Academic Journal of Applied Mathematical Sciences
ISSN(e): 2415-2188, ISSN(p): 2415-5225
Vol. 4, Issue. 10, pp: 119-128, 2018
URL: http://arpgweb.com/?ic=journal&journal=17&info=aims

Academic Research Publishing Group

Original Research

# Determining the Impact of Variation of Harvesting Effort on the Qualitative Behaviour of a Coexistence Steady State Solution and Its Stability in Prey-Predator Fishery Model

**Agwu I. A.***
Department of Mathematics, Abia State Polytechnics, Aba, Abia State, Nigeria

**Inyama S. C.**
Department of Mathematics, Federal University of Technology, Owerri, Imo State, Nigeria

**Umana R. A.**
Department of Mathematics, Federal University of Technology, Owerri, Imo State, Nigeria

**Omame A.**
Department of Mathematics, Federal University of Technology, Owerri, Imo State, Nigeria

**Ukanwoke N.**
Department of Mathematics, Federal University of Technology, Owerri, Imo State, Nigeria

**Ofomata A.**
Department of Mathematics and Statistics, Federal Polytechnics, Nekede, Owerri, Imo State, Nigeria

**Mbachu H. I.**
Department of Statistics, Imo State University, Owerri, Imo State, Nigeria

**Udofia E. S.**
Department of Mathematics and Statistics, Akwa Ibom University, Nigeria

**Uwakwe J. I.**
Department of Mathematics, Alvan Ikoku Federal College of Education, Owerri, Imo State, Nigeria

## Abstract

In this paper, we consider a prey-predator fishery model in a three –patch aquatic habitat with selective harvesting of predator and prey populations. We attempt to study the qualitative behaviour of stability and co-existence steady state solution in an interaction between prey and predator populations due to variation of the harvesting effort when other model parameters are fixed using the method of numerical simulation. The innovation of this simulation technique has been used to determine the fraction of harvest and un-harvest resource biomass for prey and predator populations. Explicit expressions and values of the maximum sustainable yield (MSY) and the corresponding populations' level are obtained. Some sort of control is suggested to avoid over exploitation of resource biomass. Graphical solutions of the model are provided.

**Keywords:** Prey-predator; Co-existence steady state solution; Stability; Aquatic habitat; Harvesting.



## 1. Introduction

The marine ecosystem needs protection, as well as individual species and habitats [1]. Over-fishing, use of destructive fishing methods and pollution are all taking a toll on marine biodiversity [2]. The effectiveness of a protected area, restricted from fishing, depends on a complex set of interactions between biological, economic and institutional factors as it provides protection for critical habitats and cultural heritage sites and some cases conserve biodiversity as a tool to enhance fishery management. Ecological benefits within marine protected areas (MPA'S) are realised as they increase fish abundance in adjacent fishable area [3] which makes up for the lost areas that could have been used for open-access fishery.

Other works done on the effects of harvesting on population growth from the context of predator-prey interaction include those of Brauer and Sondack [4], Dai and Tang [5], Chaudhuri and Ray [6],Kar and Swarnakamal [7], Zhang, *et al.* [8], Kar [9], Khamis, *et al.* [1]. Inyama [10], Dubey and Patra [11], Mellachervn, *et al.* [12], Brauer and Sondack [13], Kar and Pahari [14], Dubey, *et al.* [15], May [16], Clark [17], Khamis, *et al.* [1] and Das and Chaudhuri [18] and so on.

Closing off an area that historically contributed a significant catch would prohibit or reduce the total catch at least in the short run, as population levels begin to recover in the MPA (Marine Protected Area) and spill over to the

*Corresponding Author

fishable areas increase [19]. On the basis of a detailed literature review on our propose fishery model, the issue of dynamics of fisheries proposed in the context of the exploitation of fishery resources consisting of three zones remain to be an open problem.

## 2. Mathematical Model

Consider an aquatic ecosystem consisting of unreserved and reserved zones. Modelling the fishery habitat, we consider that no fishing and other recreational activities are allowed in the reserved zone while the unreserved zone is an open access fishery zone. The fishery habitat under consideration consist of three patchy zones assumed to be homogeneous and combines a formulation of marine protected area and selective harvesting of biomass resources. The model considers two species, the prey's fish and the predator fish species and the growth of the prey's fish population follows the logistic form of the Lotka-Volterra prey –predator equation with the presence of predator in both zones. Preys migrate between zones randomly at different rates$\tau_1$, $\tau_2$ and $\tau_3$ for first, second and third prey fish populations respectively. The model which was developed in Agwu, *et al.* [20] is re-presented in (2.1) below.

$$\frac{dP_1}{dt} = r_1P_1\left(1-\frac{P_1}{k_1}\right) - \tau_1P_1 + \tau_2P_2 + \tau_3P_3 - \phi_1P_1P$$
$$\frac{dP_2}{dt} = r_2P_2\left(1-\frac{P_2}{k_2}\right) + \tau_1P_1 - \tau_2P_2 + \tau_3P_3 - \phi_2P_2P \qquad (2.1)$$
$$\frac{dP_3}{dt} = r_3P_3\left(1-\frac{P_3}{k_3}\right) + \tau_1P_1 + \tau_2P_2 - \tau_3P_3 - \phi_3P_3P$$
$$\frac{dP}{dt} = P(-\mu_1 - \mu_2P + \epsilon_1P_1 + \epsilon_2P_2 + \epsilon_3P_3)$$

This system (2.1) evolve on the basis of initial conditions

$P_1(0) > 0, P_2(0) > 0, P_3(0) > 0$ and $P(0) > 0$. Model (2.1) is biological meaningful if the following conditions are satisfied $(i)(r_1 - \tau_1) > 0, (ii)(r_2 - \tau_2) > 0$ and $(iii)(r_3 - \tau_3) > 0$

The possible steady state solutions of the model equations (2.1) are
$E_0(0,0,0,0), E_1(P_1,P_2,P_3,0), E_2(P_1,P_2,P_3,P), E_3(P_1,P_2,0,P), E_4(0,P_2,P_3,P) and\ E_5(P_1,0,P_3,P)$.
$E_0(0,0,0,0), E_1(P_1,P_2,P_3,0)\ and\ E_2(P_1,P_2,P_3,P)$ are realistic steady state solutions. For the purpose of this study, we will consider the co-existence steady-state solution otherwise called the positive interior equilibrium$E_2(P_1,P_2,P_3,P)$

## 3. Materials and Method

The application of the technique of numerical simulation was utilized to calculate the impact of variation of harvesting effort on the qualitative behaviour of a coexistence steady state solution and its stability. A hypothetical data and data provided by Khamis, *et al.* [1] and Kar [2] are the primary source of data for this computational analysis and algebraic formula with a biological meaning were derived and explored to calculate the fraction of harvest and un-harvest resource biomass of the fishery populations that remain due to harvesting and its implications for biodiversity gain and biodiversity loss. Newton–Raphson numerical scheme was utilized to calculate the approximate steady state solution using a MATLAB discretization scheme.

We considered a variation of the harvesting effort on prey and predator populations' selectively when other model parameters are fixed and utilized these variations to quantify the qualitative behaviour of stability and its impact on the co-existence steady state solution.

The process of this investigation is implemented numerically following these steps:

Firstly, defined and code the parameter values using MATLAB programming language: coexistence steady-state solutions are obtained for each variation of harvesting effort.

Secondly, the co-existence steady state solution which was obtained numerically was also coded; thirdly, the sixteen partial derivatives of the four interaction function were derived and coded. Fourthly, a Jacobian matrix of sixteen elements is constructed and coded from which four eigen-values are calculated numerically for each variation of harvesting effort. From the theory of sign method or Routh Hurwitz criteria in the study of stability of a steady state solution, the qualitative values of the eigen-values are determine which form the basis for each type of stability of a co-existence steady state solution Agwu, *et al.* [21]. If upon the evaluation of the Jacobian matrix and we obtain three positive eigen-value or eigen-values of opposite signs, then the co-existence steady state solution can be classified as being unstable. On the contrary, if four negative eigen-values are obtained, then the co-existence steady state solution is said to be stable.

### 3.1. Prey Fish Harvesting

In this section, we consider the effects of harvesting prey fish population $P_1(t)$ on model system (2.1) and testing the type of stability on the resulting co-existence steady state solution, we will consider a variation of the harvesting effort($E_1$) with catch-ability coefficient value of 0.9unit.

In this scenario, $P_1(t)$ represent the prey fish population in the free fishing zone while $P_2(t)$ and $P_3(t)$ represent the preys' fish populations in the reserve zone where fishing and other recreational activities are prohibited.

Applying the same assumptions, definition of variables and parameters governing model system (2.1), the model system (2.1) with harvesting becomes:

$$\frac{dP_1}{dt} = r_1P_1\left(1-\frac{P_2}{k_2}\right) - \tau_1P_1 + \tau_2P_2 + \tau_3P_3 - \phi_1P_1P - qE_1P_1$$

$$\frac{dP_2}{dt} = r_2P_2\left(1-\frac{P_2}{k_2}\right) + \tau_1P_1 - \tau_2P_2 + \tau_3P_3 - \phi_2P_2P$$

$$\frac{dP_3}{dt} = r_3P_3\left(1-\frac{P_3}{k_3}\right) + \tau_1P_1 - \tau_2P_2 + \tau_3P_3 - \phi_3P_3P \qquad (3.1)$$

$$\frac{dP}{dt} = P(-\mu_1 - \mu_2P + \epsilon_1P_1 + \epsilon_2P_2 + \epsilon_3P_3)$$

The model system (3.1) evolve on the basis of initial condition $P_1(0) > 0,\ P_2(0) > 0,\ P_3(0) > 0,$ $and\ \ P(0) > 0$. Model (3.1) is biological meaningful if the following conditions are satisfied

$(i)\ \ (r_1 - \tau_1 - \ qE_1) > 0,\ \ (ii)(r_2 - \tau_2) > 0, \quad and\ \ (iii)(r_3 - \tau_3) > 0,$

Where $qE_1$ is the catch rate function which is based on the catch-per-unit-effort hypothesis [9] and $E_1$ is the effort applied to harvest the prey species in the fishing zone and q is the catchability coefficient.

## 3.2. Predator Harvesting

Assuming that the predator species are also subject to a harvesting effort, in this scenario, $P_1(t), P_2(t)$ and $P_3(t)$represent the prey fish population in the reserve zone where fishing and other recreational activities are prohibited while $P(t)$ represent the predator fish populations in the open access fishery zone where fishing and other recreational activities are prohibited. Applying the same assumptions, definition of variables and parameters governing model system (2.1), we may write the model system (2.1) as

$$\frac{dP_1}{dt} = r_1P_1\left(1-\frac{P_2}{k_2}\right) - \tau_1P_1 + \tau_2P_2 + \tau_3P_3 - \phi_1P_1P$$

$$\frac{dP_2}{dt} = r_2P_2\left(1-\frac{P_2}{k_2}\right) + \tau_1P_1 - \tau_2P_2 + \tau_3P_3 - \phi_2P_2P$$

$$\frac{dP_3}{dt} = r_3P_3\left(1-\frac{P_3}{k_3}\right) + \tau_1P_1 - \tau_2P_2 + \tau_3P_3 - \phi_3P_3P \qquad (2.3)$$

$$\frac{dP}{dt} = P(-\mu_1 - \mu_2P + \epsilon_1P_1 + \epsilon_2P_2 + \epsilon_3P_3) - qE_2P$$

The model system (2.3) evolve on the basis of initial condition $P_1(0) > 0,\ P_2(0) > 0,\ P_3(0) > 0,$ $and\ \ P(0) > 0$.

In this scenario of harvesting the predator fish species and testing the type of stability on the resulting co-existence steady state solution, we will consider a variation of the harvesting effort $E_2$ with catch-ability coefficient value of 0.9unit.

The process of harvesting the predator is implemented numerically following the algorithm in Section 3.0

## 3.2. Co-Existence Steady State Solutions $E_2(P_1, P_2, P_3, P)$

**Table-3.1.** Initial values for parameters of the model system (2.1)

| Parameters | Value | Reference |
|---|---|---|
| $k_1$ | 110 | [13] |
| $k_2$ | 100 | [13] |
| $k_3$ | 90 | [Assume] |
| $r_1$ | 3 | [12] |
| $r_2$ | 1.5 | [15] |
| $r_3$ | 1.2 | [Assume] |
| $\tau_1$ | 0.5 | [12] |
| $\tau_2$ | 0.4 | [13] |
| $\tau_3$ | 0.3 | [Assume] |
| $\mu_1$ | 0.6 | [15] |
| $\mu_2$ | 0.05 | [15] |
| $\epsilon_1$ | 0.03 | [12] |
| $\epsilon_2$ | 0.02 | [13] |
| $\epsilon_3$ | 0.01 | [Assume] |
| $\phi_1$ | 0.3 | [12] |
| $\phi_2$ | 0.2 | [13] |
| $\phi_3$ | 0.1 | [Assume] |
| $q$ | 0.9 | [Assume] |

The model system (2.1) is four dimensional; the analytical study of the system is difficult to tractable [12]. To determine the co-existence steady state solution $E_2(P_1, P_2, P_3, P)$ from equation (2.1) in terms of the parameter values analytically is a daunting task, due to the presence of non-linearity term in the equation. To overcome this analytical barrier, we proposed to use Newton – Raphson numerical scheme to calculate the approximate steady state solution with MATLAB.

Using the parameters value in table3.1,we determine the solutions for the steady state solution $E_2(P_1, P_2, P_3, P)$ with the help of Newton-Raphson Numerical scheme with MATLAB , and is given by

$P_1 = 70.8828,\ P_2 = 128.3600,\ P_3 = 185.6401$ and $P = 6.9229$

**Table-3.2.** Showing 16 simulations at different initial guess values

| Example | $P_1$ | $P_2$ | $P_3$ | $P$ | Initial Guess Values |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | (0, 0, 0, 0) |
| 2 | 0.3498 | -0.0905 | -0.0938 | 0.0967 | (1, 1, 1, 1) |
| 3 | 0.6248 | -03279 | -0.1115 | 0.1651 | (5, 5, 5, 5) |
| 4 | 0.0874 | -0.5446 | 0.0659 | -0.0102 | (10, 10, 10, 10) |
| 5 | 0.1017 | -0.1017 | 0.6227 | -0.0035 | (20, 20, 20, 20) |
| 6 | 63.0642 | -12.1392 | -10;1909 | 2.1820 | (21, 21, 21, 21) |
| 7 | 70.8828 | 128.36 | 185.640 | 6.9229 | (22, 22, 22, 22) |
| 8 | 70.8828 | 128.36 | 185.640 | 6.9229 | (25, 25, 25, 25) |
| 9 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (30, 30, 30, 30) |
| 10 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (50, 50, 50, 50) |
| 11 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (100, 100, 100, 100) |
| 12 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (200, 200, 200, 200) |
| 13 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (500, 500, 500, 500) |
| 14 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (800, 800, 800, 800) |
| 15 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (1000,1000, 1000, 1000) |
| 16 | 70.8828 | 128.36 | 185.2401 | 6.9229 | (1500,1500, 1500, 1500) |

From Table 3.2, we clearly see that infinitely many co-existence steady state solutions are obtained. On the choice of initial guess values, we have obtained a robust convergence to a co-existence Steady state solution of (70.8828, 128.36, 185.2401, 6.9229) with instances of degeneracy steady state solutions, that do not have biological meaning, which occurs between initial guess values of (1,1, 1, 1) and (21, 21,21, 21).

Using this defined algorithm, the results obtained due to variation of the harvesting effort on the first prey population when other model parameters are fixed are presented and discussed in the next section.

## 4. Results and Discussion

For the first scenario of the simulation for prey fish harvesting, we have considered the harvesting effort value $E_1$=0 and catch rate value of 0.9unit. Our results in this scenario revealed 70.8828units and 0unit of un-harvest biomass and harvest biomass of prey fish respectively with a steady state solution (70.8828, 128.3600, 185.6401, and 6.9229) that is unstable

For the second scenario of the simulation for the prey fish harvesting, we have considered the harvesting effort values $E_1 = 3.5$unit and catch rate value of 0.9unit. Our results in this second scenario revealed 32.2414units and 38.641units of un-harvest biomass and harvest biomass of prey fish respectively with a steady state solution (32.2414, 129.9140, 185.0046, 6.0130) that is unstable. Other scenarios for prey harvesting, when the value of the catch rate is 0.9unit and other variations of harvesting effort ($E_1$) are displayed in table 4.1 to table 4.6. Note that the notations Ex. Stands for example, fHP1 stands for fraction of harvest P1 biomass, TOS stands for type of stability and $\lambda_i$, i= 1, 2, 3 and 4 stands for eigen values

For the first scenario of the simulation for the predator harvesting, we have considered the harvesting effort value

$E_2$ =0 and catch rate value of 0.9unit. Our results in this scenario revealed 70.8828 unharvest biomass and 0 harvest biomass of the predator P, with a steady state solution (70.8828, 128.3600, 185.6401, and 6.9229) that is unstable

For the second scenario of the simulation for predator harvesting, we have considered the harvesting effort values $E_2 = 1$unit and catch rate value of 0.9unit. Our results in this second scenario of harvesting the predator revealed 4.6031units of unharvest biomass and 2.3198units of harvest biomass of predator with a steady state solution (91.3142, 157.5838, 207.3003, and 4.6031) that is unstable

In another scenario for harvesting the predator, when the harvesting effort ($E_2$) is increased to 10units, our results revealed 1.0578units of unharvest biomass and 5.9092units of harvest biomass of predator with a steady state solution (124.9757, 203.2916, 239.5835, 1.0578) that is stable.

Other scenarios for harvesting the predator when the value of the catch rate is 0.9unit and other variations of harvesting effort ($E_2$) are displayed in Table 4.7 and Table 4.8. Note that the notations Ex. Stands for example, fHP stands for fraction of harvest P biomass, TOS stands for type of stability and $\lambda_i$, i= 1, 2, 3 and 4 stands for eigen-values.

## 4.1. The Maximum Sustainable Yield

The maximum sustainable yield (MSY) is the maximum rate of harvesting biological resource biomass after which a further harvest will lead to the depletion of resource biomass eventually to zero.

Clark [17] found that the value of MSY in the absence of any population is given as

$$h_{MSY}^{0} = \frac{rK}{4}$$

Model system (3.1) is subject to selective harvesting on population $P_1$ and P.

In this section, we will derive expression for their maximum sustainable yield. From the steady state of the first equation of (2.2), we have

$$h_1(MSY) = qE_1P_1 = r_1P_1\left(1 - \frac{P_1}{k_1}\right) - \tau_1P_1 + \tau_2P_2 + \tau_3P_3 - \phi_1P_1P = 0 \qquad (4.1a)$$

Following Dubey and Patra [11]

$$\frac{\partial h_1}{\partial h_1} = r_1 - \frac{2r_1p_1}{k_1} - \tau_1 - \phi_1P = 0 \qquad (4.1b)$$

$\Rightarrow P_1 = \frac{k_1(r_1 - \tau_1 - \phi_1P)}{2r_1}$ and

$$\frac{\partial^2 h_1}{\partial P_1^2} = \frac{-2r_1}{k_1} < 0$$

Substituting (4.1b) into (4.1a), we obtain

$$h_1(MSY) = \frac{k_1(r_1 - \tau_1 - \phi_1P)(r_1 + \tau_1 - \phi_1P)}{4r_1} + \tau_2P_2 + \tau_3P_3$$

From equation (4.1b), it is interesting to note that when P = 0 and no migration, $P_1 = \frac{k_1}{2}$ and $h_1(MSY) = \frac{r_1k_1}{4} = h^0MSY$

The expression derived for $h_1(MSY)$ is the maximum sustainable yield for the prey fish $P_1$ and it matches with the result of Clark [17]

The maximum Sustainable yield (MSY) for predator harvesting is also derive as follows

From the steady state of the fourth equation of (3.2), we have

$$h_2(MSY) = qE_2P = -\mu_1P - -\mu_2P^2 + \varepsilon_1P_1 + \varepsilon_1P_1P + \varepsilon_2P_2P + \varepsilon_3P_3P = 0 \qquad (4.1c)$$

$$\frac{\partial h_2}{\partial P} = -\mu_1 - 2\mu_2P + \varepsilon_1P_1 + \varepsilon_2P_2 + \varepsilon_3P_3 = 0$$

$$\Rightarrow P = \frac{-\mu_1 + \varepsilon_1P_1 + \varepsilon_2P_2 + \varepsilon_3P_3}{2\mu_2} \qquad (4.1d)$$

and

$$\frac{\partial^2 h_2}{\partial P^2} = -2\mu_2 < 0$$

Substituting (4.1d) into (4.1c), we obtain

$$h_2(MSY) = -\mu_1 + \varepsilon_1P_1 + \varepsilon_2P_2 + \varepsilon_3P_3$$

From equation (4.1d), it is interesting to note that when $P_1 = P_2 = P_3 = 0$, then $P = \frac{-\mu_1}{2\mu_2}$ and $h_2(MSY) = -\mu_1$ for predator harvesting.

**Table-4.1.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of Harvesting effort $E_1$ between 0 and 31.5units when q=0.9 for prey harvesting

| | | **CSS (70.8828, 128.3600, 185.6401, 6.9229)** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. | $E_1$ | $P_1$ | $P_2$ | $P_3$ | P | fH$P_1$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 1 | 0 | 70.88 | 128.36 | 185.64 | 6.92 | 0 | 3.95 | -1.79 | -4.30 | -4.92 | unstable |
| 2 | 3.5 | 32.24 | 129.91 | 185.01 | 6.01 | 38.64 | 3.32 | -2.50 | -4.57 | -4.89 | unstable |
| 3 | 7.0 | 17.99 | 130.69 | 184.88 | 5.66 | 52.90 | 3.11 | -6.62 | -2.96 | -4.73 | unstable |
| 4 | 10.5 | 12.07 | 131.06 | 184.84 | 5.52 | 58.81 | -9.32 | 3.02 | -3.08 | -4.72 | unstable |
| 5 | 14.0 | 9.01 | 131.25 | 184.83 | 5.44 | 61.87 | -12.26 | 2.97 | **-3.12** | **-4.71** | unstable |
| 6 | 17.5 | 7.17 | 131.37 | 184.83 | 5.39 | 63.71 | -15.28 | 2.94 | -3.14 | -4.71 | unstable |
| 7 | 21.0 | 5.95 | 131.46 | 184.82 | 5.36 | 64.94 | -18.35 | 2.92 | -3.92 | -4.71 | unstable |
| 8 | 24.5 | 5.08 | 131.52 | 184.82 | 5.34 | 65.81 | -21.45 | 2.90 | -3.16 | -4.71 | unstable |
| 9 | 28.0 | 4.43 | 131.56 | 184.82 | 5.32 | 66.46 | -24.55 | 2.89 | -3.16 | -4.71 | unstable |
| 10 | 31.5 | 3.92 | 131.59 | 184.82 | 5.31 | 66.96 | -27.67 | 2.88 | -3.17 | -4.70 | unstable |

**Table-4.2.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of harvesting effort $E_1$between35.0 and 66.5units when q=0.9 for prey harvesting

| | | **CSS (70.8828, 128.3600, 185.6401, 6.9229)** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex | $E_1$ | $P_1$ | $P_2$ | $P_3$ | P | fH$P_1$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 11 | 35.0 | 3.52 | 131.62 | 184.82 | 5.30 | 67.36 | -30.79 | 2.87 | -3.17 | -4.70 | unstable |
| 12 | 38.5 | 3.20 | 131.64 | 184.82 | 5.29 | 67.69 | -33.92 | 2.86 | -3.17 | -4.70 | unstable |
| 13 | 42.0 | 2.93 | 131.66 | 184.82 | 5.29 | 67.96 | -37.06 | 2.86 | -3.17 | -4.70 | unstable |
| 14 | 45.5 | 2.90 | 131.68 | 184.82 | 5.28 | 67.99 | -40.20 | 2.86 | -3.18 | -4.70 | unstable |
| 15 | 49.0 | 2.50 | 131.63 | 184.82 | 5.28 | 68.38 | -43.33 | 2.85 | -3.17 | -4.70 | unstable |
| 16 | 52.5 | 2.33 | 131.70 | 184.82 | 5.27 | 68.55 | -46.47 | 2.85 | -3.18 | -4.70 | unstable |
| 17 | 56.0 | 2.18 | 131.71 | 184.82 | 5.27 | 68.70 | -49.61 | 2.84 | -3.18 | -4.70 | unstable |
| 18 | 59.5 | 2.05 | 131.72 | 184.82 | 5.26 | 68.83 | -52.75 | 2.84 | -3.18 | -4.70 | unstable |
| 19 | 63.0 | 1.94 | 131.73 | 184.82 | 5.26 | 68.95 | -55.89 | 2.84 | -3.18 | -4.70 | unstable |
| 20 | 66.5 | 1.83 | 131.74 | 184.82 | 5.26 | 69.05 | -59.03 | 2.84 | -3.18 | -4.70 | unstable |

**Table-4.3.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of harvesting effort $E_1$between 70.0 and 101.5units when q=0.9 for prey harvesting

| | | **CSS (70.8828, 128.3600, 185.6401, 6.9229)** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex | $E_1$ | $P_1$ | $P_2$ | $P_3$ | P | fH$P_1$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 21 | 70.0 | 1.74 | 131.75 | 184.82 | 5.26 | 69.14 | -62.18 | 2.84 | -3.18 | -4.70 | unstable |
| 22 | 73.5 | 1.66 | 131.75 | 184.82 | 5.25 | 69.23 | -65.32 | 2.83 | -3.18 | -4.70 | unstable |
| 23 | 77.0 | 1.58 | 131.75 | 184.82 | 5.25 | 69.30 | -68.47 | 2.83 | -3.18 | -4.70 | unstable |
| 24 | 80.5 | 1.51 | 131.76 | 184.82 | 5.25 | 69.37 | -71.61 | 2.81 | -3.18 | -4.70 | unstable |
| 25 | 84.0 | 1.45 | 131.77 | 184.82 | 5.25 | 69.44 | -74.76 | 2.83 | -3.18 | -4.70 | unstable |
| 26 | 87.5 | 1.39 | 131.77 | 184.82 | 5.25 | 69.49 | -77.90 | 2.83 | -3.18 | -4.70 | unstable |
| 27 | 91.0 | 1.34 | 131.77 | 184.82 | 5.25 | 69.55 | -81.05 | 2.83 | -3.18 | -4.70 | unstable |
| 28 | 94.5 | 1.29 | 131.78 | 184.82 | 5.25 | 69.60 | -84.20 | 2.83 | -3.18 | -4.70 | unstable |
| 29 | 98.0 | 1.24 | 131.78 | 184.82 | 5.24 | 69.64 | -87.35 | 2.83 | -3.18 | -4.70 | unstable |
| 30 | 101.5 | 1.20 | 131.78 | 184.82 | 5.24 | 69.69 | -90.49 | 2.83 | -3.18 | -4.70 | unstable |

**Table-4.4.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of harvesting effort $E_1$between 105.0 and 129.5 units when q=0.9 for prey harvesting

| | | **CSS (70.8828, 128.3600, 185.6401, 6.9229)** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. | $E_1$ | $P_1$ | $P_2$ | $P_3$ | P | fH$P_1$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 31 | 105.0 | 1.16 | 131.79 | 184.82 | 5.24 | 69.73 | -93.64 | 2.82 | -3.18 | -4.70 | unstable |
| 32 | 108.5 | 1.12 | 131.79 | 184.82 | 5.24 | 69.77 | -96.79 | 2.82 | -3.18 | -4.70 | unstable |
| 33 | 112.0 | 1.08 | 131.79 | 184.82 | 5.24 | 69.80 | -99.93 | 2.82 | -3.18 | -4.70 | unstable |
| 34 | 115.5 | 1.05 | 131.79 | 184.82 | 5.24 | 69.83 | -103.08 | 2.82 | -3.18 | -4.70 | unstable |
| 35 | 119.0 | 1.02 | 131.80 | 184.82 | 5.24 | 69.86 | -106.23 | 2.82 | -3.18 | -4.70 | unstable |
| 36 | 121.1 | 1.00 | 131.80 | 184.82 | 5.24 | 69.87 | -108.12 | 2.82 | -3.18 | -4.70 | unstable |
| 37 | 121.18 | 1.00 | 131.80 | 184.82 | 5.24 | 69.88 | -108.19 | 2.82 | -3.18 | -4.70 | unstable |
| 38 | 122.5 | 0.99 | 131.80 | 184.82 | 5.24 | 69.89 | -109.38 | 2.82 | -3.18 | -4.70 | unstable |
| 39 | 126.0 | 0.96 | 131.80 | 184.82 | 5.24 | 69.92 | -112.53 | 2.82 | -3.18 | -4.70 | unstable |
| 40 | 129.5 | 0.94 | 131.80 | 184.82 | 5.24 | 69.95 | -115.68 | 2.82 | -3.18 | -4.70 | unstable |

**Table-4.5.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of harvesting effort $E_1$between 133.0 and 164.5units when q=0.9 for prey harvesting

| | | **CSS (70.8828, 128.3600, 185.6401, 6.9229)** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. | $E_1$ | $P_1$ | $P_2$ | $P_3$ | P | fH$P_1$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 41 | 133.0 | 0.91 | 131.80 | 184.82 | 5.24 | 69.97 | -118.82 | 2.82 | -3.18 | -4.70 | unstable |
| 42 | 136.5 | 0.89 | 131.80 | 184.82 | 5.24 | 69.99 | -121.97 | 2.82 | -3.18 | -4.70 | unstable |
| 43 | 140.0 | 0.87 | 131.81 | 184.82 | 5.23 | 70.02 | -128.22 | 2.79 | -3.18 | -4.70 | unstable |
| 44 | 143.5 | 0.84 | 131.81 | 184.82 | 5.23 | 70.04 | -128.22 | 2.79 | -3.18 | -4.70 | unstable |
| 45 | 147.0 | 0.82 | 131.81 | 184.82 | 5.23 | 70.06 | -131.37 | 2.79 | -3.18 | -4.70 | unstable |
| 46 | 150.5 | 0.80 | 131.81 | 184.82 | 5.23 | 70.08 | -134.52 | 2.79 | -3.18 | -4.70 | unstable |
| 47 | 154.0 | 0.79 | 131.81 | 184.82 | 5.23 | 70.10 | -137.67 | 2.79 | -3.18 | -4.70 | unstable |
| 48 | 157.5 | 0.77 | 131.81 | 184.82 | 5.23 | 70.12 | -140.82 | 2.79 | -3.18 | -4.70 | unstable |
| 49 | 161.0 | 0.75 | 131.82 | 184.82 | 5.23 | 70.13 | -143.97 | 2.79 | -3.18 | -4.70 | unstable |
| 50 | 164.5 | 0.74 | 131.82 | 184.82 | 5.23 | 70.15 | -147.12 | 2.79 | -3.18 | -4.70 | unstable |

**Table-4.6.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of harvesting effort $E_1$between 168.0 and 182.0units when q=0 for prey harvesting

| | | CSS (70.8828, 128.3600, 185.6401, 6.9229) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. | $E_1$ | $P_1$ | $P_1P_2$ | $P_3$ | P | fH$P_1$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 61 | 168.0 | 0.72 | 131.82 | 184.82 | 5.23 | 70.16 | -150.27 | 2.79 | -3.18 | -4.70 | unstable |
| 62 | 171.5 | 0.71 | 131.82 | 184.82 | 5.23 | 70.18 | -153.42 | 2.79 | -3.18 | -4.70 | unstable |
| 63 | 175.0 | 0.69 | 131.82 | 184.82 | 5.23 | 70.19 | -156.57 | 2.79 | -3.18 | -4.70 | unstable |
| 64 | 178.5 | 0.68 | 131.82 | 184.82 | 5.23 | 70.21 | -159.72 | 2.79 | -3.18 | -4.70 | unstable |
| 85 | 182.0 | 0.66 | 131.82 | 184.82 | 5.23 | 70.22 | -162.87 | 2.79 | -3.18 | -4.70 | unstable |

What do we learn from Table 4.1 to Table 4.6?

In this scenario of harvesting a prey fish$P_1$, the harvesting effort is varied by 3.5 units, we observed in column1 of Table 4.1 a decrease in resource biomass of $P_1$ population. When the harvesting effort is 121.18 units and catch rate value of 0.9 units, a maximum sustainable yield $h_1(MSY)$ value of 69.88 is reached as shown in column 5 of Table 4.4, the MSY Steady state solution is (1, 131.80, 184.82, and 5.24) and is unstable. We also observed that, there is no much significant increase in the resource biomass for $P_1$ *and* $P_3$ in the reserve zones. The simulation results show that between harvesting effort value of 0 to 182 units, the $P_3$ resource biomass droped from 185.6401 to 184.82 and between harvesting effort value of 0 to 182 units, the $P_2$ resource biomass from 128.3600 increased slightly to a neighbourhood value of 131.80 which agrees with the realization of ecological benefits of marine protected areas as they increase fish abundance (resource biomass) in adjacent fishable area Kar and Swarnakamal [7].

**Table-4.7.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of harvesting effort $E_2$between 0 and 9units when q=0.9 for predator harvesting

| | | CSS (70.8828, 128.3600, 185.6401, 6.9229) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. | $E_2$ | $P_1$ | $P_2$ | $P_3$ | P | $(fHP)$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 1 | 0 | 70.88 | 128.36 | 185.64 | 6.92 | 0 | 3.95 | -1.79 | -4.30 | -4.92 | unstable |
| 2 | 1 | 91.32 | 157.58 | 207.30 | 4.60 | 2.32 | 5.12 | -2.66 | -4.72 | -5.28 | unstable |
| 3 | 2 | 102.53 | 173.07 | 218.42 | 2.39 | 4.53 | 5.60 | -2.99 | -4.73 | -5.35 | unstable |
| 4 | 3 | 109.36 | 182.36 | 225.00 | 2.67 | 4.25 | 5.03 | -3.30 | -5.09 | -5.57 | unstable |
| 5 | 4 | 113.89 | 188.48 | 229.29 | 2.20 | 4.72 | 4.54 | -3.46 | -5.19 | -5.64 | unstable |
| 6 | 5 | 117.11 | 192.80 | 232.31 | 1.87 | 5.06 | 3.93 | -3.56 | -5.25 | -5.69 | unstable |
| 7 | 6 | 119.49 | 195.99 | 234.53 | 1.62 | 5.30 | 3.23 | -3.63 | -5.30 | -5.73 | unstable |
| 8 | 7 | 121.34 | 198.45 | 236.24 | 1.43 | 5.49 | 2.47 | -3.67 | -5.34 | -5.76 | unstable |
| 9 | 8 | 122.80 | 200.40 | 237.59 | 1.28 | 5.64 | 1.67 | -3.70 | -5.37 | -5.78 | unstable |
| 10 | 9 | 123.99 | 201.98 | 238.68 | 1.16 | 5.77 | 0.83 | -3.69 | -5.40 | -5.80 | unstable |

**Table-4.8.** Calculating the qualitative stability of resulting co-existence steady state solution due to variation of harvesting effort $E_2$between 10 and 10.751 units when q=0.9 for predator harvesting

| | | CSS (70.8828, 128.3600, 185.6401, 6.9229) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. | $E_2$ | $P_1$ | $P_2$ | $P_3$ | P | $(fHP)$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | TOS |
| 11 | 10 | 124.98 | 203.29 | 239.58 | 1.06 | 5.87 | -0.05 | -3.66 | -5.42 | -5.81 | stable |
| 12 | 10.5 | 125.41 | 203.87 | 239.98 | 1.01 | 5.91 | -0.51 | -3.63 | -5.42 | -5.82 | stable |
| 13 | 10.7 | 125.57 | 204.08 | 240.13 | 1.00 | 5.93 | -0.70 | -3.61 | -5.43 | -5.82 | stable |
| 14 | 10.75 | 125.61 | 204.13 | 240.16 | 0.99 | 5.93 | -0.74 | -3.60 | -5.43 | -5.82 | stable |
| 15 | 10.751 | 125.61 | 204.14 | 240.17 | 0.99 | 5.93 | -0.74 | -3.60 | -5.43 | -5.82 | stable |

What do we learn from Table 4.7 and Table 4.8?

In this scenario, harvesting a predator fish P, *the* $P_1, P_2$ *and* $P_3$ becomes the reserve zones. Harvesting effort is varied by 1 unit. We observed in column 4 of Table 4.7, a decrease in resource biomass of P population, when the harvesting effort is increased to 10.7 units with catchability coefficient of 0.9 unit, a maximum sustainable yield $h_2(MSY)$ value of 5.93 is reached, as shown in column 7 of Table 4.8.

The MSY Steady state solution is (125.4076, 203.8650, 239.9791, 1.0137) and its stable with eigen values $\lambda_1 = -0.5057, \lambda_2 = -3.6271, \lambda_3 = -5.4239, \lambda_4 = -5.819$. We also observed that there is a very big increase in the resource biomass of $P_1, P_2$ *and* $P_3$ in the reserve zones, where fishing is prohibited. Between harvesting effort value of 0 units to 10.751 units, the $P_1, P_2$ *and* $P_3$ resource biomass increase sharply from 70.8828, 128.3600 and 185.6401 to 125.6113, 204.1352 and 240.1655 respectively.

# 5. Graphical Presentation

For the graphical example of the model systems, we choose the following set of parameter values as specified in Table 3.1

$r_1$ =3, $r_2$=1.5, $r_3$ =1.2, $k_1$=110, $k_2$ =100, $k_3$=90, $\tau_1$ = 0.5, $\tau_2$=0.4 , $\tau_3$=0.3, $\epsilon_1$= 0.03, $\epsilon_2$=0.02, $\epsilon_3$=0.01, $\phi_1$=0.3, $\phi_2$=0.2, $\phi_3$=0.1, $\mu_1$=0.6, $\mu_2$=0.05, q=0.9 and variation of harvesting effort $E_1$ and $E_2$ in appropriate units.

Using these parameter values,we plot the graphical representation of the harvest and unharvest biomass of the first prey species and predator species are shown in the figures 5.1-5.4. The figures 5.1 and 5.2 show the decaying behaviour of solution trajectories due to exploitation of the prey and predator resource biomass while figures 5. 3 and 5.4 show the harvest biomass of the first prey species and the predator species being an outcome of the effect of the exploitation.

**Fig-5.1.** Plot of fraction of $P_1$ unharvest biomass versus Effort

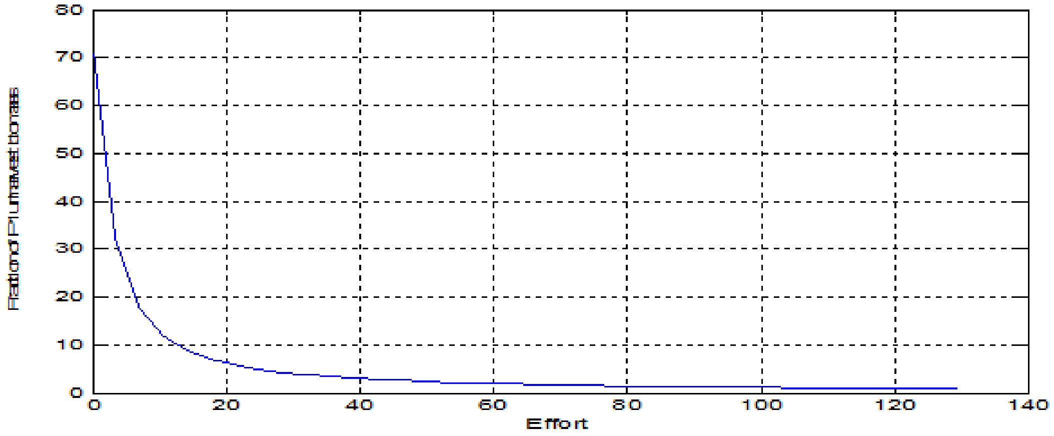



**Fig-5.2.** Plot of fraction of P unharvest biomass versus Effort

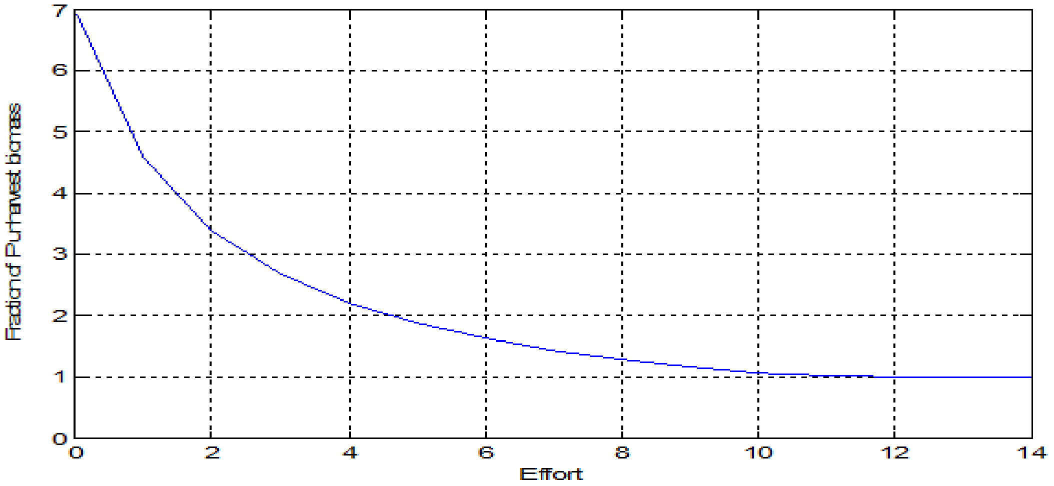



**Fig-5.3.** Plot of fraction of $P_1$ harvest biomass versus Effort

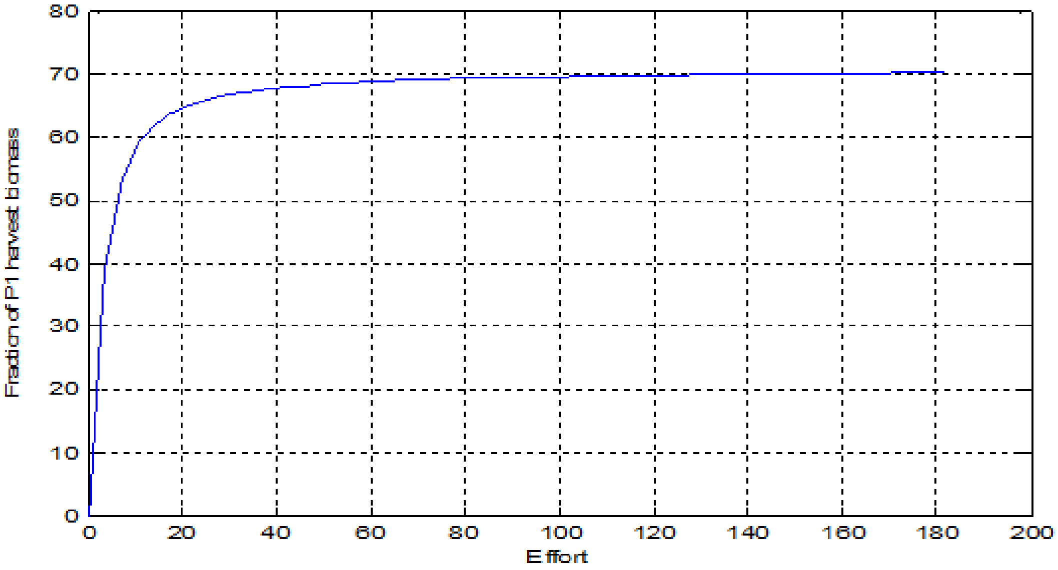
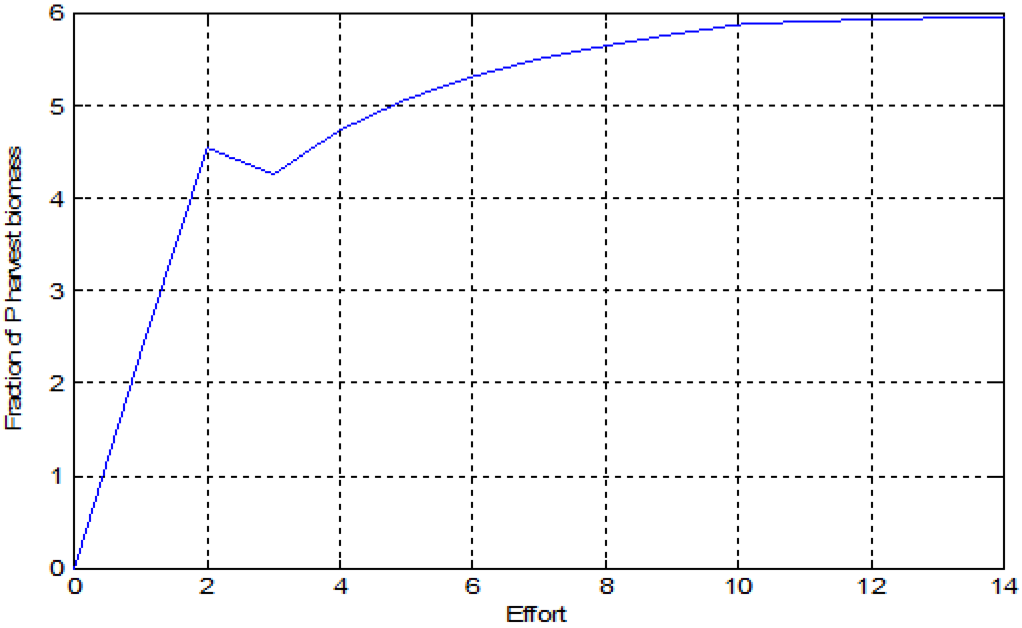


**Fig-5.4.** Plot of fraction of P harvest biomass versus Effort




## 6. Conclusion

On the basis of this simulation analysis, we found that stability is gain for the resulting coexistence steady state solution due to a variation of the harvesting effort( $E_2$) and stability is dominantly lost for the resulting coexistence steady state solution due to a variation of the harvesting effort( $E_1$). Comparative results for the two types of selective harvesting show that the predator harvesting protects $P_1, P_2$ *and* $P_3$ in inside the reserve zone as the resource biomass increases sharply which attracts prospect for fish abundance in adjacent fishing zones compare to the scenario of prey harvesting, where the resource biomass $P_2$ and $P_3$ in the reserve zones dropped. Our over-all observation is that, harvesting the predator fish species introduces onset of stability to the ecological system that is previously unstable, this implies that, a reduction in the rate of predation of the prey species by harvesting the predator stabilize the ecological system. Our overall observation is that, the fish in the reserve zone rewards the effect of harvesting on the free fishing zones as the resource biomass in the reserve zone are seen to increase. This implies that marine reserve is a good tool for managing fishery in comparison with the situation without reserve Khamis, *et al.* [1]. We conclude that if h>$h_1(MSY)$,we have over exploitation of the first prey fish biomass or otherwise and if h>$h_2(MSY)$, we have over exploitation of the predator biomass or otherwise.

Our key contribution to knowledge in this paper is that, we have successfully applied a mathematical reasoning to solve a complex ecological problem which was not considered in the work of Khamis, *et al.* [1] or any other published works.

We would expect this contribution whose results agrees with the realization of ecological benefits of marine protected areas and sustained biodiversity gain in the reserve zones and sustained biodiversity loss in the free fishing zone to provide further insight into ecosystem planning and stability. For future consideration, we will be interested in studying the optimal harvesting policy using pontryagins maximum principle.

## Acknowledgement

We the entire members of Mathematical Modelling Research Group (MMRG) of the Federal University of Technology want to use this medium to mourn the death of one of us, Mr. Agwu Ikoro who was one of us who had this paper as part of his Ph.D. Thesis, who was snatched from us by death a month to his defence of his Ph. D. thesis in Applied Mathematics. He toiled for four years and could not be awarded the Ph. D. due to death. We want to immortalize your name by publishing this paper in the ever green memory of you. Sleep on our dear colleague till we meet in that resurrection morning to part no more. Adiu Mr. Agwu! Adiu Ikoro!! Adiu the grate Mathematician (Doctor Agwu).

## References

[1] Khamis, S. A., Tchuenche, J. M., Lukka, M., and Heilio, M., 2011. "Dynamics of fisheries with prey reserve and harvesting." *International Journal of Computer Mathematics*, vol. 88, pp. 1775-1802.

[2] Kar, T. K., 2006. "A model for fisheries resources with reserve area and facing prey-predator interaction." *J. Can, Appl. Math. Q.*, vol. 14, pp. 385-399.

[3] Kribs-zaleta, C. M., 2009. "Sharpness of saturation in harvesting and predator." *Math. Biosci. Eng.*, vol. 6, pp. 719-742.

[4] Brauer, F. and Sondack, A. C., 1979a. "Stability Regions and Transition phenomena for Harvested predator-prey systems." *Journal of mathematical Biology*, vol. 7, pp. 317-337.

[5] Dai, G. and Tang, M., 1998. "Coexistence region and global dynamics of a harvested predator prey system." *SIAM Journal on Applied Mathematical,* vol. 58, pp. 193-210.
[6] Chaudhuri, K. S. and Ray, S. S., 1996. "On the combined harvesting of a predator prey system." *Journal of Biological Systems,* vol. 4, pp. 373-389.
[7] Kar, T. K. and Swarnakamal, M., 2006. "Influence of prey reserve in a prey-predator fishery." *Non-linear Analysis,* vol. 65, pp. 1725-1735.
[8] Zhang, R., Sun, J. F., and Yang, H. X., 2007. "Analysis of a prey-predator fishery model with prey reserve." *Applied Mathematical Science,* vol. 50, pp. 2481-2492.
[9] Kar, T. K., 2003. "Selective harvesting in a prey-predator fishery with time delay." *Mathematical and Computer Modelling,* vol. 38, pp. 449-458.
[10] Inyama, S. C., 2003. "Prey-predator mathematical model using van de pol's equation." *Journal of the Nigerian Association of Mathematical Physics,* vol. 12, pp. 435-438.
[11] Dubey, B. and Patra, A., 2013. "Optimal management of a renewable resource utilized by a population with taxation as a control variable." *Nonlinear Analysis Modelling and Control,* vol. 18, pp. 37-52.
[12] Mellachervn, N. S., Mantripragata, A. S. S., Das, K., and Gazi, N. H., 2011. "Prey-predator fishery model with stage structure in two patchy Marine Aquatic Environment." *Applied Mathematics,* vol. 2, pp. 1405-1416.
[13] Brauer, F. and Sondack, A. C., 1979b. "Stability regions in predator-prey systems with constant-Rate Prey Harvesting." *Journal of Mathematical Biology,* vol. 8, pp. 55-71.
[14] Kar, T. K. and Pahari, U. K., 2007. "A model for prey-predator fishery with marine reserve." *J. Fish, Aquatic, Science,* vol. 2, pp. 195-205.
[15] Dubey, B., Chandra, P., and Sinha, P., 2003. "A model for fishery resource with reserve area, non-linear analysis." *Real World Applications,* vol. 4, pp. 625-637.
[16] May, R. M., 1974. *Stability and complexity in model ecosystem*. Princeton New Jessy, USA: Princeton University Press.
[17] Clark, C. W., 1976. *Mathematical Bio-economics the optimal management of renewable resources*. New York: Wiley.
[18] Das, T. M. and Chaudhuri, 2009. "Bionomic harvesting of a prey-predator fishery." *Journal of Biological Dynamics,* vol. 3, pp. 447-462.
[19] Sanchirico, J. N., Cochran, K. A., and Emerson, P. M., 2002. "Marine protected areas, Economic and social implications." *Resources for the Future,* pp. 1-24. Available: http://www.rff.org/research/publications/marine-protected-areas-economic-and-social-implications
[20] Agwu, Inyama, S. C., Umana, R. A., Omame, A., Ukanwoke, N., and Ofomata, A., 2017. "Mathematical analysis of a prey-predator fishery model in three patch aquatic habitat." *Quest Journals Journal of Research in Applied Mathematics,* vol. 3, pp. 1-23. Available: www.questjournals.org
[21] Agwu, Ekaka-a, E. N., and Nafo, N. M., 2014. "Effect of the intra-species competition parameters on the onset of stability, Instability and degeneracy of co-existence steady state solutions between competing populations." *Journal of the Nigerian Association of Mathematical Physics,* vol. 25, pp. 313-316.