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Modeling Heteroscedasticity in the Presence of Outliers in Discrete-Time Stochastic Series

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Abstract

The aim of this study is to examine the effects of outliers on the specification and efficiency of heteroscedastic models fitted to the daily closing share price returns series of two outstanding banks in Nigeria from January 3, 2006 to November 24, 2016. The series consists of 2690 observations for each bank. The data were obtained from the Nigerian Stock Exchange. GARCH(2,0) model with respect to student-t error distribution and GARCH(1,1) model under normal error distribution were successfully fitted to the outlier contaminated series of Diamond bank and United bank for Africa accordingly. On the contrary, EGARCH(1,1) model with respect to student-t error distribution adequately captured the changing variance in the outlier adjusted series of the two banks considered. Substantial evidence revealed that the presence of outliers in returns series leads to model misspecification and adjusting for such outliers ensures model efficiency.

Keywords: ARCH effects; Model efficiency; Nigerian banking sector; Volatility.

1. Introduction

Heteroscedasticity simply means changing variance. It usually occurred when the assumption of constant variance in time series is violated. According to Rosopa, *et al.* [1], the assumption of constant variance is required to ensure the accuracy of standard errors and asymptotic covariances among estimated parameters. It could be remarked that a major setback to linear stationary models when applying to financial data (returns series) is their failure to account for changing variance. One way to account for changing variance is to entertain heteroscedastic models mainly generalized autoregressive conditional heteroscedastic (GARCH-type) models. Meanwhile, stock returns often occur in clusters indicating the presence of outliers which is a very common attribute in time series data. By definition, outliers are observations that are distinct from the main body of the data and are incompatible with the rest of the data [2]. As noted by Alih and Ong [3], outliers in homoscedastic model make the model heteroscedastic. Carnero, *et al.* [4] further affirmed and maintained that outliers affect the identification of conditional heteroscedasticity and the estimation of GARCH models. Also, it is evident in Rana [5] that outliers have great impact on the existing heteroscedasticity tests and the estimators of heteroscedastic model and such impact of outliers on the diagnostic tools for heteroscedasticity is well defined in van Dijk, *et al.* [6]. They showed that both the asymptotic size and power properties of Lagrange (LM) test for ARCH/GARCH are adversely affected by outliers, particularly, additive outliers. Meanwhile, Grossi and Laurini [7] found that order of identification, t-statistics and corresponding p-values of the estimates of GARCH parameters are affected by outliers in an unexpected manner. Therefore, it could be argued that it is gainful to take into consideration the presence of outliers whenever heteroscedasticity is modeled.

This study is aimed at examining the effects of outliers on the specification and efficiency of heteroscedastic models. The motivation is derived from the fact that prior studies in Nigeria did take to account the effects of outliers while modeling the heteroscedasticity in the stock returns of Nigerian banks. The fact that previous studies in Nigeria have failed to consider the presence of outliers while modeling heteroscedasticity in stock returns has provided a novel ground for this study. For instance, Onwukwe, *et al.* [8] investigated the time series behaviours of daily stock returns of four firms listed in the Nigerian Stock Market from January 2, 2002 to December, 31 2006 using three different models of heteroscedastic process, namely; GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models respectively. The four firms whose share prices were used in the analysis are United Bank for Africa, Unilever, Guinness and Mobil. All return series exhibit leverage effect, leptokurtosis, volatility clustering and negative skewness which are common to most economic financial time series. The estimated results revealed that the GJR-GARCH(1,1) gives a better fit to the data and are found to be superior both in-sample and out-sample forecasts evaluation.

Onwukwe, *et al.* [9] studied the modeling and forecasting of daily returns volatility of Nigerian Banks Stocks using data from January 4, 2005 to August 31, 2012 Three symmetric models ARCH(1), ARCH(2) and

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GARCH(1,1) and two asymmetric models EGARCH(1,1) and TARCH(1,1) were used in capturing the volatility pattern of the banks stocks. The findings of the study revealed that the return series were stationary but not normally distributed with the presence of ARCH effect. Furthermore, the results of post estimation evaluation revealed that asymmetric conditional heteroscedastic models are more suitable for modeling daily returns volatility of Nigerian Banks stocks compared with symmetric heteroscedastic models.

Akpan, *et al.* [10] looked at a possible combination of both ARMA and ARCH-type models to form a single model such as ARMA-ARCH that will completely model the linear and non-linear features of financial data. Daily closing share prices of First Bank of Nigeria plc from January 4, 2000 to December 31, 2013were considered. The study provided evidence to show that ARMA (2,2) model was adequate in modeling the linear dependence in the returns while ARCH(1) model was adequate in modeling the changing conditional variance in the returns. Hence, ARMA(2,2)-ARCH(1) model completely modeled the returns series of First Bank of Nigeria.

Akpan and Moffat [11] detected and modeled the asymmetric GARCH effects in a discrete-time series by exploring the share price returns of Zenith bank plc obtained from the Nigerian Stock Exchange from January 4, 2006 to May26, 2015. The study applied sign and size test to identify the asymmetric GARCH effects and modeled by EGARCH and TGARCH respectively with respect to normal distribution. The findings of the study revealed that the asymmetric effect was adequately captured modeled by EGARCH(0,1) and TGARCH(0,1) models.

The organization of the remaining part of the paper is as follows: section 2 treats the materials and methods; results are presented in section 3; discussion of results is covered in section 4 while section 5 accommodates the conclusion.

2. Materials and Methods

2.1. Return

The return series R_t can be obtained given that P_t is the price of a unit share at time, t and P_{t-1} is the share price at time t−1.

$$\boldsymbol{R_t = \nabla lnP_t = (1 - B)lnP_t = ln\,P_t - ln\,P_{t-1}} \quad (1)$$

The R_t in equation (1) is regarded as a transformed series of the share price, P_t meant to attain stationarity, that is, both mean and variance of the series are stable [11]. The letter $\boldsymbol{B}$ is the backshift operator.

2.2. Autoregressive Integrated Moving Average (Arima) Model

Box, *et al.* [12] considered the extension of ARMA model to deal with homogenous non-stationary time series in which X_t, itself is non-stationary but its d^{th}difference is a stationary ARMA model. Denoting the d^{th} difference of X_t by

$$\varphi(B) = \phi(B)\nabla^d X_t = \theta(B)\varepsilon_t \quad (2)$$

where $\varphi(B)$ is the nonstationary autoregressive operator such that d of the roots of $\varphi(B) = 0$ are unity and the remainder lie outside the unit circle. $\phi(B)$ is a stationary autoregressive operator.

2.3. Heteroscedastic Models

2.3.1. Autoregressive Conditional Heteroscedastic (ARCH) Model

The first model that provides a systematic framework for modeling heteroscedasticity is the ARCH model of Engle [13]. Specifically, an ARCH (q) model assumes that,

$$R_t = \mu_t + a_t, \quad a_t = \sigma_t e_t,$$
$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2. \quad (3)$$

where $[e_t]$ is a sequence of independent and identically distributed (i.i.d.) random variables with mean zero, that is E(e_t) = 0 and variance 1, that is E(e_t^2) = 1, $\omega > 0, and \alpha_1, .., \alpha_q \geq 0$ [14]. The coefficients α_i, for $i > 0$, must satisfy some regularity conditions to ensure that the unconditional variance of $a_t$is finite.

2.3.2. Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of a share price return. Some alternative models must be sought. Bollerslev [15] proposed a useful extension known as the generalized ARCH (GARCH) model. For a return series, R_t, let $a_t = R_t - \mu_t$ be the innovation at time t. Then, a_t follows a GARCH (q, p) model if

$$a_t = \sigma_t e_t,$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 . \quad (4)$$

where again e_t is a sequence of i.i.d. random variance with mean, 0, and variance, 1, $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $and \sum_{i=1}^{max(p,q)} (\alpha_i + \beta_i) < 1$ [16].

Here, it is understood that $\alpha_i = 0$, for $i > p$, and $\beta_i = 0$, for $i > q$. The latter constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of a_t is finite, whereas its conditional variance σ_t^2, evolves over time.

2.3.3. Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) Model

The EGARCH model represents a major shift from ARCH and GARCH models [17]. Rather than modeling the variance directly, EGARCH models the natural logarithm of the variance, and so no parameter restrictions are required to ensure that the conditional variance is positive. The EGARCH (q, p) is defined as,

$$R_t = \mu_t + a_t, \quad a_t = \sigma_t e_t,$$

$$ln\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \left| \frac{a_{t-i}}{\sqrt{\sigma_{t-i}^2}} \right| + \sum_{k=1}^{r} \gamma_k \left(\frac{a_{t-k}}{\sqrt{\sigma_{t-k}^2}} \right) + \sum_{j=1}^{p} \beta_j \, ln\sigma_{t-j}^2. \quad (5)$$

where again, e_t is a sequence of i.i.d. random variance with mean, 0, and variance, 1, and γ_k is the asymmetric coefficient.

2.3.4. Glosten, Jagannathan and Runkle (GJR-GARCH) Model

The GJR-GARCH (q, p) model proposed by Glosten, *et al.* [18] is a variant, represented by

$$a_t = \sigma_t e_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{i=1}^{p} \gamma_i I_{t-i} a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \quad (6)$$

where I_{t-1} is an indicator for negative a_{t-i} , that is,

$$I_{t-1} = \begin{cases} 0 & if \;\; a_{t-i} < 0, \\ 1 & if \;\; a_{t-i} \geq 0, \end{cases}$$

and α_i, γ_i, and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models. Also the introduction of indicator parameter of leverage effect, I_{t-1} in the model accommodates the leverage effect, since it is supposed that the effect of a_{t-i}^2 on the conditional variance σ_t^2 is different accordingly to the sign ofa_{t-i}.

2.4. Outliers in Time Series

Generally, a time series might contain several, say k outliers of different types and we have the following general outlier model;

$$Y_t = \sum_{j=1}^{k} \tau_j V_j(\mathrm{B}) I_t^{(T)} + X_t, \quad (7)$$

where $X_t = (\theta(B)) / (\varphi(B)) \, a_t$, $V_j(\mathrm{B}) = 1$ for an AO, and $V_j(\mathrm{B}) = \frac{\theta(B)}{\varphi(B)}$ for an IO at $t = T_j$, $V_j(\mathrm{B}) = (1 - B)^{-1}$ for a LS, $V_j(\mathrm{B}) = (1 - \delta B)^{-1}$ for an TC, and τ is the size of outlier. For more details on the types of outliers and estimation of the outliers effects [12, 19-23].

Moreover, in financial time series, the residual series, a_t is assumed to be uncorrelated with its own past, so additive, innovative , temporary change and level shift outliers coincide, and where both the mean and variance equations evolves together, we have for example GARCH(1,1) model:

$$R_t - \mu_t = \tilde{a}_t + \tau I_t^{(T)}. \quad (8)$$

$$\tilde{a}_t = \sigma_t e_t. \quad (9)$$

$$\sigma_t^2 = \omega + \alpha_1 \tilde{a}_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \quad (10)$$

where $\tilde{a}_t$ is the outliers contaminated residuals.

2.4.1. Methods of Outliers Detection in Heteroscedasticity

One approach for correcting the series for outliers is using standard criteria and then estimates the conditional variance. This approach involves detecting and correcting of outliers before estimating the conditional variance [24].

2.4.2. Efficiency of Heteroscedastic Models

Efficiency is a measure of quality of an estimator of a model. It is often expressed using variance or mean square error. For the purpose of this study which looks at a unified effect of outliers, unconditional variance is considered as the measure of efficiency of estimator of heteroscedastic model.

For ARCH(q) model which is equivalent to GARCH(q, 0) model, the unconditional variance is given as follows:

$$\sigma^2 = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i} \quad (11)$$

For GARCH(q,p) model, the unconditional variance is expressed thus:

$$\sigma^2 = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_i} \quad (12)$$

For EGARCH(q,p) model, the unconditional variance is expressed as follows:

$$\sigma^2 = e^{\left(\frac{\omega}{1 - \sum_{j=1}^{p} \beta_j} \right)} \quad (13)$$

Where e is natural exponential function

For GJR-GARCH(q,p) model

$$\sigma^2 = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{i=1}^{q} \gamma_i/2 - \sum_{j=1}^{p} \beta_j} \quad (14)$$

Hence, a model with the smallest unconditional variance is considered superior in terms of efficiency.

3. Results

Figures 1 - 2 represent the share price series for the two giant banks in Nigeria. It could be observed that the share prices of all the banks do not fluctuate around a common mean which clearly indicate the presence of a stochastic trend in the share prices, implying non-stationarity.

Figure-1. Share Price Series of Diamond Bank

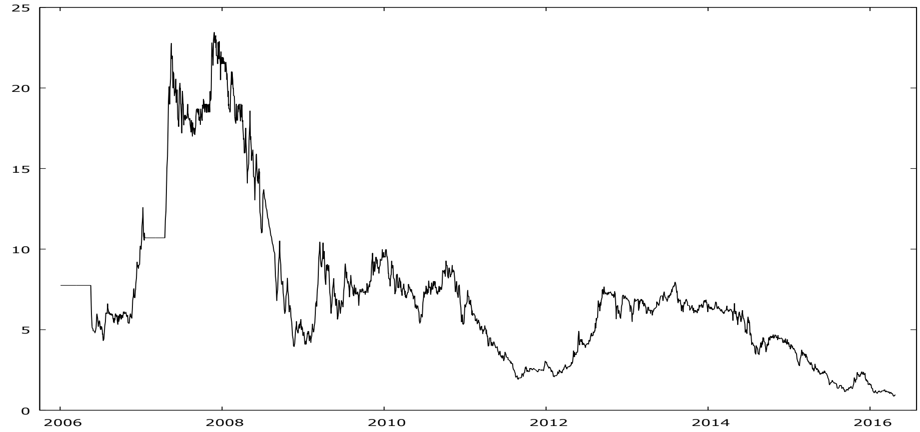


Figure-2. Share Price Series of United Bank for Africa

Since the share price series is found to be non-stationary, the first difference of the natural logarithm of the series is taken to have a stationary (returns) series. The inclusion of the log transformation is to normalize the variance. Figures 3-4 show that the returns series appear to be stationary which suggests that volatility clustering is quite evident in the series.

Figure-3. Return Series of Diamond Bank

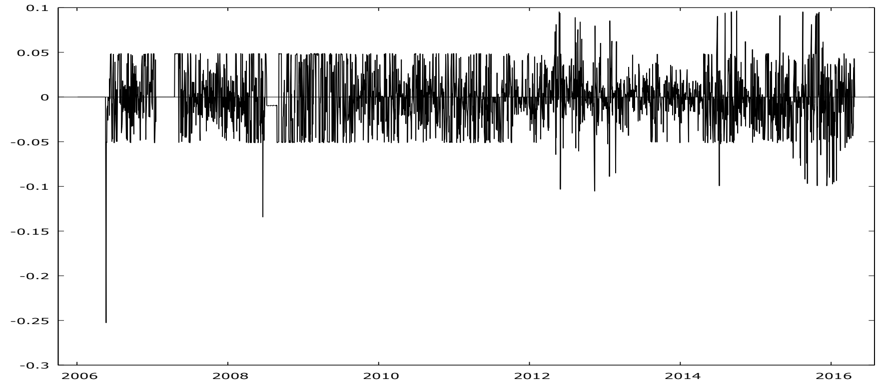
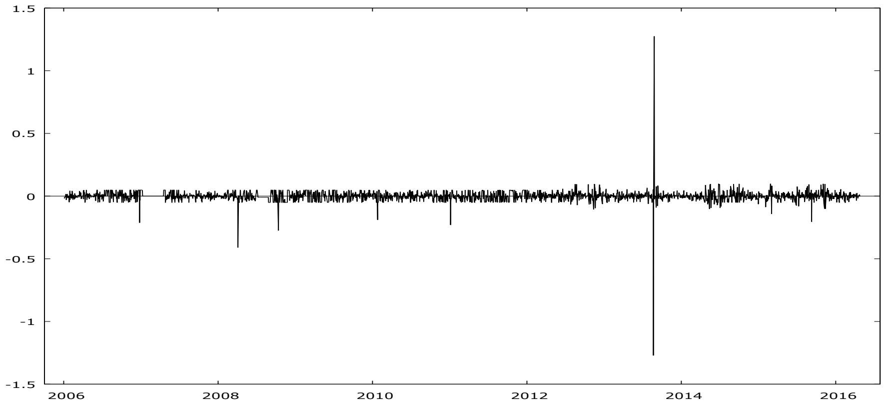


Figure-4. Return Series of United Bank for Africa Diamond Bank



From Figures 5 and 6, both ACF and PACF indicate that mixed model could be entertained. The following models; ARIMA(1,1,1), ARIMA(1,1,2) and ARIMA (2,1,1) are considered tentatively.

Figure-5. ACF of Return Series of Diamond Bank

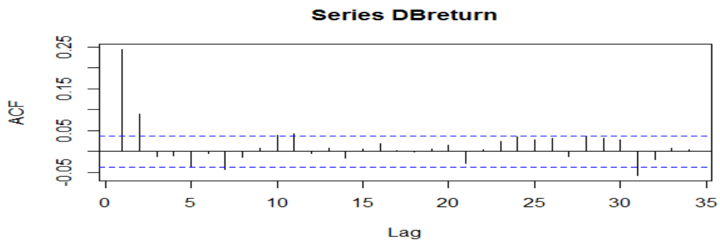
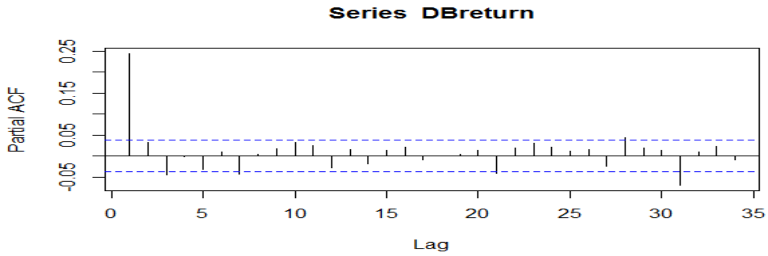


Figure-6. PACF of Return Series of Diamond Bank



From Table I, ARIMA(2,1,1) model is selected based on the grounds of significance of the parameters and minimum AIC.

Table-I. ARIMA Models for Return Series of Diamond Bank

Model	Parameter				Akaike Information Criteria (AIC)
	φ_1	φ_2	θ_1	θ_2	
ARIMA(1,1,1)	0.3349^{***}		-0.0957		-11357.69
ARIMA(1,1,2)	-0.0476		0.2858	0.1093^{*}	-11360.79
ARIMA(2,1,1)	-0.5029^{***}	0.2199^{***}	0.7404^{***}		-11360.86

*** significance at 5% level ; * significance at 1% level

Evidence from Ljung-Box Q-statistics shows that ARIMA(2,1,1) model is adequate at 5% level of significance given the Q-statistic at Lags 1, 4, 8 and 24, that is, Q(1) = 0.0084, Q(4) = 1.5075, Q(8) = 6.3308 and Q(24) = 25.476 with corresponding ($P = 0.9268$), ($P = 0.8253$), ($P = 0.6102$) and ($P = 0.3803$) respectively.

On the other hand, evidence from ACF and PACF in Figures 7 and 9; Portmanteau-Q (PQ) statistics and Lagrange-Multiplier (LM) test statistics in Table II shows that heteroscedasticity exists.

Figure-7. ACF of Squared Residuals of ARIMA(2,1,1) Model

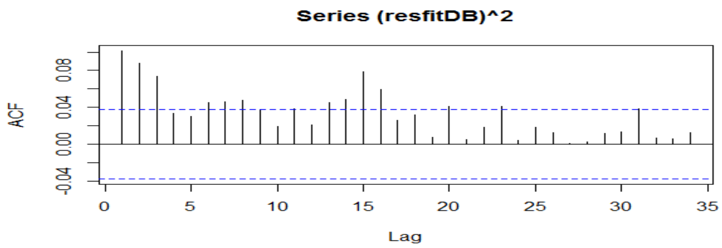


Figure-8. PACF of Squared Residuals of ARIMA(2,1,1) Model

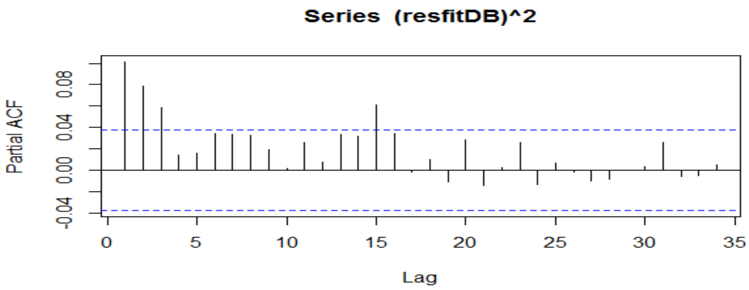


Table-II. ARCH Heteroscedasticity Test for Residuals of ARIMA(2,1,1) Model fitted to Return Series of Diamond Ban

Lag(Order)	**Portmanteau-Q Test**	**p-value**	**Lagrange -Multiplier Test**	**p-value**
4	66.1	$1.53e-13^{***}$	2021	0.0000^{***}
8	85.9	$3.11e-15^{***}$	992	0.0000^{***}
12	95.7	$3.89e-15^{***}$	651	0.0000^{***}
16	133.9	0.0000^{***}	472	0.0000^{***}
20	143.1	0.0000^{***}	373	0.0000^{***}
24	148.6	0.0000^{***}	307	0.0000^{***}

*** significance at 5% level

With the presence of heteroscedasticity in the residual series of ARIMA(2,1,1) model confirmed, we moved to entertain heteroscedastic models that would account for the failure of the ARIMA(2,1,1) model in capturing the changing variance. Tentatively, the following models are considered: GARCH(1,0), GARCH(2,0), GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) with respect to Normal (norm) and Student-t (std) distributions. The following model candidates; GARCH(1,0)-norm, GARCH(2,0)-norm, GARCH(1,1)-std, and GJR-GARCH(1,1)-std were not successful due to convergence problem, a situation where the estimator of the parameter is not equal to the true parameter, that is, the estimator of the parameter does not converge to the true parameter. On the other, GARCH(1,0)-std, GARCH(2,0)-std, GARCH(1,1)-norm, EGARCH(1,1)-norm, EGARCH(1,1)-std, and GJR-GARCH(1,1)-norm were successful.

Comparing the values of the information criteria of the six models as indicated in Table III, it is observed that the information criteria for GARCH(2,0)-std model is the smallest. Hence, based on the ground of smallest information criteria, GARCH(2,0)-std model is selected as the appropriate heteroscedastic model for the return series of Diamond bank.

Table-III. Output of Heteroscedastic Models of Return Series of Diamond Bank

Model	**Parameter**					**Information Criteria**		
						Akaike	**Bayes**	**Hannan-Quinn**
GARCH (1,0)-std	μ	φ_1	φ_2	θ_1				
	$-9.93e^{-4***}$	0.6479^{***}	0.0115	-0.7192^{***}				
	ω	α_1	α_2	β_1	γ_1			
	$4.9e^{-4***}$	0.5380^{***}				-4.3202	-4.3049	-4.3147
GARCH (2,0)-std	μ	φ_1	φ_2	θ_1				
		-0.2748^{***}	0.1899^{***}	0.2976^{***}				
	ω	α_1	α_2	β_1	γ_1			
		0.5085^{***}	0.4899^{***}			-5.0430	-5.0255	-5.0367
GARCH (1,1)-norm	μ	φ_1	φ_2	θ_1				
	$-1.89e^{-4***}$	0.7177^{***}	0.0116	-0.7663^{***}				
	ω	α_1	α_2	β_1	γ_1			
	$5.0e^{-6***}$	0.1499^{***}		0.8491^{***}		-4.3997	-4.3843	-4.3941
EGARCH (1,1)-norm	μ	φ_1	φ_2	θ_1				
	$-1.325e^{-3***}$	-0.6678^{***}	-0.0247	0.6243^{***}				
	ω	α_1	α_2	β_1	γ_1			
	-1.8914^{***}	-0.0003^{***}		0.7326^{***}	0.3446^{***}	-4.3056	-4.2881	-4.2993
EGARCH (1,1)-std	μ	φ_1	φ_2	θ_1				
		-0.2876^{***}	0.0023	0.2356^{***}				
	ω	α_1	α_2	β_1	γ_1			
	-0.8316^{***}	-0.0537^{***}		0.8820^{***}	0.9400^{***}	-4.4228	-4.4031	-4.4157
GJR-GARCH (1,1)-norm	μ	φ_1	φ_2	θ_1				
	$-2.15e^{-4***}$	0.7240^{***}	0.0128	-0.7735^{***}				
	ω	α_1	α_2	β_1	γ_1			
	$5.0e^{-6***}$	0.1400^{***}		0.8486^{***}	0.0208	-4.3994	-4.3819	-4.3931

*** significance at 5% level

The estimated GARCH(2,0) model under student-t distribution is shown in equation (15)

$$R_t = -0.2748R_{t-1} + 0.1899R_{t-2} + 0.2976\,a_{t-1} + a_t$$
s.e: (0.1017) (0.0250) (0.0988)
t-ratio: (−2.7030) (7.5938) (3.0112)
p-value: (0.0069) (0.0000) (0.0026) (15)
$$\sigma_t^2 = 0.5085a_{t-1}^2 + 0.4899a_{t-2}^2$$
s.e: (0.0215) (0.0216)
t-ratio: (23.6094) (22.6980)
p-value: (0.0000) (0.0000)

The model is found to be adequate given that the p-values corresponding to weighted Ljung-Box Q statistics at lags 1, 8 and 14 on Standardized Residuals, weighted Ljung-Box Q statistics at lags 1, 5 and 9 on Standardized Squared Residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7 are all greater than 5% level of significance [see Table IV].

Table-IV. Diagnostic Checking for Heteroscedastic Models of Return Series of Diamond Bank

Model	**Standardized Residuals**		**Standardized Squared Residuals**		**ARCH Tests**	
	Lag	p-value of Weighted LB	Lag	p-value of Weighted LB	Lag	p-value of Weighted LM
GARCH(2,0)-std	1	0.9903	1	0.9835	3	0.9835
	8	1.0000	5	1.0000	5	1.0000
	14	1.0000	9	1.0000	7	1.0000

However, about seventeen (17) different outliers were identified to have contaminated the residual series of ARIMA(21,1) model using the critical value, C =4 on the condition that n≥ 450; Four (4) innovation outliers (IO), ten (10) additive outliers and three (3) temporary change. The outliers at a given are indicated as follows: IO (t = 99), IO (t = 642), IO (t = 1671), IO (t = 1791), AO (t = 1656), AO (t = 1723), AO (t = 1739), AO (t = 1770), AO (t = 1843), AO (t = 2263), AO (t = 2281), AO (t = 2562), AO (t = 2626), TC (t = 98), AO (t = 2559), TC (t = 1667) and TC(t = 2554) [Excepts Table V].

Table-V. Types of Outliers Identified

	Type	**Ind(time)**	**Coefhat(estimate)**	**Tstat**
1	IO	99	-0.25260558	-10.126263
2	IO	642	-0.14020952	-5.620614
4	IO	1671	-0.09994872	-4.006669
5	IO	1791	0.10771031	4.317810
11	AO	1656	0.10786904	4.453514
12	AO	1723	0.09917004	4.094364
13	AO	1739	0.09980816	4.120710
14	AO	1790	-0.09871167	-4.075440
16	AO	1843	0.09746866	4.024121
17	AO	2263	0.10332795	4.266029
18	AO	2281	0.10065028	4.155478
19	AO	2562	0.10020395	4.137050
20	AO	2626	-0.10386669	-4.288272
21	TC	98	-0.09207813	-4.344299
3	AO	2559	-0.09636362	-4.043664
51	TC	1667	0.09378011	4.497073
23	TC	2554	0.08697699	4.190962

Notably, in financial time series, it is assumed that the error is uncorrelated with it past value, and then all the outliers are classified as innovation outliers with a unified effect. An outlier adjusted series is obtained by removing the effects of outliers from the return series. For the purpose of argument, ARIMA(2,1,1) model is fitted to the outlier adjusted series. Evidence from ACF and PACF in Figures 9 and 10, Portmanteau-Q (PQ) statistics and Lagrange-Multiplier (LM) test statistics in Table VI show that heteroscedasticity exist.

Figure-9. ACF of Squares of Residuals of ARIMA(2,1,1) Model fitted to Outlier Adjusted Return Series of Diamond Bank

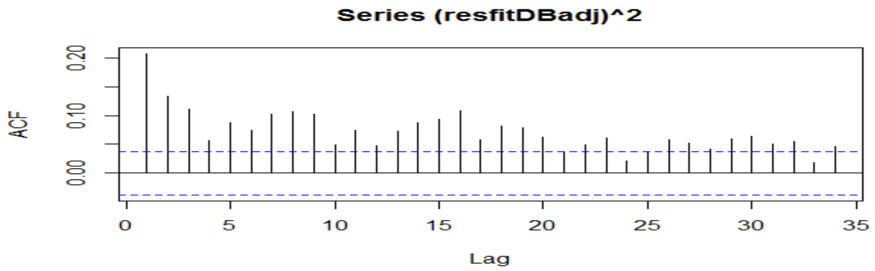


Figure-10. PACF of Squares of Residuals of ARIMA(2,1,1) Model fitted to Outlier

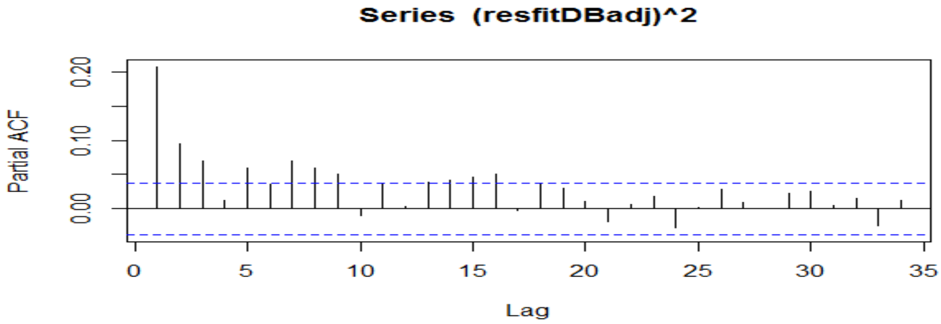


Table-VI. ARCH Heteroscedasticity Test for Residuals of ARIMA(2,1,1) Model fitted to Outlier Adjusted Series of Diamond Bank

Lag(Order)	Portmanteau-Q Test	p-value	Lagrange -Multiplier Test	p-value
4	205	0.0000^{***}	608.2	0.0000^{***}
8	298	0.0000^{***}	291.4	0.0000^{***}
12	353	0.0000^{***}	187.9	0.0000^{***}
16	443	0.0000^{***}	137.3	0.0000^{***}
20	496	0.0000^{***}	107.4	$2.38e^{-14***}$
24	518	0.0000^{***}	88.8	$1.14e^{-09***}$

*** significance at 5% level

For the outlier adjusted series, the following models are considered: GARCH(1,0), GARCH(2,0), GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) with respect to Normal(norm) and Student-t (std) distributions. GARCH(2,0)std and GJR-GARCH(1,1)-std were not successful in that the estimators of the parameters are not equal to the true parameters.

Comparing the values of the information criteria of the eight models as indicated in Table VII, it is observed that GARCH(1,1)-std model has the smallest information criteria but its parameters are not significant. EGARCH(1,1)-norm model has the second smallest information criteria but the second order of the autoregressive part of the mean equation is not significant. The next model with the smallest information criteria is EGARCH(1,1)-std having all its parameters significant. Based on the grounds of significance of the parameters and third smallest information criteria, EGARCH(1,1) model with respect to student-t distribution is chosen as the best fitting model for the outlier adjusted return series of Diamond bank.

Table-VII. Output of Heteroscedastic Models of Outlier Adjusted Return Series of Diamond Bank

Model	Parameter					Information Criteria		
						Akaike	Bayes	Hannan-Quinn
GARCH(1,0) -norm	μ	φ_1	φ_2	θ_1		−4.3114	−4.2983	−4.3067
	$-5.32e^{-4}$	0.9384^{***}	0.0602^{***}	-1.0000^{***}				
	ω	α_1	α_2	β_1	γ_1			
	$4.27e^{-4***}$	0.9990^{***}						
GARCH(1,0) -std	μ	φ_1	φ_2	θ_1		−4.3806	−4.3653	−4.3751
	$-1.041e^{-3***}$	0.7235^{***}	0.0023	-0.7939^{***}				
	ω	α_1	α_2	β_1	γ_1			
	$4.95e^{-4***}$	0.4058^{***}						
GARCH(2,0) -norm	μ	φ_1	φ_2	θ_1		−2.4014	−2.3861	−2.3959
	-0.0585	-0.2388^{***}	-0.1723^{***}	0.7170^{***}				
	ω	α_1	α_2	β_1	γ_1			
	$1.0e^{-6}$	0.3235^{***}	0.6755^{***}					
GARCH (1,1)-norm	μ	φ_1	φ_2	θ_1		−4.4722	−4.4568	−4.4666
	$-3.18e^{-4***}$	0.7421^{***}	0.00386	-0.7890^{***}				
	ω	α_1	α_2	β_1	γ_1			

	$9.0e^{-6***}$	0.1512^{***}		0.8479^{***}				
GARCH(1,1)-std	μ	φ_1	φ_2	θ_1				
		-0.0394	0.0079	-0.0128				
	ω	α_1	α_2	β_1	γ_1			
		0.2970		0.6966		-5.0285	-5.0109	-5.0285
EGARCH (1,1)-norm	μ	φ_1	φ_2	θ_1				
	$-1.6e^{-5}$	-0.4275^{*}	-0.0110	0.3754^{***}				
	ω	α_1	α_2	β_1	γ_1			
	-0.6894^{***}	-0.0099^{**}		0.9033^{***}	0.3678^{***}	-4.4886	-4.4711	-4.4823
EGARCH(1,1)-std	μ	φ_1	φ_2	θ_1				
	$-1.77e^{-4***}$	-0.2736^{***}	0.0167^{***}	0.2381^{***}				
	ω	α_1	α_2	β_1	γ_1			
	-0.2002^{***}	-0.1405^{**}		0.9731^{***}	0.3375^{***}	-4.6156	-4.5959	-4.6085
GJR-GARCH(1,1)-norm	μ	φ_1	φ_2	θ_1				
	-0.0004^{***}	0.7459^{***}	0.0048	-0.7936^{***}				
	$\boldsymbol{\omega}$	$\boldsymbol{\alpha_1}$	$\boldsymbol{\alpha_2}$	$\boldsymbol{\beta_1}$	$\boldsymbol{\gamma_1}$			
	$9.0e^{-5***}$	0.1410^{***}		0.8472^{***}	0.0216	-4.4719	-4.4544	-4.4655

*** significance at 5% level

The estimated EGARCH(1,1) model with respect to student-t distribution is presented in equation (16):

$$R_t = -1.77e^{-4} - 0.2736R_{t-1} + 0.0167R_{t-2} + 0.2381\, a_{t-1} + a_t$$

s.e: $(5.0e^{-6})$ (0.0235) (0.0064) (0.0232)
t-ratio: (-34.3575) (-11.6466) (2.6185) (10.2434)
p-value: (0.0000) (0.0000) (0.0088) (0.0000) (16)

$$ln\sigma_t^2 = -0.2002 + 0.3375a_{t-1} - 0.1405\left(|a_{t-1}| - \frac{2\sqrt{v-2}\,\Gamma(v+1)/2}{(v-1)\Gamma(v/2)\sqrt{\pi}}\right) + 0.9731ln\sigma_{t-1}^2$$

s.e: (0.0056) (0.0137) (0.0218) (4.4×10^{-5})
t-ratio: (-35.7613) (24.7142) (-6.4568) (22280.9466)
p-value: (0.0000) (0.0000) (0.0000) (0.0000)

The selected model is adequate since all the p-values corresponding to weighted Ljung-Box Q statistics at lags 1, 8 and 14 on Standardized Residuals, weighted Ljung-Box Q statistics at lags 1, 5 and 9 on Standardized Squared Residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7 are greater than 5% level of significance [see Table VIII].

Table-VIII. Diagnostic Checking for Heteroscedastic Models of Outlier Adjusted Return Series of Diamond Bank

Model	**Standardized Residuals**		**Standardized Squared Residuals**		**ARCH Tests**	
	Lag	p-value of Weighted LB	Lag	p-value of Weighted LB	Lag	p-value of Weighted LM
	1	0.7966	1	0.9500	3	0.9493
	8	1.0000	5	1.0000	5	0.9995
EGARCH(1,1)-std	14	1.0000	9	1.0000	7	1.0000

3.1. United Bank for Africa

From Figures 11 and 12, both ACF and PACF indicate that mixed model could be entertained. The following models, ARIMA(0,1,1), ARIMA(0,1,2), ARIMA(1,1,1), ARIMA (2,1,1) and ARIMA(2,1,2) are fitted tentatively.

Figure-11. ACF of Return Series of United Bank for Africa

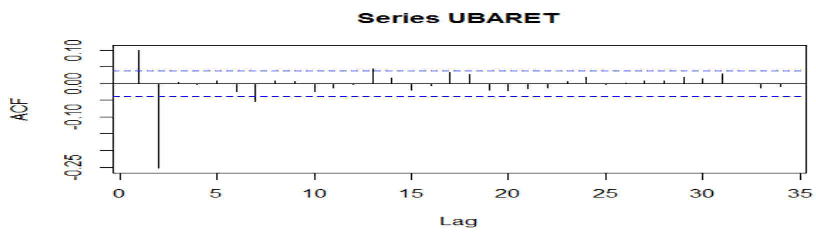
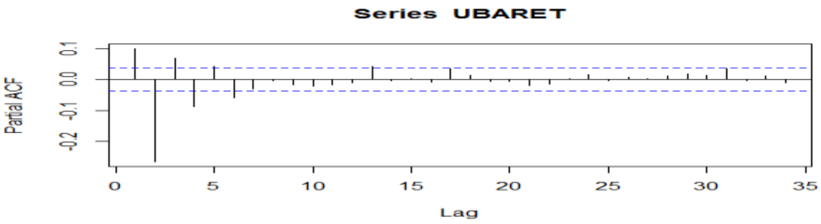


Figure-12. PACF of Return Series of United Bank for Africa



From Table IX, ARIMA(0,1,2) model is selected based on the grounds of significance of the parameters and smallest AIC.

Table-IX. ARIMA Models for Return Series of United Bank for Africa

Model	Parameter				Akaike Information Criteria (AIC)
	φ_1	φ_2	θ_1	θ_2	
ARIMA(0,1,1)			0.2134^{***}		−8862.15
ARIMA(0,1,2)			0.1559^{***}	-0.2838^{***}	-9067.16
ARIMA(1,1,0)	0.1001^{***}				−8831.33
ARIMA(1,1,1)	-0.5283^{***}	0.7563^{***}			−8989.25
ARIMA(2,1,0)	0.1267^{***}	-0.2654^{***}			−9025.98
ARIMA(2,1,2)	0.0313	0.0386	0.1244	-0.3250^{***}	−9063.38

*** significance at 5% level ; * significance at 1% level

Evidence from Ljung-Box Q-statistics shows that ARIMA(0,1,2) model is adequate at 5% level of significance given the Q-statistic at Lags 1, 4, 8 and 24, that is, Q(1) = 0.0002, Q(4) = 0.1807, Q(8) = 8.9625 and Q(24) = 23.662 with corresponding (*P = 0.9882*), (*P = 0.9962*), (*P = 0.3455*) and (*P = 0.4811*) respectively.

Contrariwise, evidence from ACF and PACF in Figures 13 and 14, Portmanteau-Q (PQ) statistics and Lagrange-Multiplier (LM) test statistics in Table X shows that heteroscedasticity exists.

Figure-13. ACF of Squared Residuals of ARIMA(0,1,2) Model

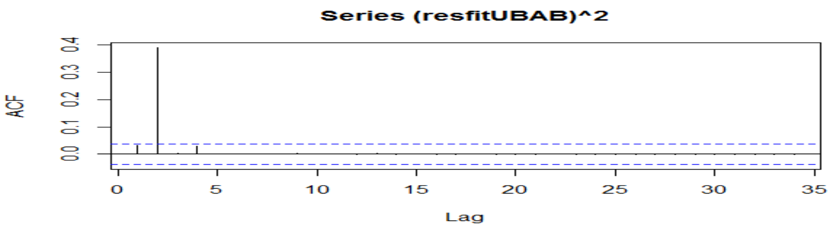


Figure-14. PACF of Squared Residuals of ARIMA(0,1,2) Model

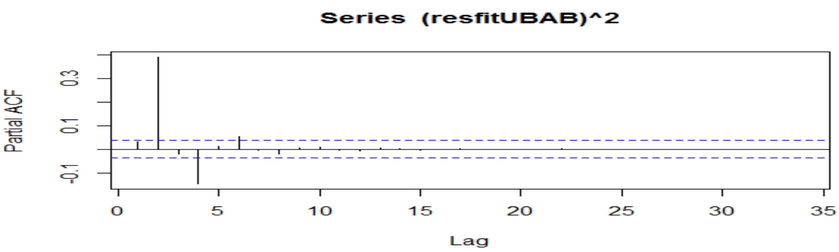


Table-X. ARCH Heteroscedasticity Test for Residuals of ARIMA(0,1,2) Model fitted to Outlier

Lag(Order)	Portmanteau-Q Test	p-value	Lagrange -Multiplier Test	p-value
4	416	0.0000^{***}	3604	0.0000^{***}
8	416	0.0000^{***}	1547	0.0000^{***}
12	416	0.0000^{***}	1025	0.0000^{***}
16	416	0.0000^{***}	766	0.0000^{***}
20	416	0.0000^{***}	611	0.0000^{***}
24	416	0.0000^{***}	508	0.0000^{***}

*** significance at 5% level

With the presence of heteroscedasticity identified in the residual series of ARIMA(0,1,2) model, heteroscedastic models such as GARCH(1,0), GARCH(2,0), GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) with respect to Normal (norm) and Student-t (std) distributions are entertained. Only GARCH(2,0)-norm was not successful due to convergence problem.

Comparing the values of the information criteria of the successful models as indicated in Table XI, it is observed that among the models with significant parameters, GARCH(1,1) model under normal distribution has the smallest information criteria and it is selected as the appropriate heteroscedastic model for the return series of United bank for Africa.

Table-XI. Output of Heteroscedastic Models of Return Series of United Bank for Africa

Model	Parameter						Information Criteria		
							Akaike	Bayes	Hannan Quinn
GARCH(1,0)-norm	μ	φ_1	φ_2	θ_1	θ_2		−3.8921	−3.8812	−3.8882
	$-0.0025e^{-4***}$			-0.2809^{***}	0.2392^{***}				
	ω	α_1	α_2	β_1		γ_1			
	$0.0005e^{-6***}$	0.9990^{***}							
GARCH(1,0)-std	μ	φ_1	φ_2	θ_1	θ_2		−4.2841	−4.2709	−4.2793
	$-0.0005e^{-4}$			-0.0868^{***}	0.3045^{***}				
	ω	α_1	α_2	β_1		γ_1			
	0.0005^{***}	0.6454^{***}							
GARCH(2,0)-std	μ	φ_1	φ_2	θ_1	θ_2		−4.3103	−4.2949	−4.3047
	-0.0006			0.0470	0.2773^{***}				
	ω	α_1	α_2	β_1		γ_1			
	0.0004^{***}	0.5006^{***}	0.2736^{***}						
GARCH (1,1)-norm	μ	φ_1	φ_2	θ_1	θ_2		**−4.1635**	**−4.1504**	**−4.1588**
	$\mathbf{3.0e^{-4***}}$			$\mathbf{0.0550^{***}}$	$\mathbf{0.2716^{***}}$				
	ω	α_1	α_2	β_1		γ_1			
	$\mathbf{2.0e^{-5***}}$	$\mathbf{0.1969^{***}}$		$\mathbf{0.8007^{***}}$					
GARCH(1,1)-std	μ	φ_1	φ_2	θ_1	θ_2		−4.4699	−4.4545	−4.4643
				0.0387	0.2996^{***}				
	ω	α_1	α_2	β_1		γ_1			
		0.2845^{***}		0.7129^{***}					
EGARCH (1,1)-norm	μ	φ_1	φ_2	θ_1	θ_2		−4.0782	−4.0628	−4.0726
				0.1444^{***}	0.2884^{***}				
	ω	α_1	α_2	β_1		γ_1			
	-3.184^{***}	0.2766^{***}		0.5201^{***}		1.2149^{***}			
EGARCH (1,1)-std	μ	φ_1	φ_2	θ_1	θ_2		−4.3778	−4.3603	−4.3715
				-0.0394	0.2975^{***}				
	ω	α_1	α_2	β_1		γ_1			
	-0.4881^{***}	0.0643^{***}		0.9314^{***}		0.4454^{***}			
GJR-GARCH(1,1)-norm	μ	φ_1	φ_2	θ_1	θ_2		−4.1630	−4.1476	−4.1574
	$1.0e^{-4***}$			0.0489^{***}	0.2936^{***}				
	ω	α_1	α_2	β_1		γ_1			
	$2.1e^{-5***}$	0.1927^{***}		0.7930^{***}		0.0201^{***}			
GJR-GARCH(1,1)-std	μ	φ_1	φ_2	θ_1	θ_2		−4.4746	−4.4570	−4.4682
				0.0398	0.2973^{***}				
	ω	α_1	α_2	β_1		γ_1			
		0.2723^{***}		0.7060^{***}		0.0379			

*** significance at 5% level

The estimated GARCH(1,1) model with respect to normal distribution is shown in equation (17)

$$R_t = 3.0e^{-4} + 0.0550a_{t-1} + 0.2716a_{t-2} + a_t$$

s.e: (0.0000) (0.0000) (0.0001)

t-ratio: (3626.2) (3625.2) (3624.9)

p-value: (0.0000) (0.0000) (0.0000) (17)

$$\sigma_t^2 = 2.0e^{-5} + 0.1969a_{t-1}^2 + 0.8007\sigma_{t-1}^2$$

s.e: (0.0000) (0.0001) (0.0002)

t-ratio: (3627.6) (3618.6) (3609.5)

p-value: (0.0000) (0.0000) (0.0000)

The selected model is adequate since all the p-values corresponding to weighted Ljung-Box Q statistics at lags 1, 5 and 9 on Standardized Residuals, weighted Ljung-Box Q statistics at lags 1, 5 and 9 on Standardized Squared Residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7 are greater than 5% level of significance [see Table XII]. The unconditional variance is computed to be 0.0083.

Table-XII. Diagnostic Checking for Heteroscedastic Models of Return Series of United Bank for Africa

Model	Standardized Residuals		Standardized Squared Residuals		ARCH Tests	
	Lag	p-value of Weighted LB	Lag	p-value of Weighted LB	Lag	p-value of Weighted LM
GARCH (1,1)-norm	1	0.7702	1	0.958	3	0.9233
	5	0.1357	5	1.000	5	0.9992
	9	0.2781	9	1.000	7	1.0000

About twenty seven (27) different outliers were found to have contaminated the residuals series of ARIMA(0,1,2) model using the critical value, C =4 on the condition that n≥ 450; one (1) innovation outliers (IO), eleven (11) additive outliers (AO) and fifteen (15) temporary change (TC). The outliers at a given time are indicated as follows: IO (t = 1992), AO (t = 255), AO (t = 588), AO (t = 590), AO (t = 724), AO (t = 1059), AO (t = 1306), AO (t = 1990), AO (t = 1994), AO (t = 2391), AO (t = 2526), AO (t = 2001), TC (t = 258), TC (t = 586), TC (t = 720), TC (t = 722) and TC(t = 743), TC(t = 745), TC(t =747), TC(t = 1057), TC(t = 1507), TC(t = 1727), TC(t = 1988), TC(t = 1993), TC(t = 2212), TC(t = 2217), TC(t = 817), TC(t = 1986), TC(t = 726) and TC(t = 1989) [Excepts Table XIII].

Table-XIII. Types of Outliers Identified

	type	ind(time)	coefhat	tstat
7	IO	1992	-1.28502933	-50.180476
12	AO	255	-0.22086607	-9.215670
14	AO	588	-0.42240432	-17.624884
15	AO	590	-0.18565429	-7.746453
16	AO	724	-0.26195514	-10.930118
18	AO	1059	-0.21501218	-8.971416
19	AO	1306	-0.20172914	-8.417179
21	AO	1990	-0.30146512	-12.578678
24	AO	1994	0.85624927	35.727130
27	AO	2391	-0.12980376	-5.416082
28	AO	2526	-0.18732654	-7.816228
29	TC	258	0.08938516	5.150659
32	TC	586	-0.17371521	-10.010027
36	TC	720	-0.09820632	-5.658963
37	TC	722	-0.12146766	-6.999356
40	TC	743	-0.08686036	-5.005173
41	TC	745	-0.08974508	-5.171399
42	TC	747	-0.08980803	-5.175027
43	TC	1057	-0.10201697	-0.10201697
46	TC	1507	0.08901146	5.129125
48	TC	1727	0.09046794	5.213053
51	TC	1988	-0.14516399	-8.364815
55	TC	1993	0.32569376	18.767519
58	TC	2212	0.09073975	5.228715
181	AO	2001	-0.12316896	-5.194574
431	TC	2217	-0.08968031	-5.223304
44	TC	2386	0.08645455	5.035424
34	TC	817	0.08559737	5.031276
35	TC	1986	0.08566229	5.035092
30	TC	726	0.08504413	5.007596
141	TC	1989	0.12691902	6.052727

In financial time series, it is assumed that the error is uncorrelated with it past value, and then all the outliers are classified as innovation outliers with a unified effect. With the effects of outliers removed from the return series, a new series called outlier adjusted is obtained. For the purpose of argument, ARIMA(0,1,2) model did not fit well to the outlier adjusted series given that second order of parameter of the model is not significant. Consequently, ARIMA(2,1,0) model fitted well to the outlier adjusted series. Evidence from ACF and PACF in Figures 15 and 16, Portmanteau-Q (PQ) statistics and Lagrange-Multiplier (LM) test statistics in Table XIV show that heteroscedasticity exist.

Figure-15. ACF of Squares of Residuals of ARIMA(2,1,0) Model fitted to Outlier Adjusted Return Series of United Bank for Africa

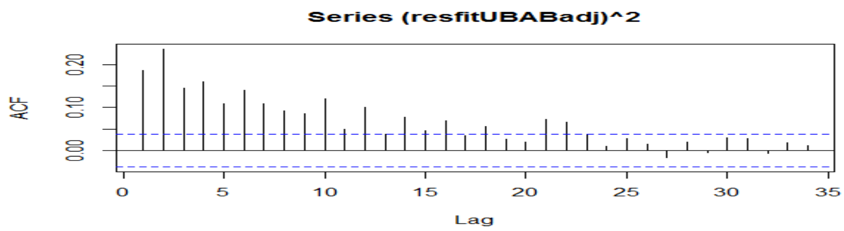


Figure-16. PACF of Squares of Residuals of ARIMA(2,1,0) Model fitted to Outlier Adjusted Return Series of United Bank for Africa

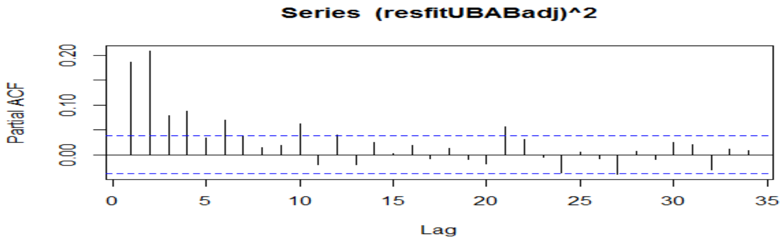


Table-XIV. ARCH Heteroscedasticity Test for Residuals of ARIMA(2,1,0) Model fitted to Outlier Adjusted Series of United Bank for Africa

Lag(Order)	Portmanteau-Q Test	p-value	Lagrange -Multiplier Test	p-value
4	371	0.0000^{***}	683	0.0000^{***}
8	511	0.0000^{***}	321	0.0000^{***}
12	603	0.0000^{***}	210	0.0000^{***}
16	642	0.0000^{***}	155	0.0000^{***}
20	657	0.0000^{***}	124	$2.38e^{-14^{***}}$
24	686	0.0000^{***}	101	$9.49e^{-12^{***}}$

*** significance at 5% level

Having confirmed the presence of heteroscedasticity in the residual series of ARIMA(2,1,0) model fitted to outlier adjusted series of United Bank for Africa, heteroscedastic models such as GARCH(1,0), GARCH(2,0), GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) with respect to Normal (norm) and Student-t (std) distributions are entertained. GARCH(1,0)-norm was not successful due to convergence problem, a situation where the estimator of the parameter is not equal to the true parameter, that is, the estimator of the parameter does not converge to the true parameter.

Comparing the values of the information criteria of the successful models as indicated in Table XV, it is observed that GJR-GARCH(1,1)-std has the smallest information criteria followed by GARCH(1,1)-std and EGARCH(1,1)-std respectively. However, the parameters of the mean equations of GJR-GARCH(1,1)-std and GARCH(1,1)-std are not significant while all the parameters of EGARCH(1,1)-std are significant. Therefore, based on the grounds of significance of the parameters, EGARCH(1,1) model with respect to student-t distribution is chosen as the appropriate heteroscedastic model for the outlier adjusted return series of United bank for Africa.

Table-XV. Output of Heteroscedastic Models of Outlier Adjusted Return Series of United Bank for Africa

Model	**Parameter**						**Information Criteria**		
							Akaike	**Bayes**	**Hannan-Quinn**
GARCH (1,0)-std	μ	φ_1	φ_2	θ_1	θ_2				
	0.0005^{***}	0.9453^{***}	0.0537^{***}	-1.0000^{***}					
	ω	α_1	α_2	β_1		γ_1			
	$4.8e^{-4***}$	0.4579^{***}					-4.4070	-4.3916	-4.4014
GARCH (2,0)-norm	μ	φ_1	φ_2	θ_1	θ_2				
	-0.0002	0.4585	0.0244	-0.5237					
	ω	α_1	α_2	β_1		γ_1			
	$4.5e^{-4***}$	0.2609^{***}	0.1626^{***}				-4.4169	-4.4015	-4.4113
GARCH (2,0)-std	μ	φ_1	φ_2	θ_1	θ_2				
	-0.0004	-1.0828^{***}	-0.0860^{***}	0.0004^{***}					
	ω	α_1	α_2	β_1		γ_1			
	$3.9e^{-4***}$	0.3651^{***}	0.1981^{***}				-4.4313	-4.4137	-4.4249
GARCH (1,1)-norm	μ	φ_1	φ_2	θ_1	θ_2				
	-0.0002	0.0728	0.0234	-0.1356					
	ω	α_1	α_2	β_1		γ_1			
	$3.9e^{-5}$	0.1685^{***}		0.7871^{***}			-4.4608	-4.4455	-4.4553
GARCH (1,1)-std	μ	φ_1	φ_2	θ_1	θ_2				
		0.0402	0.0374	-0.0966					
	ω	α_1	α_2	β_1		γ_1			
		0.2301^{***}		0.7668^{***}			-4.5781	-4.5606	-4.5718
EGARCH (1,1)-norm	μ	φ_1	φ_2	θ_1	θ_2				
	$-1.5e^{-5}$	0.1713^{***}	0.0162	-0.2177^{***}					
	ω	α_1	α_2	β_1		γ_1			
	-0.7800^{***}	-0.0009		0.8915^{***}		0.3369^{***}	-4.4686	-4.4511	-4.4623
EGARCH(1,1)-std	μ	φ_1	φ_2	θ_1	θ_2				
	$\mathbf{-1.3e^{-4}}^{***}$	0.1668^{***}	0.0292^{***}	-0.2254^{***}					
	ω	α_1	α_2	β_1		γ_1			
	-0.3731^{***}	$\mathbf{-0.0426}^{***}$		$\mathbf{0.9486}^{***}$		$\mathbf{0.4125}^{***}$	**−4.5170**	**−4.4972**	**−4.5098**
GJR-GARCH (1,1)-norm	μ	φ_1	φ_2	θ_1	θ_2				
	-0.0004	0.9153^{***}	0.066^{***}	-0.9781^{***}					
	ω	α_1	α_2	β_1		γ_1			
	$4.0e^{-5***}$	0.1648^{***}		0.7856^{***}		0.0098^{***}	-4.4606	-4.4430	-4.4542
GJR-	μ	φ_1	φ_2	θ_1	θ_2		-4.5832	-4.5634	-4.5760

GARCH (1,1)-std		0.0412	0.0357	−0.0975					
	ω	α_1	α_2	β_1		γ_1			
		0.2170***		0.7579***		0.0434			

*** significance at 5% level

The estimated EGARCH(1,1) model under student-t distribution is presented in (18):

$R_t = -1.3e^{-4} + 0.1668R_{t-1} + 0.0291R_{t-2} - 0.2254a_{t-1} + a_t$

s.e: (0.0000) (0.0297) (0.0057) (0.0310)

t-ratio: (−6.2398) (5.6063) (5.1346) (−7.2660)

p-value: (0.0000) (0.0000) (0.0000) (0.0000) (18)

$ln\sigma_t^2 = -0.3731 + 0.4125\, a_{t-1} - 0.0426\left(|a_{t-1}| - \frac{2\sqrt{v-2}\Gamma(v+1)/2}{(v-1)\Gamma(v/2)\sqrt{\pi}}\right) + 0.9485 ln\sigma_{t-1}^2$

s.e: (0.0055) (0.0190) (0.0182) (0.0006)

t-ratio: (−67.2928) (21.6638) (−2.3423) (1646.3180)

p-value: (0.0000) 0.0000) (0.0192) (0.0000)

The selected model is adequate since all the p-values corresponding to weighted Ljung-Box Q statistics at lags 1, 8 and 14 on Standardized Residuals, weighted Ljung-Box Q statistics at lags 1, 5 and 9 on Standardized Squared Residuals and weighted Lagrange Multiplier statistics at lags 3, 5 and 7 are greater than 5% level of significance [see Table XVI]. The computed unconditional variance is 0.0007.

Table-XVI. Diagnostic Checking for Heteroscedastic Models of Outlier Adjusted Return Series of United Bank for Africa

Model	**Standardized Residuals**		**Standardized Squared Residuals**		**ARCH Tests**	
	Lag	p-value of Weighted LB	Lag	p-value of Weighted LB	Lag	p-value of Weighted LM
EGARCH(1,1)-std	1	0.3388	1	0.9020	3	0.8636
	8	1.0000	5	0.9992	5	0.9928
	14	0.9976	9	1.0000	7	0.9995

4. Discussion

ARIMA(2,1,1) model is found to be adequate in modeling the linear dependence in both the outlier contaminated and outlier adjusted return series of Diamond bank. With the presence of heteroscedasticity detected in the residuals of ARIMA(2,1,1) model, GARCH(2,0) model with respect to student-t distribution appeared to capture the heteroscedasticity in the outlier contaminated series and on the contrary, EGARCH(1,1) model is successful in capturing the heteroscedasticity in the outlier adjusted return series. Hence, it is evident that the presence of outliers in the return series of Diamond Bank has substantial effects on the specification of heteroscedastic model. Specifically, on the efficiency of the model, it is observed that the conditional variance of GARCH(2,0) model collapses to 0.0000 instead of the unconditional variance, reason being that the constant term, $\omega = 0.0000$. While the conditional variance of EGARCH(1,1) model actually converges to unconditional variance of 0.0006. Thus EGARCH(1,1) model appears to be more efficient than GARCH(2,0) model. The practical implication of ARIMA(2,1,1) model is that the return at present day depends on its past two day's values and the previous day error term. Also, the presence of heteroscedasticity in the residual series of ARIMA(2,1,1) model implies that assumption of constant variance is violated thereby providing a more pragmatic reason for entertaining heteroscedastic model. For EGARCH(1,1) model, The coefficient of ARCH parameter (also called coefficient of volatility clustering) is negative implying that negative changes are followed by negative changes. The coefficient of GARCH parameter (also known as persistence parameter) which is about 0.9731 is very high and close to one, implying that volatility clustering will continue for a long time. The coefficient of the leverage effect being significant indicates that leverage effect exists in the return series and its positive nature implies that any unexpected increase in the return series would increase the volatility more than any unexpected decrease of the same magnitude.

For the return series of the United Bank for Africa, ARIMA(0,1,2) model is adequate in modeling the linear dependence in the outlier contaminated while ARIMA(2,1,0) model was appropriate for outlier adjusted return series of United Bank for Africa. However, with the presence of heteroscedasticity in the residual series of ARIMA(2,1,0) model, GARCH(1,1) model with respect to normal distribution successfully captured the heteroscedasticity in the outlier contaminated series while on the other hand, EGARCH(1,1) model with respect to student-t distribution adequately modeled the heteroscedasticity in the outlier adjusted series. Therefore, it is apparent that the presence of outliers in the return series of United Bank for Africa has substantial effects on the model specification. Particularly, on the efficiency of the model, it is found that the unconditional variance of EGARCH(1,1) model, 0.0007 is far smaller than the unconditional variance of GARCH(1,1) model, 0.0083; indicating that EGARCH(1,1) model is more efficient than GARCH(1,1) model. The practical implication of ARIMA(2,1,0) model is that the current day return depends on its past two day's values and the previous day shock. But the presence of heteroscedasticity in its residual series implied the violation of assumption of constant variance. For EGARCH(1,1) model, The coefficient of ARCH parameter (also called coefficient of volatility clustering) is positive implying that positive changes are followed by positive changes. The coefficient of GARCH parameter (also known as persistence parameter) which is about 0.9485 is very high and close to one, implying that volatility clustering will continue for a long time. The coefficient of the leverage effect being significant indicates that leverage effect exists in the return series and its negative nature implies that any unexpected decrease in the return series would decrease the volatility more than any unexpected decrease of the same magnitude.

Generally, the implication of considering the presence of outliers in this study is to ensure proper and correct model specification and efficiency.

The findings of this study agree with the works of [8-11] that heteroscedasticity exists in the stock returns of Nigerian banks and could be captured by GARCH-type models but differ in a way that it considers the presence of outliers and shows that with outliers, heteroscedastic models are misspecified and their estimators are less efficient.

5. Conclusion

Outliers being common attributes of every time series adversely influenced the detection and modeling of heteroscedasticity. It is in this view that this study traced the effects of outliers on heteroscedastic models specification and efficiency of estimators. Based on the results of our findings, for return series of Diamond bank; GARCH(2,0) model with respect to student-t error distribution fitted well to the outlier contaminated series while EGARCH(1,1) model with respect to student-t error distribution fitted successfully to the outlier adjusted series. Also, with the unconditional variance being the measure of efficiency, the conditional variance of GARCH(2,0) model failed to converge to the unconditional variance, instead converges to zero. Conversely, the conditional variance of EGARCH(1,1) model actually converges to the unconditional variance thereby showing to be superior in efficiency to GARCH(2,0) model.

For the return series of United bank for Africa; GARCH(1,1) model under normal error distribution adequately expressed the heteroscedasticity in the outlier contaminated series. On the other hand, EGARCH(1,1) under student-t error distribution was suitable for outlier adjusted series. The conditional variances for each of GARCH(1,1) model and EGARCH(1,1) model converge to respective unconditional variances with EGARCH(1,1) model showing to be more efficient. Therefore, it could be deduced that the presence of outliers in time series have an adverse effect in heteroscedasticity modeling.

The effects of outliers on heteroscedasticity forecasting should be considered for further study. Also, The approach of estimating initially a model for the conditional variance and then obtains the conditional outliers using the resulting estimated conditional standard deviations is recommended.

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