

On Two Covariates Cosine and Sine Noisy-Wave Trigonometry Regression of Heartbeats

Olanrewaju Rasaki Olawale

Department of Statistics, Faculty of Science, University of Ibadan, Ibadan, Oyo State, Nigeria

Abstract

This paper proposes and describes the acumen on alternate two covariates linear Cosine and Sine regression functions that possessed a noisy-wave or tone frequencies via wave-trend of actualized observations of regressors and responsive variable needed in fitting a wavy equation of trigonometry regression. The method of maximum likelihood was used in estimating parameters associated to the Cosine and Sine alternate functions via vector coefficients as well as their distributional and residual properties. The estimations obtained via the method were enthralled to the noisy-wave mesokurtic observations of babies' rate of heartbeats exactly an hour after birth (HR_1), two hours after birth (HR_2) and three hours after birth (HR_3). The implementation and illustrative application was via R using the heartbeat dataset. It was gleaned that the trigonometry equation line of $\alpha + \beta_1 \cos(HR_2) + \beta_2 \sin(HR_1)$ optimally captured the wave observations and robustly outstripped the alternate Cosine and Sine equation line of $\alpha + \beta_1 \sin(HR_2) + \beta_2 \cos(HR_1)$.

Keywords: Cosine; Equation line; Noisy-wave; Regression functions; Rates of heartbeat; Sine.



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1. Introduction

Regression analysis is a technique use in modeling the relationship(s) between response variable and predictor(s) or among predictors. This unknown connection could either be a linear or non-linear relationship depending on the transfer function [1, 2]. It is termed "simple regression" if the dependent variable is constrained to only a predictor and "multiple regression" if the formal is subjugated to two or more predictors [3]. The conventional methods of statistic and parameter (regression coefficients, model performance indexes, residual indexes, prediction error indexes, etc.) estimation ranges from Maximum Likelihood (ML), Least Squares (LS), Quasi- Likelihood (QL), Generalized Linear Model (GLM) etc. for parametric approach; method of sieves, difference sequence method, Ordinary differential Equations (ODEs) etc. for non-parametric approach and some amalgamated methods of both parametric and non-parametric that resulted in semi-parametric approach [4-7]. The main purpose of regression modeling is for generalization of studied relationship(s), prediction making, decision-making, diagnosis and to ascertain statistical property of the studied system [8, 9].

According to Hanley [10], a number of extensive studies had been carried-out on different forms of regression estimators to accommodate and recodify the assumptions of normality, independence and attached time factors to covariates. Among the few forms are ridge regression, seasonality regression analysis, Fourier regression, trigonometric series regression analysis, and smoothing splines regression [11-14]. All these mentioned forms are for demonstrating the dummy variables for estimation of seasonal effects in a time series, to penalize estimators in situation where the number of parameters estimated is strictly greater than the sample size, and to free the distributional property of the observations in non-parametric settings [15-17].

Rigdon, *et al.* [18], propounded a Fourier trigonometric like regression and applied it to uniform time-varying public health surveillance disease data with the assertion of normality assumption, seasonality, and independence ascertained as well as the stationarity of the first and second order- Fourier regression like model.

This paper presents a conspectus diversify approach by considering noisy-wave or tone frequencies observations of covariates without seasonality, uniform time varying (unequal spaced time intervals of unordered sequence of set of observations) of recording observations via a Gaussian density function. A two alternate Cosine and Sine linear equation functions (a trigonometry regression approach) will be formulated such the parametric method of maximum likelihood will adopted in estimating the Cosine and Sine alternate equations vector coefficient noisy-wave mesokurtic observations as well as its distributional and residual traits.

2. Material and Method

2.1. The Two Covariates Alternate Cosine and Sine Function Trigonometry Regression

Given a linear regression model function with random error variables ω_i ;

$$Y = f(X\beta) + \omega_i \tag{1}$$

For ω_i are uncorrelated noisy-wave standardized random variables with mean zero and unity variance.

And,

$$f(X\beta) = \alpha + \beta_1 \cos X_{i1} + \beta_2 \sin X_{i2} \tag{2}$$

Then,

$$y_i = \alpha + \beta_1 \cos X_{i1} + \beta_2 \sin X_{i2} + \omega_i \tag{3}$$

Alternatively;

$$y_i = \alpha + \beta_1 \sin X_{i1} + \beta_2 \cos X_{i2} + \omega_i$$

Where,

$y_i \Rightarrow$ Is a n by 1 vector of responses.

$X_{i1}, X_{i2} \Rightarrow$ Is a n by $p+1$ is full rank design matrix of the model.

$\beta = \alpha, \beta_1, \beta_2 \Rightarrow$ Is a p by 1 vector of coefficients.

$\omega_i \Rightarrow$ Is a n by 1 vector of random errors.

Cos & Sin \Rightarrow Are the trigonometry noisy-waves for the two covariates.

$$\omega_i = y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2}$$

In matrix form;

$$Y = HB + \omega \tag{4}$$

B Is the column vector of parameter to be estimated; H is the coefficient matrix of a square matrix; where the

error term $\omega \sim (0, I\sigma^2)$

2.2. Maximum Likelihood Estimation Method of the Two Covariates Alternate Cosine and Sine Function

Considering error random variables (ω_i) that are assumed independent and normally distributed with zero mean and unity variance, adopting the maximum likelihood estimation gives

$$f(\omega) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(\omega)^2\right] \tag{5}$$

Then,

$$f(\omega) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2})^2\right] \tag{6}$$

The maximum likelihood gives,

$$L = f(\omega_1) \times f(\omega_2) \times \dots \times f(\omega_n)$$

$$L = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left[\sum_{i=1}^n \left[-\frac{1}{2\sigma^2}(y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2})^2\right]\right] \tag{7}$$

$$\ln L = -n \ln \sigma - \frac{1}{2} \ln(2\pi) - \frac{1}{2} \sum_{i=1}^n \left[\frac{(y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2})^2}{\sigma}\right] \tag{8}$$

Taking the log of equation

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^n \left[\frac{(y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2})}{\sigma}\right]$$

Equating to zero gives,

$$\sum_{i=1}^n (y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2}) = 0 \tag{9}$$

$$\frac{\partial \ln L}{\partial \beta_1} = \cos X_{i1} \sum_{i=1}^n \left[\frac{(y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2})}{\sigma}\right]$$

Equating to zero gives,

$$\sum_{i=1}^n \cos X_{i1} (y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2}) = 0 \tag{10}$$

$$\frac{\partial \ln L}{\partial \beta_2} = \sin X_{i2} \sum_{i=1}^n \left[\frac{(y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2})}{\sigma} \right]$$

Equating to zero gives,

$$\sum_{i=1}^n \sin X_{i2} (y_i - \alpha - \beta_1 \cos X_{i1} - \beta_2 \sin X_{i2}) = 0 \tag{11}$$

Expanding equations (7), (8) and (9) gives the system of equations;

$$\sum_{i=1}^n y_i - n\alpha - \beta_1 \sum_{i=1}^n \cos X_{i1} - \beta_2 \sum_{i=1}^n \sin X_{i2} = 0$$

$$\sum_{i=1}^n (y_i \cos X_{i1}) - \alpha \sum_{i=1}^n (\cos X_{i1}) - \beta_1 \sum_{i=1}^n (\cos^2 X_{i1}) - \beta_2 \sum_{i=1}^n (\cos X_{i1})(\sin X_{i2}) = 0$$

$$\sum_{i=1}^n (y_i \sin X_{i2}) - \alpha \sum_{i=1}^n (\sin X_{i2}) - \beta_1 \sum_{i=1}^n (\sin X_{i2})(\cos X_{i1}) - \beta_2 \sum_{i=1}^n (\sin^2 X_{i2}) = 0$$

Re-arranging and converting to matrix form gives,

$$\begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \cos X_{i1}) \\ \sum_{i=1}^n (y_i \sin X_{i2}) \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta_1 \\ \beta_2 \end{pmatrix} \begin{pmatrix} n & \sum_{i=1}^n \cos X_{i1} & \sum_{i=1}^n \sin X_{i2} \\ \sum_{i=1}^n (\cos X_{i1}) & \sum_{i=1}^n (\cos^2 X_{i1}) & \sum_{i=1}^n (\cos X_{i1})(\sin X_{i2}) \\ \sum_{i=1}^n (\sin X_{i2}) & \sum_{i=1}^n (\sin X_{i2})(\cos X_{i1}) & \sum_{i=1}^n (\sin^2 X_{i2}) \end{pmatrix} \tag{12}$$

Where,

$$B = \begin{pmatrix} \alpha \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} n & \sum_{i=1}^n \cos X_{i1} & \sum_{i=1}^n \sin X_{i2} \\ \sum_{i=1}^n (\cos X_{i1}) & \sum_{i=1}^n (\cos^2 X_{i1}) & \sum_{i=1}^n (\cos X_{i1})(\sin X_{i2}) \\ \sum_{i=1}^n (\sin X_{i2}) & \sum_{i=1}^n (\sin X_{i2})(\cos X_{i1}) & \sum_{i=1}^n (\sin^2 X_{i2}) \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \cos X_{i1}) \\ \sum_{i=1}^n (y_i \sin X_{i2}) \end{pmatrix} \tag{13}$$

$$B = (H^T H)^{-1} H^T y \tag{14}$$

$$(H^T H)^{-1} = \begin{pmatrix} n & \sum_{i=1}^n \cos X_{i1} & \sum_{i=1}^n \sin X_{i2} \\ \sum_{i=1}^n (\cos X_{i1}) & \sum_{i=1}^n (\cos^2 X_{i1}) & \sum_{i=1}^n (\cos X_{i1})(\sin X_{i2}) \\ \sum_{i=1}^n (\sin X_{i2}) & \sum_{i=1}^n (\sin X_{i2})(\cos X_{i1}) & \sum_{i=1}^n (\sin^2 X_{i2}) \end{pmatrix}^{-1} \tag{15}$$

Where,

$$H^T y = \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \cos X_{i1}) \\ \sum_{i=1}^n (y_i \sin X_{i2}) \end{pmatrix} \tag{16}$$

Since cosine of x_i varies from sine of x_i , that is $\cos(x_i) \neq \sin(x_i)$

$$y_i = \alpha + \beta_1 \sin X_{i1} + \beta_2 \cos X_{i2} + \omega_i$$

$$\omega_i = y_i - \alpha - \beta_1 \sin X_{i1} - \beta_2 \cos X_{i2}$$

When $y_i = \alpha + \beta_1 \sin X_{i1} + \beta_2 \cos X_{i2} + \omega_i$, the estimates of equation (13) becomes equation (17) below;

$$B = \begin{pmatrix} \alpha \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} n & \sum_{i=1}^n \sin X_{i1} & \sum_{i=1}^n \cos X_{i2} \\ \sum_{i=1}^n (\sin X_{i1}) & \sum_{i=1}^n (\sin^2 X_{i1}) & \sum_{i=1}^n (\sin X_{i1})(\cos X_{i2}) \\ \sum_{i=1}^n (\cos X_{i2}) & \sum_{i=1}^n (\cos X_{i2})(\sin X_{i1}) & \sum_{i=1}^n (\cos^2 X_{i2}) \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \sin X_{i1}) \\ \sum_{i=1}^n (y_i \cos X_{i2}) \end{pmatrix} \quad (17)$$

2.3. Distributional Properties of the Two Covariates Trigonometric Regression

From equation (14)

$$B = (H^T H)^{-1} H^T y$$

Recall from equation (4), $y = HB + \omega$

$$\begin{aligned} \text{So,} \quad B &= (H^T H)^{-1} H^T (HB + \omega) \\ &= (H^T H)^{-1} (H^T H)B + H^T \omega \end{aligned}$$

$$= B + H^T \omega \quad (18)$$

Taking expectation gives,

$$E(B) = E(B) + H^T E(\omega)H \quad (19)$$

Recall $E(\omega) = 0$,

$$E(B) = B$$

So, mean of the estimate B is nothing but B

Subtracting " B " from both sides of equation (14) gives

$$\begin{aligned} B - B &= H^T \omega \quad \text{But, } y - HB = \omega \\ B - B &= H^T (y - HB) \\ B - B &= H^T y - H^T HB = (H^T H)^{-1} H \omega \end{aligned}$$

2.4. The Dispersion Matrix of B

$$\begin{aligned} V(B) &= E[B - B]^T [B - B] \\ &= E\left[(H^T H)^{-1} H \omega \right] \\ &= E\left[(H^T H)^{-1} H \omega \omega^T H (H^T H)^{-1} \right] \\ &= \left[(H^T H)^{-1} H E(\omega \omega^T) H (H^T H)^{-1} \right] \end{aligned}$$

Recall $E(\omega \omega^T) = \sigma_\omega^2$;

$$= \left[(H^T H)^{-1} H H (H^T H)^{-1} \right] \sigma_\omega^2$$

$$V(B) = (H^T H)^{-1} \sigma_\omega^2 \quad (20)$$

Where σ_ω^2 is the variance of the error term

This implies that,

$$B \square \left(B, (H^T H)^{-1} \sigma_\omega^2 \right)$$

$$V(B) = \left(\begin{array}{ccc} n & \sum_{i=1}^n \cos X_{i1} & \sum_{i=1}^n \sin X_{i2} \\ \sum_{i=1}^n (\cos X_{i1}) & \sum_{i=1}^n (\cos^2 X_{i1}) & \sum_{i=1}^n (\cos X_{i1})(\sin X_{i2}) \\ \sum_{i=1}^n (\sin X_{i2}) & \sum_{i=1}^n (\sin X_{i2})(\cos X_{i1}) & \sum_{i=1}^n (\sin^2 X_{i2}) \end{array} \right)^{-1} \sigma_\omega^2$$

Such tha

$$\text{For } y_i = \alpha + \beta_1 \cos X_{i1} + \beta_2 \sin X_{i2} + \omega_i \tag{21}$$

While

$$V(B) = \left(\begin{array}{ccc} n & \sum_{i=1}^n \sin X_{i1} & \sum_{i=1}^n \cos X_{i2} \\ \sum_{i=1}^n (\sin X_{i1}) & \sum_{i=1}^n (\sin^2 X_{i1}) & \sum_{i=1}^n (\sin X_{i1})(\cos X_{i2}) \\ \sum_{i=1}^n (\cos X_{i2}) & \sum_{i=1}^n (\cos X_{i2})(\sin X_{i1}) & \sum_{i=1}^n (\cos^2 X_{i2}) \end{array} \right)^{-1} \sigma_\omega^2$$

$$\text{For } y_i = \alpha + \beta_1 \sin X_{i1} + \beta_2 \cos X_{i2} + \omega_i \tag{22}$$

2.5. Variance of the Error Term for the Two the Two Covariates Trigonometric Regression

$$\sigma_\omega^2 = \frac{E(\omega^T \omega)}{n - p} = \frac{y^T y - BH^T y}{n - p}$$

Where "n" is the number of observations and "p" is the number of parameter to be estimated.

$$\sigma_\omega^2 = \frac{\left(\begin{array}{c} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \cos X_{i1}) \\ \sum_{i=1}^n (y_i \sin X_{i2}) \end{array} \right)}{n - p} \tag{23}$$

For $y_i = \alpha + \beta_1 \cos X_{i1} + \beta_2 \sin X_{i2} + \omega_i$

$$\sigma_\omega^2 = \frac{\left(\begin{array}{c} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \sin X_{i1}) \\ \sum_{i=1}^n (y_i \cos X_{i2}) \end{array} \right)}{n - p} \tag{24}$$

For $y_i = \alpha + \beta_1 \sin X_{i1} + \beta_2 \cos X_{i2} + \omega_i$

2.6. Coefficient of Determination for the two Covariates Trigonometric Regression

The coefficient of determination being denoted by;

$$R^2 = \frac{BH^T y - n\bar{y}^{-2}}{y^T y - n\bar{y}^{-2}}$$

$$R^2 = \frac{(\alpha, \beta_1, \beta_2) \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \cos X_{i1}) \\ \sum_{i=1}^n (y_i \sin X_{i2}) \end{pmatrix} - n\bar{y}^{-2}}{y^T y - n\bar{y}^{-2}} \tag{25}$$

For $y_i = \alpha + \beta_1 \cos X_{i1} + \beta_2 \sin X_{i2} + \omega_i$

$$R^2 = \frac{(\alpha, \beta_1, \beta_2) \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (y_i \sin X_{i1}) \\ \sum_{i=1}^n (y_i \cos X_{i2}) \end{pmatrix} - n\bar{y}^{-2}}{y^T y - n\bar{y}^{-2}} \tag{26}$$

For $y_i = \alpha + \beta_1 \sin X_{i1} + \beta_2 \cos X_{i2} + \omega_i$

3. Results

The secondary dataset used in validating the obtained estimations above was the readings of rate of heartbeats of newly born babies in Lagos University Teaching Hospital (LUTH), a federal government owned hospital in Lagos state, Nigeria. These rate of heartbeats' readings variability were recorded in three different time-frames (in hours); rate of heartbeats exactly after an hour after birth (HR_1), rate of heartbeats exactly after two hours after birth (HR_2) and rate of heartbeats exactly after three hours after birth (HR_3). These readings were recorded for nine hundred and fifty (950) babies in the year 2017. These readings were examined and recorded via Electrocardiogram (ECG). HR_1 and HR_2 are considered the two covariates (independent variables) because of the fact that the responses of HR_3 rely solely upon the improved heartbeats of the first two hours after birth.

Figure-1. The Noisy-Wave of Heartbeats of HR_1 and HR_2

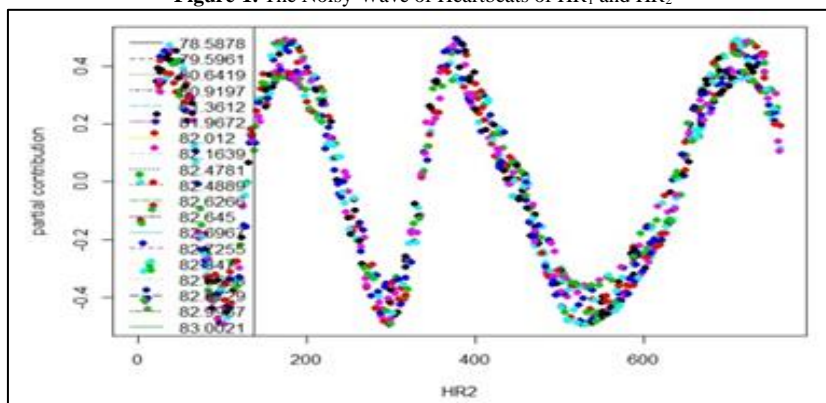
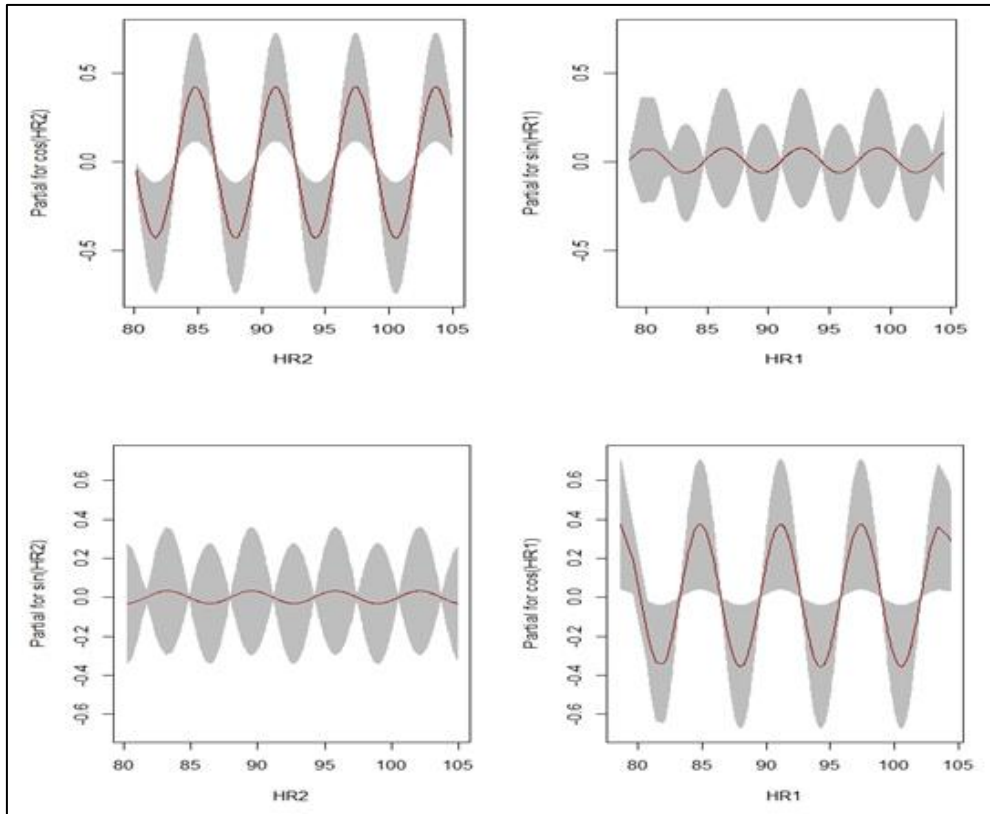
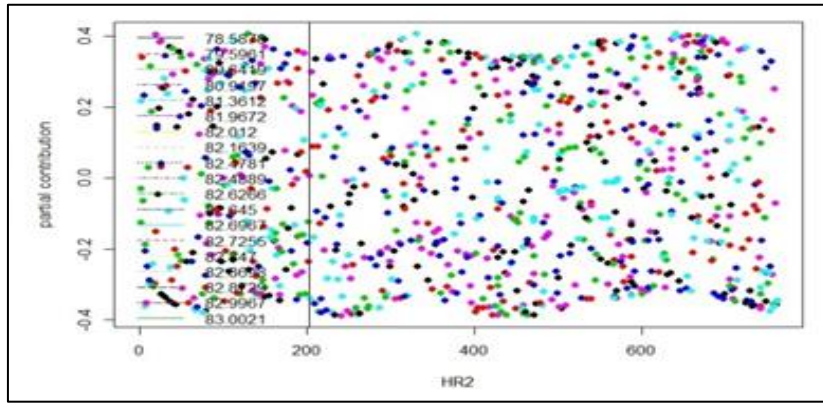


Figure-2. The Partial Cosine and Sine Wave Trend of HR_1 and HR_2



From Figure 1 and 2, the trend of the actual readings of the rate of heartbeats exactly one and two hours after birth, that is, HR₁ and HR₂ for the same level of four mesokurtics (Normal bell-curves) nature possessed. The Sine and Cosine plots of the two readings (the two covariates) revealed and actualized the possessed noisy-wave (Sine and Cosine waves) of the two examined observations of babies' heartbeats. This suggested a wave particle duality of the heartbeats. In other words, the HR₁, HR₂ and HR₃ heartbeats are noise or tone frequencies, that is, noisy data (noisy-wave) that requested a trigonometry (Cosine and Sine) transformation or Fourier transformation as an alternative to smoothing process or modeling.

Table-1. Fitted Cosine and Sine equation of $\alpha + \beta_1 \cos(\text{HR}_2) + \beta_2 \sin(\text{HR}_1)$

Parameter	Estimate	Std. Error	t-value	Pr.(> t)
α	58.6749	0.1112	527.647	< 0.0021
β_1	-0.4261	0.1539	-2.769	0.0057
β_2	-0.0692	0.1525	-0.454	0.6501
σ_ω	1.2220	0.0229	53.27	<0.0021

Global Deviance: 17.802

AIC: 5025.802

SBC: 5045.228

log Lik: -2508.901

The Maximum Likelihood estimator is 281.132

$$\text{HR}_3 = 58.6749 - 0.4261 \cos(\text{HR}_2) - 0.0692 \sin(\text{HR}_1)$$

Such that, $\sigma_\omega \in (1.2220, 0.00052)$

Table-2. Fitted Cosine and Sine equation of $\alpha + \beta_1 \sin(HR_2) + \beta_2 \cos(HR_1)$

Parameter	Estimate	Std. Error	t-value	Pr.(> t)
α	58.6837	0.1105	531.246	< 0.0021
β_1	0.0325	0.1586	0.205	0.8378
β_2	-0.3650	0.1619	-2.255	0.0244
σ_ω	1.2235	0.0229	53.33	<2e-16

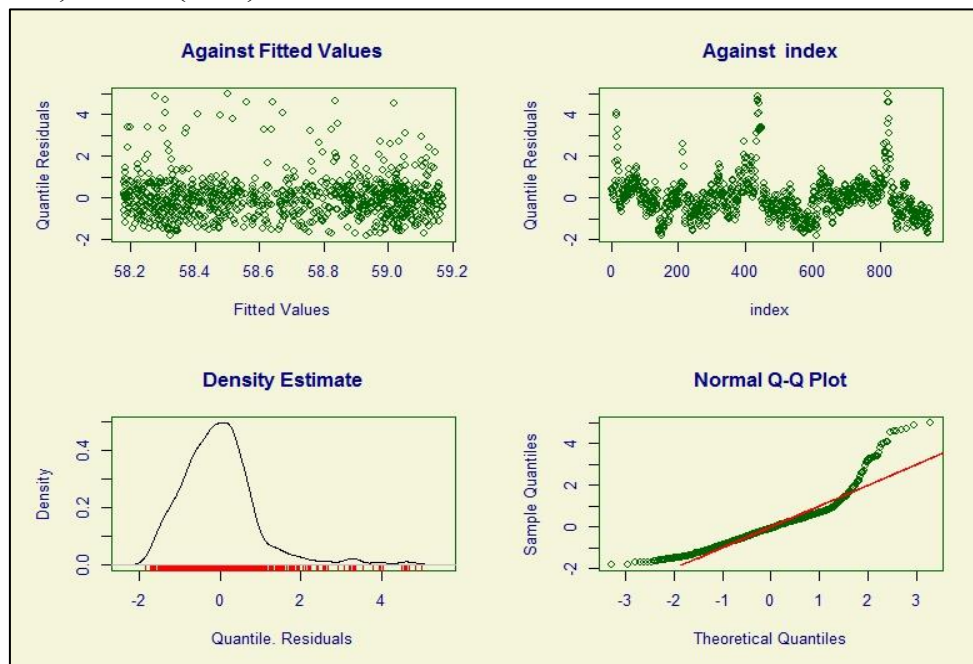
Global Deviance: 20.719
 AIC: 5028.719
 SBC: 5048.144
 log Lik: -2510.359
 The Maximum Likelihood estimator is 43.47365

$$HR_3 = 58.6837 + 0.0325 \sin(HR_2) - 0.3650 \cos(HR_1)$$

Such that $\sigma_\omega \square (1.2235, 0.00052)$

Table.1 was subjected to the fitted function of $\alpha + \beta_1 \cos(HR_2) + \beta_2 \sin(HR_1)$, while table. 2 was based on $\alpha + \beta_1 \sin(HR_2) + \beta_2 \cos(HR_1)$. It was deduced that the formal fitted equation robustly accommodated the wave like nature with improved model performance of (AIC: 5025.802; SBC: 5045.228) compare to a less model performance of (AIC: 5028.719; SBC: 5048.144) by the latter. Furthermore, the global aberrances from normal non-trigonometry fitted line of the two alternate Cosine and Sine equations were relatively miniature in the two fitted equations, with a lesser miniature of global deviance of 17.802 in fitted $\alpha + \beta_1 \cos(HR_2) + \beta_2 \sin(HR_1)$ compare to a global deviance of 17.802 in fitted $\alpha + \beta_1 \sin(HR_2) + \beta_2 \cos(HR_1)$. In collaboration with the stated claims, the coefficient of β_1 , which was the estimated coefficient of the rate of heartbeats exactly after an hour after birth (HR_1) in the formal equation hinted to be the most significant co-variate in the contributing factor to the next stability of heartbeats of babies in the next three hours and more after birth. This is due to its P-value=0.0057 being strictly far away from the 5% chance of error. In the latter, it was the coefficient of β_2 for rate of heartbeats exactly after an hour after birth (HR_2) with P-value=0.0244 that was greater than the P-value=0.0057 of the latter.

Figure-3. The Residual Deviance of the fitted Equations of $\alpha + \beta_1 \cos(HR_2) + \beta_2 \sin(HR_1)$ and $\alpha + \beta_1 \sin(HR_2) + \beta_2 \cos(HR_1)$



It was noted that the two alternate fitted functions of Cosine and Sine yielded the same residual indexes in terms of the estimated quantiles density, QQ-plot and approximately the same the observed and estimated frequencies.

Table-3. Centile of the Cosine and Sine alternate equation lines

Centiles	$\alpha + \beta_1 \cos(\text{HR}_2) + \beta_2 \sin(\text{HR}_1)$	$\alpha + \beta_1 \sin(\text{HR}_2) + \beta_2 \cos(\text{HR}_1)$
% of cases below 0.4 centile is	0	0
% of cases below 10 centile is	6	6.1053
% of cases below 50 centile is	54.4211	54.42105
% of cases below 90 centile is	92.5263	92.8421
% of cases below 99.6 centile is	97.1579	96.9474

Centile otherwise known as percentile has been one of the values of a statistical variable that divides the distribution of the variable into 100 groups having equal frequencies. The 99.6 percent of the values in the function $\alpha + \beta_1 \cos(\text{HR}_2) + \beta_2 \sin(\text{HR}_1)$ lies at 97.1579 centile, capturing and explaining the wave nature of the covariates above the 99.6 percent of the values in the function $\alpha + \beta_1 \sin(\text{HR}_2) + \beta_2 \cos(\text{HR}_1)$ that lies at 96.9474 centile in capturing and explaining the wave nature of the system.

Table-4. Summary of the Quantile Residuals Cosine and Sine alternate equation Functions

$\alpha + \beta_1 \cos(\text{HR}_2) + \beta_2 \sin(\text{HR}_1)$		$\alpha + \beta_1 \sin(\text{HR}_2) + \beta_2 \cos(\text{HR}_1)$	
Parameter	Estimate	Parameter	Estimate
Mean	0.0057	Mean	-0.0056
Variance	1.0010	Variance	1.0011
coef. of skewness	1.5899	coef. of skewness	1.5633
FCC	0.9437	FCC	0.9449
Cox-Snell	0.85	Cox-Snell	0.54
Cragg-Uhler	0.85	Cragg-Uhler	0.54

Keys: FCC=Filliben Correlation Coefficient

Table 4 divulged the approximately equivalence of the residual variance of the two alternate wave nature of Cosine and Sine equations as well as the same residual location parameter of positive effect by $\alpha + \beta_1 \cos(\text{HR}_2) + \beta_2 \sin(\text{HR}_1)$, whereas $\alpha + \beta_1 \sin(\text{HR}_2) + \beta_2 \cos(\text{HR}_1)$ adopted the negative effect of location parameter. In addition, the two alternate equations were not affected by skewedness (outliers), since their skewedness coefficients of 1.5899 and 1.5633 respectively are < 3 . The Filliben Correlation Coefficient (FCC), which is use as test statistic for normal probability correlation coefficient of composite hypothesized for normality (non-normal) test; since its coefficient $r \approx 0.940$ for the two equations, it implies the noisy data indicated a length of lower tail (symmetric shorter-tailed) of 94% with 5% level of significant as maintained by Filliben [19]. The Cox-Snell residual and Cragg-Uhler coefficients of (0.85 and 0.54) and (0.85 and 0.54) respectively for assessing the goodness-of-fit for the heartbeats' regression hinted the formal fitted function accommodated the wave nature of the heartbeat of the babies to 85% and the latter was able to explain to 54%. The two residual indexes are alternate index for the Pseudo- R^2 .

4. Conclusion

From the anteceding, it is necessary to do ascertain the wavy trend of regressors (covariates) to throee the level of noisy and frequency tone. By doing so, it allows a clear-cut whether the distributional property of the noise is a trigonometry (Cosine, Sine) of two or more covariates linear regression. The rate of heartbeats exactly after an hour, exactly after two hours and exactly three hours after birth followed a noisy Cosine and Sine wave nature trigonometry regression. The alternate Cosine and Sine two covariates was subjected to the heartbeats' observations

such that the fitted equation of $\alpha + \beta_1 \cos(\text{HR}_2) + \beta_2 \sin(\text{HR}_1)$ captured the wave nature than the alternate function of $\alpha + \beta_1 \sin(\text{HR}_2) + \beta_2 \cos(\text{HR}_1)$. It is to be noted that this research could be extended to more than two co-variates linear trigonometry regression, that is generalized multiple trigonometry regression.

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