said to be interval-valued intuitionistic fuzzy β-filter on X, if it satisfies for all $x, y \in X$.

(1) $\tilde{\mu}_A(x\Delta y) \geq \min\{\tilde{\mu}_A(x), \tilde{\mu}_A(x+y)\}$ and

$\tilde{\lambda}_A(x\Delta y) \leq \max\{\tilde{\lambda}_A(x), \tilde{\lambda}_A(x+y)\}$

(2) $\tilde{\mu}_A(x\nabla y) \geq \min\{\tilde{\mu}_A(x), \tilde{\mu}_A(x-y)\}$ and

$\tilde{\lambda}_A(x\nabla y) \leq \max\{\tilde{\lambda}_A(x), \tilde{\lambda}_A(x-y)\}$

(3) $\tilde{\mu}_A(y) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(y) \leq \tilde{\lambda}_A(x)$ if $x \leq y$.

Definition 3.2 Let X be a β-algebra and A be an interval-valued intuitionistic fuzzy β-subalgebra. A is said to be interval-valued intuitionistic L-fuzzy β-filter on X, if it satisfies for all $x, y \in X$.

(1) $\tilde{\mu}_A(x\Delta y) \geq \{\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(x+y)\}$ and

$\tilde{\lambda}_A(x\Delta y) \leq \{\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(x+y)\}$

(2) $\tilde{\mu}_A(x\nabla y) \geq \{\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(x-y)\}$ and

$\tilde{\lambda}_A(x\nabla y) \leq \{\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(x-y)\}$

(3) $\tilde{\mu}_A(y) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(y) \leq \tilde{\lambda}_A(x)$ if $x \leq y$.

Example 3.3 Let $X = \{0,1,2,3\}$ be a β-algebra with constant 0 and two binary operation + and – defined on X with the Cayley's table. Let $[0, 0] \leq \tilde{t}_1 \leq \tilde{t}_2 \leq [1, 1]$, where $t_1 \leq t_2 \in L$.

+	0	1	2	3
0	0	0	0	0
1	1	2	1	0
2	1	2	2	3
3	0	1	3	3

-	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

$A = \{2,3\}$ is an interval-valued intuitionistic L-fuzzy β-subalgebra, defined by,

$$\tilde{\mu}_A = \begin{cases} \tilde{t}_1, if x = 2 \\ \tilde{0}, otherwise \end{cases} \text{ and } \tilde{\lambda}_A = \begin{cases} \tilde{t}_2, if x = 2 \\ \tilde{0}, otherwise \end{cases}$$

One can observe that A is not an interval-valued intuitionistic L-fuzzy strong β-filter on X.

Definition 3.4 Let X be a β-algebra and A be an interval-valued Intuitionistic L-fuzzy β-subalgebra. A is said to be an interval-valued Intuitionistic L-fuzzy strong β-filter on X, if it satisfies for all $x, y \in X$.

(1) $\tilde{\mu}_A(x\Delta y) = \tilde{\mu}_A(x\nabla y)$

(2) $\tilde{\lambda}_A(x\Delta y) = \tilde{\lambda}_A(x\nabla y)$

(3) $\tilde{\mu}_A(y) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(y) \leq \tilde{\lambda}_A(x)$ if $x \leq y$.

Example 3.5 Let $X = \{0,1,2,3\}$ be a β-algebra with constant 0 and two binary operations + and – defined on X with the Cayley's table. Let $[0,0] \leq \tilde{t}_1 \leq \tilde{t}_2 \leq [1,1]$.

+	0	1	2	3
0	0	0	0	0
1	1	2	1	3
2	0	3	2	2
3	3	1	3	3

-	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

$A=\{2,3\}$ is an interval-valued intuitionistic L-fuzzy β-subalgebra, defined by the membership function and non membership function:

$$\tilde{\mu}_A=\begin{cases}\tilde{0}, & \text{if } x=2\\ \tilde{t}_1, & \text{if } x=3\end{cases} \text{ and } \tilde{\lambda}_A=\begin{cases}\tilde{1}, & \text{if } x=2\\ \tilde{t}_2, & \text{if } x=3\end{cases}$$

Then A is an interval-valued L-fuzzy β-filter on X.

Theorem 3.6 Every interval-valued L-fuzzy strong β-filter is also an interval-valued intuitionistic L-fuzzy β-subalgebra.

Proof: Directly follows from our definition of interval-valued intuitionistic L-fuzzy strong β-filter.

The following example shows that the converse part of the above theorem need not be true.

Example 3.7 Using the example 3. 3, A is an interval-valued intuitionistic L-fuzzy β-subalgebra but an intuitionistic L-fuzzy strong β-filter on X.

Since $\tilde{\mu}_A(2\Delta 3)\neq\tilde{\mu}_A(2\nabla 3)\Rightarrow\tilde{\mu}_A(3)\neq\tilde{\mu}_A(2)\Rightarrow\tilde{0}\neq\tilde{t}_1$

Theorem 3.8 Every interval-valued L-fuzzy strong β-filter is also an interval-valued intuitionistic L-fuzzy β-filter.

Proof: Obvious

Remark 3.9 The converse part of the above theorem need not be true, as seen in example 3.3 which is an interval-valued intuitionistic L-fuzzy β-filter but not an interval-valued intuitionistic L-fuzzy strong β-filter.

Lemma 3.10 Intersection of any two interval-valued intuitionistic L-fuzzy β-filters is an interval-valued intuitionistic L-fuzzy strong β-filter.

Theorem 3.11 If A is an interval-valued intuitionistic L-fuzzy β-filter on X, then $\tilde{\mu}_A(x\Delta y)\geq\tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x\nabla y)\leq\tilde{\lambda}_A(y)$, where $x\geq y$.

Proof: Since A is an interval-valued intuitionistic L-fuzzy β-filter of X,

$$\tilde{\mu}_A(x\Delta y)\geq\{\tilde{\mu}_A(x)\wedge\tilde{\mu}_A(x+y)\}$$
$$=\{\tilde{\mu}_A(x)\wedge\{\tilde{\mu}_A(x)\wedge\tilde{\mu}_A(y)\}\}, \text{ using the theorem 3.6}$$

Thus,

$$\tilde{\mu}_A(x\Delta y)=\{\tilde{\mu}_A(x)\wedge\tilde{\mu}_A(y)\} \qquad \text{Since } x\leq y\Rightarrow\tilde{\mu}_A(y)\geq\tilde{\mu}_A(x)$$
$$=\tilde{\mu}_A(y)$$

Similarly, we can prove that,

$$\tilde{\lambda}_A(x\nabla y)\leq\{\tilde{\lambda}_A(x)\vee\tilde{\lambda}_A(y)\} \quad \text{Since } x\leq y\Rightarrow\tilde{\lambda}_A(y)\leq\tilde{\lambda}_A(x)$$
$$=\tilde{\lambda}_A(y)$$

Theorem 3.12 Let f be an onto β-homomorphism from X to Y. If B is an interval-valued intuitionistic L-fuzzy strong β-filter of Y, then $f^{-1}(B)$ is also an interval-valued intuitionistic L-fuzzy strong β-filter on X.

Proof: Let B is an interval-valued intuitionistic L-fuzzy strong β-filter of Y, $x,y\in X$,

$$f^{-1}(\tilde{\mu}_B(x\Delta y))=f^{-1}(\tilde{\mu}_B(x+(x+y)))$$
$$\geq\{\tilde{\mu}_B(f(x))\wedge\tilde{\mu}_B(f(x+y))\}$$
$$=f^{-1}(\tilde{\mu}_B(x))\wedge f^{-1}(\tilde{\mu}_B(x+y))$$

Let $x,y\in X$ be such that $x\geq y$. Since B is an Interval-valued L-fuzzy strong β-filter, we have $f^{-1}(\tilde{\mu}_B(y))=\tilde{\mu}_B(f(y))\geq\tilde{\mu}_B(f(x))=f^{-1}(\tilde{\mu}_B(x))$ such that $f^{-1}(\tilde{\mu}_B(y))\geq f^{-1}(\tilde{\mu}_B(x))$. Similarly

$$f^{-1}(\tilde{\lambda}_B(x\Delta y)) = f^{-1}(\tilde{\lambda}_B(x+(x+y)))$$
$$\leq \{\tilde{\lambda}_B(f(x)) \vee \tilde{\lambda}_B(f(x+y))\}$$
$$= f^{-1}(\tilde{\lambda}_B(x)) \vee f^{-1}(\tilde{\lambda}_B(x+y))$$

Let $x, y \in X$ be such that $x \geq y$. Since B is an Interval-valued L-fuzzy strong β-filter, we have $f^{-1}(\tilde{\lambda}_B(y)) = \tilde{\lambda}_B(f(y)) \leq \tilde{\lambda}_B(f(x)) = f^{-1}(\tilde{\lambda}_B(x))$ such that $f^{-1}(\tilde{\lambda}_B(y)) \leq f^{-1}(\tilde{\lambda}_B(x))$.

Thus we can conclude that $f^{-1}(B)$ is an interval-valued intuitionistic L-fuzzy β-filter on X.

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