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New Continuous Hybrid Constant Block Method for the Solution of Third Order Initial Value Problem of Ordinary Differential Equations

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Abstract

In this study, a new one step continuous hybrid constant block method is developed using interpolation and collocation of power series approximate solution to solve initial -value problems involving third -order ordinary differential equations. The one step block method was augmented by the introduction of off grid points so as to circumvent Dahquist zero stability barrier. The block method is then applied to obtain the solution of two numerical examples for demonstration of the efficiency of the new method. The results are compared with the existing ones in literature and it is concluded that results of Continuous Hybrid Constant Block Method is more accurate than when it was implemented in predictor corrector mode or using implicit Runge-Kutta method.

Keywords: Continuous block method; Collocation; Interpolation; Off grid points; Ordinary differential equations.

1. Introduction

This paper considers the numerical solution to third order initial value problems of the form.

$$y''' = f(x, y, y', y''), \quad y^{(k)}_{(x_0)} = y_0^{(k)}, \quad k = 0,1,2 \tag{1}$$

Solutions to some third order differential equations appearing in the field of Engineering and Science are due to natural phenomenon; hence they do not have a close solution Nicolette [1]; Hopkins and Kosmatov [2]. Approximate solutions are obtained by the application of numerical methods. These methods are categorised in to two: one step and Multistep methods.

Linear Multistep methods are commonly applied for solving higher order IVPs by first reducing it to an equivalent system of first-order ordinary differential equations (ODEs). This approach has been extensively discussed by notable authors such as Lambert [3], Lambert [4], Brugnano and Trigiante [5], Onumanyi, *et al.* [6], Onumanyi, *et al.* [7], Fatunla [8] and Jator [9] are cited. One disadvange of these methods is that it involves more human effort and wastage computer time as discussed in Awoyemi [10]. Recently, Jator [11], Jator and Li [12], proposed LMMs for the direct solution of the general second and third order IVPs, which were shown to be zero stable and implemented without the need for either predictors or starting values from other methods. Their method was tested on few problems and was found to be effective.

It has also been discovered that direct methods for the solution of higher order ordinary differential equation are better than the method of reduction in terms of approximation, time of execution and cost of implementation. This was discovered by scholars such as Adesanya, *et al.* [13], James, *et al.* [14], Kayode and Obaruah [15], and Jator [16].

The technique of collocation and interpolation of power series approximate solution to generate a continuous linear multi-step method has been discussed by many authors, among them are: Awoyemi and Idowu [17], Majid, *et al.* [18], Olabode and Yusuf [19], Adesanya, *et al.* [20]. These authors developed method which is implemented either in predictor-corrector method or block method. However, Block method has advantage over predictor-corrector method because it is cost effective and give better approximations. Hybrid method has also been found to have the advantage of reducing the step number of a method and still remains zero stable.

In this paper, we derive a new continuous hybrid constant block method through interpolation and collocation, see Lie and Norsett [21], Atkinson [22], Onumanyi, *et al.* [7]. The method retains the characteristics of Runge kutta method and hybrid method. The simultaneous application of this derived method is more accurate than predictor-corrector methods which are generally applied as formulas over overlapping intervals in literature. Thus, the method presented in this paper is more robust in terms of self-starting, less time of execution, cost effectiveness. The new block method derived is also zero-stable, consistent, and hence convergent. The superiority of the method in this paper over Runge-Kutta method is established through the results obtained from the numerical examples.

The paper is organized as follows. In Section 2, the methodology for the development of the method is considered. Section 3 is devoted to the explanation of the basic properties of the method developed. The efficiency of the new block method are discussed in Section 4 by testing it on some numerical examples. Finally, the conclusion of the paper is discussed in Section 5.

2. Methodology

We consider the approximate solution of the form

$$y(x)=\sum_{j=0}^{r+s-1}a_jx^j \tag{2}$$

Where r and s are the number of interpolation and collocation points respectively. $a_{j's}$ are the unknown coefficient to be determined. X is the polynomial basis function of degree j.

The third derivation of (2) gives

$$y'''(x)=\sum_{j=3}^{r+s-1}j(j-1)(j-2)a_jx^{j-3} \tag{3}$$

Substituting (3) into (1)

$$f(x,y,y',y'')=\sum_{j=3}^{r+s-1}j(j-1)(j-2)a_jx^{j-3} \tag{4}$$

Interpolating (2) at x_{n+r}, $r=0,\frac{1}{8},\frac{1}{4}$ and collocating (4) at x_{n+s}, $s=0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,$ gives a nonlinear system of equation in the form

$$XA=U \tag{5}$$

where

$$A=[a_0,a_1,a_2,a_3,a_4,a_5,a_6,a_7]^T$$

$$U=\left[y_n,y_{n+\frac{1}{8}},y_{n+\frac{1}{4}},f_n,f_{n+\frac{1}{4}},f_{n+\frac{1}{2}},f_{n+\frac{3}{4}},f_{n+1}\right]^T$$

$$X=\begin{bmatrix}1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7\\ 1 & x_{n+\frac{1}{8}} & x^2_{n+\frac{1}{8}} & x^3_{n+\frac{1}{8}} & x^4_{n+\frac{1}{8}} & x^5_{n+\frac{1}{8}} & x^6_{n+\frac{1}{8}} & x^7_{n+\frac{1}{8}}\\ 1 & x_{n+\frac{1}{4}} & x^2_{n+\frac{1}{4}} & x^3_{n+\frac{1}{4}} & x^4_{n+\frac{1}{4}} & x^5_{n+\frac{1}{4}} & x^6_{n+\frac{1}{4}} & x^7_{n+\frac{1}{4}}\\ 0 & 0 & 0 & 6 & 24x_n & 60x_n^2 & 120x_n^3 & 210x_n^4\\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{4}} & 60x^2_{n+\frac{1}{4}} & 120x^3_{n+\frac{1}{4}} & 210x^4_{n+\frac{1}{4}}\\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{2}} & 60x^2_{n+\frac{1}{2}} & 120x^3_{n+\frac{1}{2}} & 210x^4_{n+\frac{1}{2}}\\ 0 & 0 & 0 & 6 & 24x_{n+\frac{3}{4}} & 60x^2_{n+\frac{3}{4}} & 120x^3_{n+\frac{3}{4}} & 210x^4_{n+\frac{3}{4}}\\ 0 & 0 & 0 & 6 & 24x_{n+1} & 60x^2_{n+1} & 120x^3_{n+1} & 210x^4_{n+1}\end{bmatrix}$$

Solving (6) for the unknown constants and substituting into (2) gives a continuous hybrid linear multistep method in the form

$$y(x)=\alpha_0y_0+\alpha_{\frac{1}{8}}y_{n+\frac{1}{8}}+\alpha_{\frac{1}{4}}y_{n+\frac{1}{4}}+h^3\left[\sum_{j=0}^{1}\beta_jf_{n+j}+\beta_Vf_V\right]\quad \text{where v}=\frac{1}{4},\frac{1}{2},\frac{3}{4} \tag{6}$$

Which when solved for the independent solution at the grid points gives a continuous block formula of the form

$$y^m_{n+k}=\sum_{m=1}^{2}\frac{(kh)^m}{m!}y^m_n+h^3\left[\sum_{j=0}^{1}\sigma_jf_{n+j}+\sigma_vf_{n+v}\right],\quad \text{Where}\quad v=\frac{1}{4}(\frac{1}{4})\frac{3}{4} \tag{7}$$

$$\sigma_0=\frac{1}{2520}t^3(-875t+980t^2-560t^3+128t^4+420)$$

$$\sigma_{\frac{1}{4}}=\frac{-2}{315}(32t^7-126t^6+182t^5-105t^4)$$

$$\sigma_{\frac{1}{2}}=\frac{1}{210}(64t^7-224t^6+266t^5-105t^4)$$

$$\sigma_{\frac{3}{4}} = \frac{-1}{315}(64t^7 - 196t^6 + 196t^5 - 70t^4)$$

$$\sigma_1 = \frac{1}{2520}(128t^7 - 336t^6 + 308t^5 - 105t^4)$$

Evaluating (7) at t = $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$ gives a discrete block method of the form

$$A^0 Y_m^{(i)} = \sum_{i=0}^{2} h^i e_i y_n^{(i)} + h^{(3-i)}\left[d_i f(y_n) + b_i F(Y_m)\right], \quad i = 0,1,2 \tag{8}$$

$$Y_m^{(i)} = \left[y_{n+\frac{1}{8}}^{(i)}, y_{n+\frac{1}{4}}^{(i)}, y_{n+\frac{3}{8}}^{(i)}, y_{n+\frac{1}{2}}^{(i)}, y_{n+\frac{5}{8},}^{(i)} y_{n+\frac{3}{4},}^{(i)} y_{n+\frac{7}{8},}^{(i)} y_{n+1,}^{(i)} \right]^T$$

$$F(Y_m) = \left[f_{n+\frac{1}{4}}, f_{n+\frac{1}{2}}, f_{n+\frac{3}{4}}, f_{n+1}, f_n \right]^T$$

$$f(y_n) = \left[f_{n-\frac{1}{4}}, f_{n-\frac{1}{2}}, f_{n-\frac{3}{4}}, f_{n-1}, f_n \right]^T$$

$A^0 = 8X8$ Identity matrix

When $i = 0$ $e_0 = \begin{bmatrix} 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$

$$e_1 = \begin{bmatrix} 0&0&0&0&0&0&0&\frac{1}{8} \\ 0&0&0&0&0&0&0&\frac{1}{4} \\ 0&0&0&0&0&0&0&\frac{3}{8} \\ 0&0&0&0&0&0&0&\frac{1}{2} \\ 0&0&0&0&0&0&0&\frac{5}{8} \\ 0&0&0&0&0&0&0&\frac{3}{4} \\ 0&0&0&0&0&0&0&\frac{7}{8} \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0&0&0&0&0&0&0&\frac{1}{128} \\ 0&0&0&0&0&0&0&\frac{1}{32} \\ 0&0&0&0&0&0&0&\frac{9}{128} \\ 0&0&0&0&0&0&0&\frac{1}{8} \\ 0&0&0&0&0&0&0&\frac{25}{128} \\ 0&0&0&0&0&0&0&\frac{9}{32} \\ 0&0&0&0&0&0&0&\frac{49}{128} \\ 0&0&0&0&0&0&0&\frac{1}{2} \end{bmatrix} \qquad d_0 = \begin{bmatrix} 0&0&0&0&0&0&0&\frac{2599}{10321920} \\ 0&0&0&0&0&0&0&\frac{113}{71680} \\ 0&0&0&0&0&0&0&\frac{9729}{2293760} \\ 0&0&0&0&0&0&0&\frac{331}{40320} \\ 0&0&0&0&0&0&0&\frac{4625}{344064} \\ 0&0&0&0&0&0&0&\frac{1431}{71680} \\ 0&0&0&0&0&0&0&\frac{81977}{2949120} \\ 0&0&0&0&0&0&0&\frac{31}{840} \end{bmatrix}$$

$$b_0 = \begin{bmatrix} 0 & \frac{2693}{20643840} & 0 & \frac{-601}{6881280} & 0 & \frac{155}{4128768} & 0 & \frac{-143}{20643840} \\ 0 & \frac{107}{64512} & 0 & \frac{-103}{107520} & 0 & \frac{43}{107520} & 0 & \frac{-47}{645120} \\ 0 & \frac{15201}{2293760} & 0 & \frac{-7209}{2293760} & 0 & \frac{2979}{2293760} & 0 & \frac{-27}{114688} \\ 0 & \frac{83}{5040} & 0 & \frac{-1}{168} & 0 & \frac{13}{5040} & 0 & \frac{-19}{403320} \\ 0 & \frac{130625}{4128768} & 0 & \frac{-10625}{1376256} & 0 & \frac{1875}{458752} & 0 & \frac{-3125}{4128768} \\ 0 & \frac{1863}{35840} & 0 & \frac{-243}{35840} & 0 & \frac{45}{7168} & 0 & \frac{-81}{71680} \\ 0 & \frac{45619}{589824} & 0 & \frac{2401}{983040} & 0 & \frac{31213}{2949120} & 0 & \frac{-2401}{1474560} \\ 0 & \frac{34}{315} & 0 & \frac{1}{210} & 0 & \frac{2}{105} & 0 & \frac{-1}{504} \end{bmatrix}$$

When $i=1$

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad e_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4081}{737280} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{367}{23040} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2181}{81920} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{53}{1440} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{6925}{147456} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{147}{2560} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{50029}{737280} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{90} \end{bmatrix} \qquad b_1 = \begin{bmatrix} 0 & \frac{121}{30720} & 0 & -\frac{313}{122880} & 0 & \frac{5}{4608} & 0 & -\frac{49}{245760} \\ 0 & \frac{3}{128} & 0 & \frac{-47}{3840} & 0 & \frac{29}{5760} & 0 & -\frac{7}{7680} \\ 0 & \frac{297}{5120} & 0 & \frac{-891}{40960} & 0 & \frac{93}{10240} & 0 & -\frac{27}{16384} \\ 0 & \frac{1}{10} & 0 & \frac{-1}{48} & 0 & \frac{1}{90} & 0 & -\frac{1}{480} \\ 0 & \frac{875}{6144} & 0 & \frac{-125}{24576} & 0 & \frac{125}{9216} & 0 & -\frac{125}{49152} \\ 0 & \frac{117}{640} & 0 & \frac{27}{1280} & 0 & \frac{3}{128} & 0 & -\frac{9}{2560} \\ 0 & \frac{343}{1536} & 0 & \frac{5831}{122880} & 0 & \frac{4459}{92160} & 0 & -\frac{343}{81920} \\ 0 & \frac{4}{15} & 0 & \frac{1}{15} & 0 & \frac{4}{45} & 0 & 0 \end{bmatrix}$$

When $i=2$

$$e_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{847}{11520} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{251}{2880} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{213}{2560} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{29}{360} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{95}{1152} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{27}{320} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1883}{23040} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{90} \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 0 & \frac{1969}{23040} & 0 & \frac{-397}{7680} & 0 & \frac{499}{23040} & 0 & -\frac{91}{23040} \\ 0 & \frac{323}{1440} & 0 & \frac{-11}{120} & 0 & \frac{53}{1440} & 0 & -\frac{19}{2880} \\ 0 & \frac{813}{2560} & 0 & \frac{-117}{2560} & 0 & \frac{63}{2560} & 0 & -\frac{3}{640} \\ 0 & \frac{31}{90} & 0 & \frac{1}{15} & 0 & \frac{1}{90} & 0 & -\frac{1}{360} \\ 0 & \frac{1525}{4608} & 0 & \frac{275}{1536} & 0 & \frac{175}{4608} & 0 & -\frac{25}{4608} \\ 0 & \frac{51}{160} & 0 & \frac{9}{40} & 0 & \frac{21}{160} & 0 & -\frac{3}{320} \\ 0 & \frac{7693}{23040} & 0 & \frac{1421}{7680} & 0 & \frac{6223}{23040} & 0 & \frac{49}{11520} \\ 0 & \frac{16}{25} & 0 & \frac{2}{15} & 0 & \frac{16}{45} & 0 & \frac{7}{90} \end{bmatrix}$$

3. Basic Properties of the Method Developed

3.1. Order of the Block

Define a Linear Operator $L\{y(x):h\}$ on (7) as

$$L\{y(x):h\} = A^0 y_m^{(i)} - \sum_{i=0}^{2} h^i e_i y_n^{(i)} - h^{3-i}\left[df(y_n) + bF(y_m)\right] \tag{9}$$

Expanding y_{n+j} and f_{n+j} in Taylor series and comparing the coefficients of h gives

$$L\{y(x):h\} = C_0 y(x) + C_1 y'(x) + \ldots + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + C_{p+2} h^{p+2} y^{p+2}(x) + \ldots.$$

Definition 1 The linear operator L and associated block method are said to be of order p if $C_0 = C_1 = \ldots = C_p = C_{p+1} = 0$ $\quad C_{p+2}$ called the error constant and implies that the truncation error is given by $t_{n+k} = C_{p+2} h^{p+2} y^{p+2}(x) + 0(h^{p+3})$

By comparing the coefficient of h, order of the method is six with error constant of

$$\left[\frac{89}{16911433728}, \frac{139}{2642411520}, \frac{12609}{75161927680}, \frac{1}{2949120}, \frac{75125}{135291469824}, \frac{243}{293601280}, \frac{112847}{96636764160}, \frac{1}{645120} \right]^T$$

3.2. Consistency

A block method is said to be consistent if it has order $p \geq 1$. Hence the block method developed is consistent.

3.3. Zero Stability

A block method is said to be zero stable if as $h \to 0$, the roots $r_j = 1(1)k$ of the first characteristics polynomial $\rho(r) = 0$ that is $\rho(r) = \det\left[\sum A^{(0)} R^{K-1}\right] = 0$ satisfying $|r| \leq 1$, must have multiplicity equal to unity

For the block method derived

$$\rho(r) = \left\| \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right\| = 0$$

$r^7(r-1) = 0$ Implying that $r = [0,0,0,0,0,0,0,1]$, hence the method is zero stable

3.4. Convergence

A block method is said to be convergent if and only if it is consistent and zero stable. It is obvious that the block method is convergent.

4. Numerical Examples

Problem 1

$y''' = e^x, \qquad y(0) = 3, y'(0) = 1, y''(0) = 5$

Theoretical Solution: $y(x) = 2 + 2x^2 + e^x$,

The result is given in Table 1

Source: Taparki, *et al.* [23]

Error in NM : Error in the New Method

Table-1. Comparison of Error in Problem 1

x	**Theoretical Solution**	**Numerical Solution**	**Error in Taparki, *et al.* [23] Runge-Kutta Method**	**Error in NM**
0.1	3.12517091807564770	3.12517091807564730	2.280200e-05	4.440892e-16
0.2	3.30140275816016970	3.30140275816016880	1.673740e-04	8.881784e-16
0.3	3.52985880757600330	3.52985880757600110	5.571310e-04	2.220446e-15
0.4	3.81182469764127020	3.81182469764126660	1.328468e-03	3.552714e-15
0.5	4.14872127070012820	4.14872127070012290	2.632129e-03	5.329071e-15
0.6	4.54211880039050890	4.54211880039050090	4.634706e-03	7.993606e-15
0.7	4.99375270747047750	4.99375270747046420	7.520313e-03	1.332268e-14
0.8	5.50554092849246860	5.50554092849244900	1.148243e-02	1.953993e-14
0.9	6.07960311115695080	6.07960311115692330	1.674593e-02	2.753353e-14
1.0	6.71828182845904640	6.71828182845901270	2.355935e-02	3.375078e-14

Problem 2

$y''' = 3\cos x, \qquad y(0) = 1, y'(0) = 0, y''(0) = 2$

Theoretical Solution: $y(x) = x^2 + 3x + 1 - 3\sin x$

The result is given in Table 2

Source: Taparki, *et al.* [23]

Error in NM : Error in the New Method

Table-2. Comparison of Error in Problem 2

x	Theoretical Solution	Numerical Solution	Error in Taparki, *et al.* [23] Runge-Kutta Method	Error in NM
0.1	1.01049975005951560	1.01049975005951560	2.480000e-07	0.000000e+00
0.2	1.04399200761481660	1.04399200761481680	7.374000e-06	2.220446e-16
0.3	1.10343938001598120	1.10343938001598210	6.054200e-05	8.881784e-16
0.4	1.19174497307404840	1.19174497307404990	2.547870e-04	1.554312e-15
0.5	1.31172338418739100	1.31172338418739390	7.760160e-04	2.886580e-15
0.6	1.46607257981489350	1.46607257981489880	1.926125e-03	5.329071e-15
0.7	1.65734693828692680	1.65734693828693440	4.150540e-03	7.549517e-15
0.8	1.88793172730143200	1.88793172730144240	8.363734e-03	1.043610e-14
0.9	2.16001927111755030	2.16001927111756450	1.477375e-02	1.421085e-14
1.0	2.47558704557631200	2.47558704557633020	2.470199e-02	1.820766e-14

4.1. Discussion of Results

Two examples are considered to illustrate the efficiency of the newly derived method. It is evident that the new block method is superior to the method given in Taparki, *et al.* [23] numerically.

Tables 1 and 2 show that the New Continuous Hybrid Constant Block Method is better in terms of accuracy and convergence to theoretical solution when compared to Taparki, *et al.* [23] where they used an implicit Runge- Kutta method to solve the two problems considered in this paper.

5. Conclusion

In this paper, we have shown the efficiency of the New Continuous Hybrid Constant Block Method over an implicit Runge- Kutta method for solving general third-order ODEs.

Results from the numerical examples revealed that the performance of the developed method is better in terms of maximum errors and converges more closely to the exact solution especially with the reduced step size used in generating the scheme used in this paper.

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