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# Intuitionistic Fuzzy Soft Multisets Theory

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## Abstract

Soft set theory was originally proposed as a general mathematical tool for dealing with uncertainty. Soft multiset and fuzzy soft multiset are generalization concepts obtained from soft set theory. As a generlazation of fuzzy soft multiset we introduce the definition of intuitionistic fuzzy soft multiset and its operations and study some of their properties. Finally, we give an application in decision making problems.
**Keywords:** Fuzzy soft set; Intuitionistic fuzzy soft set; Soft multiset, Fuzzy soft multiset; Intuitionistic fuzzy soft multiset.



## 1. Introduction

In 1999 Molodtsov [1] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. [2, 3], have further studied the theory of soft sets and used this theory to solve some decision-making problems. Alkhazaleh, *et al.* [4], introduced soft multiset as a generalization of Molodtsov's soft set. Maji, *et al.* [5], introduced the concept of fuzzy soft set and studied its properties and also Roy and Maji used this theory to solve some decision-making problems [6]. Alkhazaleh, *et al.* [7], defined the concepts of possibility fuzzy soft set and gave their applications in decision making and medical diagnosis. After that Bashir and Salleh [8], defined the concept of possibility intuitionistic fuzzy soft set and gave their applications in decision making and medical diagnosis. Maji, *et al.* [9], defined the concept of intuitionistic fuzzy soft set and Maji [10], defined some new operations on intuitionistic fuzzy soft sets and studied some results relating to the properties of these operations. In 2011 Alkhazaleh and Salleh [11] introduced a concept of fuzzy soft multiset theory which is a combination of fuzzy set and soft multiset. In this paper, we generalize the concept of fuzzy soft multiset to the intuitionistic fuzzy soft multiset.We also introduce its basic operations, namely complement, union and intersection, and their properties. An application of this theory in a decision making problem is given.

## 2. Preliminaries

In this section, we recall some basic notions in soft multiset theory,and fuzzy soft multiset theory.

Alkhazaleh, *et al.* [4] defined the soft multiset and Alkhazaleh and Salleh [11], defend a fuzzy soft multiset in the following way.

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \varnothing$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} P(U_i)$ where $P(U_i)$ denotes the power set of $U_i$, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. Alkhazaleh et al. define the soft multiset as follows:

**Definition 2.1.** [4] A pair $(F, A)$ is called a *soft multiset* over $U$, where $F$ is a mapping given by $F : A \to U$.

In other words, a soft multiset over $U$ is a parameterized family of subsets of $U$. For $\varepsilon \subset A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon$ - approximate elements of the soft multiset $(F, A)$.

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Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \varnothing$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} FS(U_i)$ where $FS(U_i)$ denotes the all fuzzy subsets of $U_i$, $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$

**Definition 2.2.** [11] A pair $(F, A)$ is called a *fuzzy soft multiset* over $U$, where $F$ is a mapping given by $F : A \to U$.

In other words, a fuzzy soft multiset over $U$ is a parameterized family of fuzzy subsets of $U$. For $\varepsilon \subset A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon$ - approximate elements of the fuzzy soft multiset $(F, A)$.

**Definition 2.3.** For any fuzzy soft multiset $(F, A)$, a pair $\left(e_{U_i,j}, F_{e_{U_i,j}}\right)$ is called a $U_i - fuzzy\ soft\ multiset\ part$ $\forall e_{U_i,j} \in a_k$ and $F_{e_{U_i,j}} \subseteq F(A)$ is a *fuzzy approximate value set*, where $a_k \in A$, $k = \{1,2,3,...,n\}$, $i \in \{1,2,3,...,m\}$ and $j \in \{1,2,3,...,r\}$.

**Definition 2.4.** For two fuzzy soft multisets $(F, A)$ and $(G, B)$ over $U$, $(F, A)$ is called a fuzzy *soft multisubset* of $(G, B)$ if

(1) $A \subseteq B$ and

(2) $\forall e_{U_i,j} \in a_k$, $\left(e_{U_i,j}, F_{e_{U_i,j}}\right)$ is a fuzzy subset of $\left(e_{U_i,j}, G_{e_{U_i,j}}\right)$

where $a_k \in A$, $k = \{1,2,3,...,n\}$, $i \in \{1,2,3,...,m\}$ and $j \in \{1,2,3,...,r\}$.

This relationship is denoted by $(F, A) \subseteq (G, B)$. In this case $(G, B)$ is called a *fuzzy soft multisuperset* of $(F, A)$.

**Definition 2.5.** Two fuzzy soft multisets $(F, A)$ and $(G, B)$ over $U$ are said to be *equal* if $(F, A)$ is a *fuzzy soft multisubset* of $(G, B)$ and $(G, B)$ is a *fuzzy soft multisubset* of $(F, A)$.

**Definition 2.6.** Let $E = \prod_{i=1}^{m} E_{U_i}$ where $E_{U_i}$ is a set of parameters. The *NOT set* of $E$ denoted by $\neg E$ is defined by $\neg E = \prod_{i=1}^{m} \neg E_{U_i}$ where $\neg E_{U_i} = \{\neg e_{U_i,j} = \text{not } e_{U_i,j}, \forall i, j\}$.

**Definition 2.7.** The *complement* of a fuzzy soft multiset $(F, A)$ is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \to U$ is a mapping given by $F^c(\alpha) = C(F(\neg\alpha)), \forall \alpha \subset \neg A$, where $C$ be any fuzzy complement.

**Definition 2.8.** A fuzzy soft multiset $(F, A)$ over $U$ is called a *semi-null fuzzy soft multiset* denoted by $(F, A)_{\approx 0_i}$, if at least one of a fuzzy soft multiset parts of $(F, A)$ equals $\bar{0}$.

**Definition 2.9.** A fuzzy soft multiset $(F, A)$ over $U$ is called a *null fuzzy soft multiset* denoted by $(F, A)_0$ if all of a fuzzy soft multiset parts of $(F, A)$ equals $\bar{0}$.

**Definition 2.10.** A soft multiset $(F,A)$ over $U$ is called a *semi-absolute fuzzy soft multiset* denoted by $(F,A)_{\approx 1_i}$ if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = \bar{1}_i$ for at least one $i$, $a_k \in A$, . $a_k \in A, k = \{1,2,3,...,n\}, i \in \{1,2,3,...,m\}$ and $j \in \{1,2,3,...,r\}$.

**Definition 2.11.** A fuzzy soft multiset $(F,A)$ over $U$ is called an *absolute fuzzy soft multiset* denoted by $(F,A)_1$ if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = \bar{1}_i, \forall i$.

**Definition 3.6.** The *union* of two fuzzy soft multisets $(F,A)$ and $(G,B)$ over $U$ denoted by $(F,A) \cup (G,B)$ is the soft multiset $(H,C)$ where $C = A \cup B$, and $\forall \varepsilon \subset C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \ \varepsilon \in A - B, \\ G(\varepsilon), & if\ \ \varepsilon \in B - A, \\ s(F(\varepsilon), G(\varepsilon)), & if\ \ \varepsilon \in A \cap B. \end{cases} \quad \text{where } s \text{ be any fuzzy } s\text{-}norm.$$

.

**Definition 3.7.** The *intersection* of two fuzzy soft multisets $(F,A)$ and $(G,B)$ over $U$ denoted by $(F,A) \cap (G,B)$ is the fuzzy soft multiset $(H,C)$ where $C = A \cup B$, $\forall \varepsilon \subset C$, and

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \ \varepsilon \in A - B, \\ G(\varepsilon), & if\ \ \varepsilon \in B - A, \\ t(F(\varepsilon), G(\varepsilon)), & if\ \ \varepsilon \in A \cap B. \end{cases} \quad \text{where t be any fuzzy t-norm.}$$

.

## 3. Intuitionistic Fuzzy Soft Multisets Theory

In this section, we introduce the definition of the intuitionistic fuzzy soft multiset,and itsbasic operation such as complement, union and intersection. We give example for these concepts.

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \varnothing$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} IFS(U_i)$ where $IFS(U_i)$ denotes the all intuitionistic fuzzy subsets of $U_i$, $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$.

**Definition 3.1.** A pair $(F, A)$ is called an *intuitionistic fuzzy soft multiset* over $U$, where $F$ is a mapping given by $F : A \to U$.

In other words, an intuitionistic fuzzy soft multiset over $U$ is a parameterized family of intuitionistic fuzzy subsets of $U$. For $\varepsilon \subset A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon$ - approximate elements of the intuitionistic fuzzy soft multiset $(F, A)$.

Based on the above definition, any change in the order of universes will produce a different intuitionistic fuzzy soft multiset.

**Example 3.1.** Suppose that there are three universes $U_1, U_2$ and $U_3$. Let us consider a intuitionistic fuzzy soft multiset $(F,A)$ which describes the ‘‘attractiveness of houses ’’, “cars” and “hotels” that Mr. X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2\}$ and $U_3 = \{v_1, v_2\}$.

Let $E_U = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = \text{very costly},\ e_{U_1,2} = \text{costly},\ e_{U_1,3} = \text{cheap}, e_{U_1,4} = \text{wooden},$$
$$e_{U_1,5} = \text{in green surroundings}\},$$
$$E_{U_2} = \{e_{U_2,1} = \textit{very}\ \text{costly},\ e_{U_2,2} = \text{costly},\ e_{U_2,3} = \text{cheap},,\ e_{U_2,4} = \text{white}\},$$
$$E_{U_3} = \{e_{U_3,1} = \text{very costly}, e_{U_3,2} = \text{costly},\ e_{U_3,3} = \text{cheap},\ e_{U_3,4} = \text{in Kuala Lumpur},$$
$$e_{U_3,5} = \text{majestic}\}.$$

Let $U = \prod_{i=1}^{3} IFS\,(U_i),\ E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),$$
$$a_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}), a_5 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3})\}.$$

Suppose that

$$F(a_1) = \left(\left\{\frac{h_1}{(0.2,0.4)}, \frac{h_2}{(0.4,0.1)}, \frac{h_3}{(0.8,0.1)}\right\}, \left\{\frac{c_1}{(0.8,0)}, \frac{c_2}{(0.5,0.5)}\right\}, \left\{\frac{v_1}{(0.8,0.1)}, \frac{v_2}{(0.7,0.2)}\right\}\right)$$
$$F(a_2) = \left(\left\{\frac{h_1}{(0.2,0.4)}, \frac{h_2}{(0.4,0.1)}, \frac{h_3}{(0.8,0.1)}\right\}, \left\{\frac{c_1}{(1,0)}, \frac{c_2}{(0.8,0.1)}\right\}, \left\{\frac{v_1}{(0.6,0.2)}, \frac{v_2}{(0.5,0.3)}\right\}\right)$$
$$F(a_3) = \left(\left\{\frac{h_1}{(0.7,0.2)}, \frac{h_2}{(0.7,0.1)}, \frac{h_3}{(1,0)}\right\}, \left\{\frac{c_1}{(0.8,0.1)}, \frac{c_2}{(0.6,0.3)}\right\}, \left\{\frac{v_1}{(0.5,0.3)}, \frac{v_2}{(0.4,0.4)}\right\}\right)$$
$$F(a_4) = \left(\left\{\frac{h_1}{(0.9,0)}, \frac{h_2}{(0.5,0.3)}, \frac{h_3}{(0.5,0.5)}\right\}, \left\{\frac{c_1}{(0,1)}, \frac{c_2}{(0.2,0.6)}\right\}, \left\{\frac{v_1}{(0.8,0.1)}, \frac{v_2}{(0.7,0.2)}\right\}\right)$$
$$F(a_5) = \left(\left\{\frac{h_1}{(1,0)}, \frac{h_2}{(0.8,0.1)}, \frac{h_3}{(0.7,0.2)}\right\}, \left\{\frac{c_1}{(0.7,0.1)}, \frac{c_2}{(0.8,0.1)}\right\}, \left\{\frac{v_1}{(0.5,0.5)}, \frac{v_2}{(0.5,0.2)}\right\}\right)$$

Then we can view the intuitionistic fuzzy soft multiset $(F, A)$ as consisting of the following collection of approximations:

$$(F, A) =$$

$$\left\{\left(a_1,\left(\left\{\frac{h_1}{(0.2,0.4)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.8,0.1)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.5,0.5)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.7,0.2)}\right\}\right)\right)\right.$$

$$\left(a_2,\left(\left\{\frac{h_1}{(0.2,0.4)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.8,0.1)}\right\},\left\{\frac{c_1}{(1,0)},\frac{c_2}{(0.8,0.1)}\right\},\left\{\frac{v_1}{(0.6,0.2)},\frac{v_2}{(0.5,0.3)}\right\}\right)\right)$$

$$\left(a_3,\left(\left\{\frac{h_1}{(0.7,0.2)},\frac{h_2}{(0.7,0.1)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(0.8,0.1)},\frac{c_2}{(0.6,0.3)}\right\},\left\{\frac{v_1}{(0.5,0.3)},\frac{v_2}{(0.4,0.4)}\right\}\right)\right)$$

$$\left(a_4,\left(\left\{\frac{h_1}{(0.9,0)},\frac{h_2}{(0.5,0.3)},\frac{h_3}{(0.5,0.5)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0.2,0.6)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.7,0.2)}\right\}\right)\right)$$

$$\left.\left(a_5,\left(\left\{\frac{h_1}{(1,0)},\frac{h_2}{(0.8,0.1)},\frac{h_3}{(0.7,0.2)}\right\},\left\{\frac{c_1}{(0.7,0.1)},\frac{c_2}{(0.8,0.1)}\right\},\left\{\frac{v_1}{(0.5,0.5)},\frac{v_2}{(0.5,0.2)}\right\}\right)\right)\right\}$$

Each approximation has two parts: a *predicate* and an *approximate value set*.
We can logically explain the previous example as follows:

For $F(a_1)=\left(\left\{\frac{h_1}{(0.2,0.4)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.8,0.1)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.5,0.5)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.7,0.2)}\right\}\right)$,
IF $\left\{\frac{h_1}{(0.2,0.4)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.8,0.1)}\right\}$ is the intuitionistic fuzzy set of very costely houses to Mr. X THEN the intuitionistic fuzzy set of relatively very costely cars to him is $\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.5,0.5)}\right\}$ and IF $\left\{\frac{h_1}{(0.2,0.4)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.8,0.1)}\right\}$ is the intuitionistic fuzzy set of very costely houses to Mr. X and $\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.5,0.5)}\right\}$ is the intuitionistic fuzzy set of relatively very costely cars to him THEN the intuitionistic fuzzy set of relatively very costely hotels to him is $\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.7,0.2)}\right\}$. It is clear that the relation in intuitionistic fuzzy soft multiset is a conditional relation.

**Definition 3.2.** For any intuitionistic fuzzy soft multiset $(F,A)$, a pair $\left(e_{U_i,j},F_{e_{U_i,j}}\right)$ is called a $U_i-$*intuitionistic fuzzy soft multiset part* $\forall e_{U_i,j}\in a_k$ and $F_{e_{U_i,j}}\subseteq F(A)$ is a *fuzzy approximate value set*, where $a_k\in A,\ k=\{1,2,3,...,n\},\ i\in\{1,2,3,...,m\}$ and $j\in\{1,2,3,...,r\}$

**Example 3.2.** Consider the example as the one presented in Example 3.1. Then

$$\left(e_{U_1,j}, F_{e_{U_1,j}}\right) = \left\{\left(e_{U_1,1}, \left\{\frac{h_1}{(0.2,0.4)}, \frac{h_2}{(0.4,0.1)}, \frac{h_3}{(0.8,0.1)}\right\}\right), \left(e_{U_1,1}, \left\{\frac{h_1}{(0.2,0.4)}, \frac{h_2}{(0.4,0.1)}, \frac{h_3}{(0.8,0.1)}\right\}\right),\right.$$
$$\left(e_{U_1,2}, \left\{\frac{h_1}{(0.7,0.2)}, \frac{h_2}{(0.7,0.1)}, \frac{h_3}{(1,0)}\right\}\right), \left(e_{U_1,5}, \left\{\frac{h_1}{(0.9,0)}, \frac{h_2}{(0.5,0.3)}, \frac{h_3}{(0.5,0.5)}\right\}\right),$$
$$\left.\left(e_{U_1,4}, \left\{\frac{h_1}{(1,0)}, \frac{h_2}{(0.8,0.1)}, \frac{h_3}{(0.7,0.2)}\right\}\right)\right\}$$

is a $U_1 -$ intuitionistic *fuzzy soft multiset part* of $(F, A)$.

**Definition 3.3.** For two intuitionistic fuzzy soft multisets $(F, A)$ and $(G, B)$ over $U$, $(F, A)$ is called a *intuitionistic fuzzy soft multisubset* of $(G, B)$ if

(1) $A \subseteq B$ and

(2) $\forall e_{U_i,j} \in a_k$, $\left(e_{U_i,j}, F_{e_{U_i,j}}\right)$ is an intuitionstic fuzzy subset of $\left(e_{U_i,j}, G_{e_{U_i,j}}\right)$

where $a_k \in A$, $k = \{1, 2, 3, ..., n\}$, $i \in \{1, 2, 3, ..., m\}$ and $j \in \{1, 2, 3, ..., r\}$.

This relationship is denoted by $(F, A) \subseteq (G, B)$. In this case $(G, B)$ is called an *intuitionistic fuzzy soft multisuperset* of $(F, A)$.

**Definition 3.4.** Two intuitionistic fuzzy soft multisets $(F, A)$ and $(G, B)$ over $U$ are said to be *equal* if $(F, A)$ is an *intuitionistic fuzzy soft multisubset* of $(G, B)$ and $(G, B)$ is an *intuitionistic fuzzy soft multisubset* of $(F, A)$.

**Example 3.3.** Consider the example as the one presented in Example 3.1. Let

$$A = \left\{a_1 = \left(e_{U_1,1}, e_{U_2,1}, e_{U_3,1}\right), a_2 = \left(e_{U_1,2}, e_{U_2,3}, e_{U_3,1}\right), a_3 = \left(e_{U_1,4}, e_{U_2,3}, e_{U_3,3}\right)\right\},$$

and

$$B = \left\{b_1 = \left(e_{U_1,1}, e_{U_2,1}, e_{U_3,1}\right), b_2 = \left(e_{U_1,1}, e_{U_2,2}, e_{U_3,1}\right), b_3 = \left(e_{U_1,2}, e_{U_2,3}, e_{U_3,1}\right),\right.$$
$$\left. b_4 = \left(e_{U_1,5}, e_{U_2,4}, e_{U_3,2}\right), b_5 = \left(e_{U_1,4}, e_{U_2,3}, e_{U_3,3}\right)\right\}$$

Clearly $A \subseteq B$. Let $(F, A)$ and $(G, B)$ be two intuitionistic fuzzy soft multisets over the same $U$ such that

$$(F, A) = \left\{\left(a_1, \left(\left\{\frac{h_1}{(0.2,0.4)}, \frac{h_2}{(0.4,0.1)}, \frac{h_3}{(0.8,0.1)}\right\}, \left\{\frac{c_1}{(0.8,0)}, \frac{c_2}{(0.5,0.5)}\right\}, \left\{\frac{v_1}{(0.8,0.1)}, \frac{v_2}{(0.7,0.2)}\right\}\right)\right)\right.$$
$$\left(a_2, \left(\left\{\frac{h_1}{(0.2,0.4)}, \frac{h_2}{(0.4,0.1)}, \frac{h_3}{(0.8,0.1)}\right\}, \left\{\frac{c_1}{(1,0)}, \frac{c_2}{(0.8,0.1)}\right\}, \left\{\frac{v_1}{(0.6,0.2)}, \frac{v_2}{(0.5,0.3)}\right\}\right)\right)$$
$$\left.\left(a_3, \left(\left\{\frac{h_1}{(0.7,0.2)}, \frac{h_2}{(0.7,0.1)}, \frac{h_3}{(1,0)}\right\}, \left\{\frac{c_1}{(0.8,0.1)}, \frac{c_2}{(0.6,0.3)}\right\}, \left\{\frac{v_1}{(0.5,0.3)}, \frac{v_2}{(0.4,0.4)}\right\}\right)\right)\right\}$$

and $(G,B)=$

$$\left\{\left(b_1,\left(\left\{\frac{h_1}{(0.5,0.2)},\frac{h_2}{(0.7,0)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0.9,0)},\frac{c_2}{(0.7,0.2)}\right\},\left\{\frac{v_1}{(0.9,0)},\frac{v_2}{(0.8,0.1)}\right\}\right)\right)\right.$$

$$\left(b_2,\left(\left\{\frac{h_1}{(0.4,0.1)},\frac{h_2}{(0.7,0)},\frac{h_3}{(0.9,0.1)}\right\},\left\{\frac{c_1}{(1,0)},\frac{c_2}{(0.8,0)}\right\},\left\{\frac{v_1}{(0.7,0.1)},\frac{v_2}{(0.7,0.1)}\right\}\right)\right)$$

$$\left(b_3,\left(\left\{\frac{h_1}{(0.8,0.1)},\frac{h_2}{(0.8,0)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(0.9,0)},\frac{c_2}{(0.7,0.1)}\right\},\left\{\frac{v_1}{(0.7,0.2)},\frac{v_2}{(0.6,0.2)}\right\}\right)\right)$$

$$\left.\left(b_4,\left(\left\{\frac{h_1}{(0.9,0)},\frac{h_2}{(0.6,0.1)},\frac{h_3}{(0.7,0.1)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0.5,0.3)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.8,0.1)}\right\}\right)\right)\right\}$$

Therefore $(F,A)\subseteq(G,B)$.

**Definition 3.5.** Let $E=\prod_{i=1}^{m}E_{U_i}$ where $E_{U_i}$ is a set of parameters. The *NOT set* of $E$ denoted by $\neg E$ is defined by $\neg E=\prod_{i=1}^{m}\neg E_{U_i}$ where $\neg E_{U_i}=\left\{\neg e_{U_i,j}=\text{not } e_{U_i,j},\forall i,j\right\}$.

**Example 3.4.** Consider the example as the one presented in Example 3.1. Here

$$\neg A=\left\{\neg a_1=\left(\neg e_{U_1,1},\neg e_{U_2,1},\neg e_{U_3,1}\right),\neg a_2=\left(\neg e_{U_1,1},\neg e_{U_2,2},\neg e_{U_3,1}\right),\right.$$
$$\neg a_3=\left(\neg e_{U_1,2},\neg e_{U_2,3},\neg e_{U_3,1}\right),\neg a_4=\left(\neg e_{U_1,5},\neg e_{U_2,4},\neg e_{U_3,2}\right),$$
$$\left.\neg a_5=\left(\neg e_{U_1,4},\neg e_{U_2,3},\neg e_{U_3,3}\right)\right\}$$

**Definition 3.6.** The *complement* of an intuitionistic fuzzy soft multiset $(F,A)$ is denoted by $(F,A)^c$ and is defined by $(F,A)^c=(F^c,\neg A)$ where $F^c:\neg A\to U$ is a mapping given by $F^c(\alpha)=C\left(F(\neg\alpha)\right),\forall\alpha\subset\neg A$, where $C$ be an intuitionistic fuzzy complement.

**Example 3.5.** Consider the example as the one presented in Example 3.1. by using the basic intuitionistic fuzzy complement we have

$$(F,A)^c=\left\{\left(\neg a_1,F(\neg a_1)\right),\left(\neg a_2,F(\neg a_2)\right),\left(\neg a_3,F(\neg a_3)\right),\left(\neg a_4,F(\neg a_4)\right),\right.$$
$$\left.\left(\neg a_5,F(\neg a_5)\right)\right\}$$

$$=\left\{\left(\neg a_1,\left(\left\{\frac{h_1}{(0.4,0.2)},\frac{h_2}{(0.1,0.4)},\frac{h_3}{(0.1,0.8)}\right\},\left\{\frac{c_1}{(0,0.8)},\frac{c_2}{(0.5,0.5)}\right\},\left\{\frac{v_1}{(0.1,0.8)},\frac{v_2}{(0.2,0.7)}\right\}\right)\right)\right.$$

$$\left(\neg a_2,\left(\left\{\frac{h_1}{(0.4,0.2)},\frac{h_2}{(0.1,0.4)},\frac{h_3}{(0.1,0.8)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0.1,0.8)}\right\},\left\{\frac{v_1}{(0.2,0.6)},\frac{v_2}{(0.3,0.5)}\right\}\right)\right)$$

$$\left(\neg a_3,\left(\left\{\frac{h_1}{(0.2,0.7)},\frac{h_2}{(0.1,0.7)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(0.1,0.8)},\frac{c_2}{(0.3,0.6)}\right\},\left\{\frac{v_1}{(0.3,0.5)},\frac{v_2}{(0.4,0.4)}\right\}\right)\right)$$

$$\left(\neg a_4,\left(\left\{\frac{h_1}{(0,0.9)},\frac{h_2}{(0.3,0.5)},\frac{h_3}{(0.5,0.5)}\right\},\left\{\frac{c_1}{(1,0)},\frac{c_2}{(0.6,0.2)}\right\},\left\{\frac{v_1}{(0.1,0.8)},\frac{v_2}{(0.2,0.7)}\right\}\right)\right)$$

$$\left(\neg a_5,\left(\left\{\frac{h_1}{(0,1)},\frac{h_2}{(0.1,0.8)},\frac{h_3}{(0.2,0.7)}\right\},\left\{\frac{c_1}{(0.1,0.7)},\frac{c_2}{(0.1,0.8)}\right\},\left\{\frac{v_1}{(0.5,0.5)},\frac{v_2}{(0.2,0.5)}\right\}\right)\right)\Bigg\}$$

**Definition 3.7.** An intuitionistic fuzzy soft multiset $(F,A)$ over $U$ is called a *semi-null intuitionistic fuzzy soft multiset* denoted by $(F,A)_{\approx 0_i}$, if at least one of an intuitionistic fuzzy soft multiset parts of $(F,A)$ equals $\bar{0}$.

**Example 3.6.** Suppose that there are three universes $U_1, U_2$ and $U_3$. Let us consider an intuitionistic fuzzy soft multiset $(F,A)$ which describes the ''attractiveness of stone houses '', "cars" and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. Let $U_1 = \{h_1,h_2,h_3\}$, $U_2 = \{c_1,c_2\}$ and $U_3 = \{v_1,v_2\}$.

Let $E_{U_i} = \{E_{U_1},E_{U_2},E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = \text{very costly},\ e_{U_1,2} = \text{costly},\ e_{U_1,3} = \text{cheap},\ e_{U_1,4} = \text{wooden},$$
$$e_{U_1,5} = \text{in green surroundings}\},$$
$$E_{U_2} = \{e_{U_2,1} = \textit{very}\ \text{costly},\ e_{U_2,2} = \text{costly},\ e_{U_2,3} = \text{cheap},\ e_{U_2,4} = \text{white}\},$$
$$E_{U_3} = \{e_{U_3,1} = \text{very costly},\ e_{U_3,2} = \text{costly},\ e_{U_3,3} = \text{cheap},\ e_{U_3,4} = \text{in Kuala Lumpur},$$
$$e_{U_3,5} = \text{majestic}\}.$$

Let $U = \prod_{i=1}^{3} IFS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$, such that

$$A = \{a_1 = (e_{U_1,4},e_{U_2,1},e_{U_3,1}),\ a_2 = (e_{U_1,4},e_{U_2,3},e_{U_3,1}),\ a_3 = (e_{U_1,4},e_{U_2,3},e_{U_3,3})\}.$$

The intuitionistic fuzzy soft multiset $(F,A)$ is the collection of approximations as below:

$(F,A)_{\approx 0_1}$

$$\Bigg\{\left(a_1,\left(\left\{\frac{h_1}{(0,1)},\frac{h_2}{(0,1)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.5,0.5)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.7,0.2)}\right\}\right)\right)$$
$$\left(a_2,\left(\left\{\frac{h_1}{(0,1)},\frac{h_2}{(0,1)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(1,0)},\frac{c_2}{(0.8,0.1)}\right\},\left\{\frac{v_1}{(0.6,0.2)},\frac{v_2}{(0.5,0.3)}\right\}\right)\right)$$
$$\left(a_3,\left(\left\{\frac{h_1}{(0,1)},\frac{h_2}{(0,1)},\frac{h_3}{(0,1)}\right\},\left\{\frac{c_1}{(0.8,0.1)},\frac{c_2}{(0.6,0.3)}\right\},\left\{\frac{v_1}{(0.5,0.3)},\frac{v_2}{(0.4,0.4)}\right\}\right)\right)\Bigg\}$$

Then $(F,A)_{\approx 0_1}$ is semi-null intuitionistic fuzzy soft multiset.

**Definition 3.8.** An intuitionistic fuzzy soft multiset $(F,A)$ over $U$ is called a *null intuitionistic fuzzy soft multiset* denoted by $(F,A)_0$ if all of an intuitionistic fuzzy soft multiset parts of $(F,A)$ equals $\bar{0}$.

**Example 3.7.** Suppose that there are three universes $U_1, U_2$ and $U_3$. Let us consider an intuitionistic fuzzy soft multiset $(F,A)$ which describes the ''attractiveness of stone houses '', "very cheap red cars" and "hotels in Kajang" that Mr. X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2\}$ and $U_3 = \{v_1, v_2\}$.

Let $E_{U_i} = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters as given in the above example. Let

$$A = \left\{a_1 = \left(e_{U_1,4}, e_{U_2,3}, e_{U_3,4}\right),\ a_2 = \left(e_{U_1,4}, e_{U_2,1}, e_{U_3,4}\right)\right\}.$$

The intuitionistic fuzzy soft multiset $(F,A)$ is the collection of approximations given by

$$(F,A)_0 = \left\{\left(a_1, \left(\left\{\frac{h_1}{(0,1)}, \frac{h_2}{(0,1)}, \frac{h_3}{(0,1)}\right\}, \left\{\frac{c_1}{(0,1)}, \frac{c_2}{(0,1)}\right\}, \left\{\frac{v_1}{(0,1)}, \frac{v_2}{(0,1)}\right\}\right)\right)\right.$$
$$\left.\left(a_2, \left(\left\{\frac{h_1}{(0,1)}, \frac{h_2}{(0,1)}, \frac{h_3}{(0,1)}\right\}, \left\{\frac{c_1}{(0,1)}, \frac{c_2}{(0,1)}\right\}, \left\{\frac{v_1}{(0,1)}, \frac{v_2}{(0,1)}\right\}\right)\right)\right\}$$

Then $(F,A)$ is a null intuitionistic fuzzy soft multiset.

**Definition 3.9.** An intuitionistic fuzzy soft multiset $(F,A)$ over $U$ is called a *semi-absolute intuitionistic fuzzy soft multiset* denoted by $(F,A)_{\approx 1_i}$ if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = \bar{1}_i$ for at least one $i$, $a_k \in A$, . $a_k \in A, k = \{1,2,3,\dots,n\}, i \in \{1,2,3,\dots,m\}$ and $j \in \{1,2,3,\dots,r\}$

**Example 3.8.** Suppose that there are three universes $U_1, U_2$ and $U_3$. Let us consider an intuitionistic fuzzy soft multiset $(F,A)$ which describes the ''attractiveness of wooden houses '', "cars" and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2\}$ and $U_3 = \{v_1, v_2\}$.

Let $E_{U_i} = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters as given in the above example. Let

$$A = \left\{a_1 = \left(e_{U_1,4}, e_{U_2,1}, e_{U_3,1}\right), a_2 = \left(e_{U_1,4}, e_{U_2,3}, e_{U_3,1}\right), a_3 = \left(e_{U_1,4}, e_{U_2,3}, e_{U_3,3}\right)\right\}$$

The intuitionistic fuzzy soft multiset $(F,A)$ is the collection of approximations given by

$$(F,A)_{\approx 1_1} =$$
$$\left\{\left(a_1, \left(\left\{\frac{h_1}{(1,0)}, \frac{h_2}{(1,0)}, \frac{h_3}{(1,0)}\right\}, \left\{\frac{c_1}{(0.8,0)}, \frac{c_2}{(0.5,0.5)}\right\}, \left\{\frac{v_1}{(0.8,0.1)}, \frac{v_2}{(0.7,0.2)}\right\}\right)\right)\right.$$
$$\left(a_2, \left(\left\{\frac{h_1}{(1,0)}, \frac{h_2}{(1,0)}, \frac{h_3}{(1,0)}\right\}, \left\{\frac{c_1}{(1,0)}, \frac{c_2}{(0.8,0.1)}\right\}, \left\{\frac{v_1}{(0.6,0.2)}, \frac{v_2}{(0.5,0.3)}\right\}\right)\right)$$
$$\left.\left(a_3, \left(\left\{\frac{h_1}{(1,0)}, \frac{h_2}{(1,0)}, \frac{h_3}{(1,0)}\right\}, \left\{\frac{c_1}{(0.8,0.1)}, \frac{c_2}{(0.6,0.3)}\right\}, \left\{\frac{v_1}{(0.5,0.3)}, \frac{v_2}{(0.4,0.4)}\right\}\right)\right)\right\}$$

Then $(F, A)$ is a semi-absolute intuitionistic fuzzy soft multiset.

**Definition 3.10.** An intuitionistic fuzzy soft multiset $(F,A)$ over $U$ is called an *absolute intuitionistic c fuzzy soft multiset* denoted by $(F,A)_1$ if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = \bar{1}_i, \forall i$.

**Example 3.9.** Suppose that there are three universes $U_1, U_2$ and $U_3$. Let us consider an intuitionistic fuzzy soft multiset $(F,A)$ which describes the ''attractiveness of wooden houses '', " very costly white cars" and "hotels in KL" that Mr. X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2\}$ and $U_3 = \{v_1, v_2\}$.

Let $E_{U_i} = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters as given in the above example. Let

$$A = \left\{a_1 = \left(e_{U_1,4}, e_{U_2,1}, e_{U_3,4}\right), a_2 = \left(e_{U_1,4}, e_{U_2,4}, e_{U_3,4}\right)\right\}$$

The intuitionistic fuzzy soft multiset $(F,A)$ is the collection of approximations given by

$$(F,A)_1 = \left\{\left(a_1, \left(\left\{\frac{h_1}{(1,0)}, \frac{h_2}{(1,0)}, \frac{h_3}{(1,0)}\right\}, \left\{\frac{c_1}{(1,0)}, \frac{c_2}{(1,0)}\right\}, \left\{\frac{v_1}{(1,0)}, \frac{v_2}{(1,0)}\right\}\right)\right)\right.$$
$$\left.\left(a_2, \left(\left\{\frac{h_1}{(1,0)}, \frac{h_2}{(1,0)}, \frac{h_3}{(1,0)}\right\}, \left\{\frac{c_1}{(1,0)}, \frac{c_2}{(1,0)}\right\}, \left\{\frac{v_1}{(1,0)}, \frac{v_2}{(1,0)}\right\}\right)\right)\right\}$$

Then $(F,A)$ is an intuitionistic fuzzy absolute multisoft set.

**Proposition 3.1.** *If* $(F,A)$ *is an intuitionistic fuzzy soft multiset over* $U$ *, then*

i. $\left((F,A)^c\right)^c = (F,A)$,
ii. $(F,A)^c_{\approx 0_i} = (F,A)_{\approx 1_i}$,
iii. $(F,A)^c_0 = (F,A)_1$,
iv. $(F,A)^c_{\approx 1_i} = (F,A)_{\approx 0_i}$,
v. $(F,A)^c_1 = (F,A)_0$.

**Proof:** The proof is straightforward .

## 4. Union and Intersection

In this section we define the operation of union and intersection and give some examples by using the basic intuitionistic fuzzy union and intersection.

**Definition 4.1.** The *union* of two intuitionistic fuzzy soft multisetss $(F,A)$ and $(G,B)$ over $U$, denoted by $(F,A) \cup (G,B)$, is the intuitionistic fuzzy soft multiset $(H,C)$ where $C = A \cup B$, and $\forall \varepsilon \subset C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & if\ \varepsilon \subset A - B, \\ G(\varepsilon) & if\ \varepsilon \subset B - A, \\ \bigcup_{Atan} \left(F(\varepsilon), G(\varepsilon)\right) & if\ \varepsilon \subset A \cap B, \end{cases}$$

where $\bigcup_{Atan}$ is Atanassov union.

**Example 4.2.** Consider Example 3.1. Let

$$A=\left\{a_1=\left(e_{U_1,1},e_{U_2,1},e_{U_3,1}\right),a_2=\left(e_{U_1,2},e_{U_2,3},e_{U_3,1}\right),a_3=\left(e_{U_1,4},e_{U_2,3},e_{U_3,3}\right)\right\}$$

and

$$B=\left\{b_1=\left(e_{U_1,1},e_{U_2,1},e_{U_3,1}\right),b_2=\left(e_{U_1,1},e_{U_2,2},e_{U_3,1}\right),b_3=\left(e_{U_1,2},e_{U_2,3},e_{U_3,1}\right),\right.$$
$$\left.b_4=\left(e_{U_1,1},e_{U_2,3},e_{U_3,2}\right)\right\}.$$

Suppose $(F,A)$ and $(G,B)$ are two intuitionistic fuzzy soft multisets over the same $U$ such that

$$(F,A)=\left\{\left(a_1,\left(\left\{\frac{h_1}{(0.2,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.8,0)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.5,0.2)}\right\},\left\{\frac{v_1}{(0.8,0)},\frac{v_2}{(0.7,0.1)}\right\}\right)\right)\right.$$
$$\left(a_2,\left(\left\{\frac{h_1}{(0.7,0)},\frac{h_2}{(0.7,0.2)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(0.8,0.1)},\frac{c_2}{(0.6,0)}\right\},\left\{\frac{v_1}{(0.5,0)},\frac{v_2}{(0.4,0)}\right\}\right)\right)$$
$$\left.\left(a_3,\left(\left\{\frac{h_1}{(1,0)},\frac{h_2}{(0.8,0)},\frac{h_3}{(0.7,0.1)}\right\},\left\{\frac{c_1}{(0.7,0)},\frac{c_2}{(0.8,0.1)}\right\},\left\{\frac{v_1}{(0.5,0)},\frac{v_2}{(0.4,0.1)}\right\}\right)\right)\right\}$$

and $(G,B)=$

$$\left\{\left(b_1,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.6,0.2)}\right\},\left\{\frac{v_1}{(0.9,0)},\frac{v_2}{(0.7,0.3)}\right\}\right)\right)\right.$$
$$\left(b_2,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(1,0)},\frac{c_2}{(0.9,0)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.5,0.3)}\right\}\right)\right)$$
$$\left(b_3,\left(\left\{\frac{h_1}{(0.8,0)},\frac{h_2}{(0.9,0.1)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(0.8,0.1)},\frac{c_2}{(0.8,0.1)}\right\},\left\{\frac{v_1}{(0.7,0.3)},\frac{v_2}{(0.6,0.2)}\right\}\right)\right)$$
$$\left.\left(b_4,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0.5,0.3)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.8,0.1)}\right\}\right)\right)\right\}$$

By using the Atanassov union which is the basic intuitionistic fuzzy union we have

$$(H,C)=(F,A)\cup(G,B)=$$
$$\left\{\left(c_1,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.6,0.2)}\right\},\left\{\frac{v_1}{(0.9,0)},\frac{v_2}{(0.7,0.1)}\right\}\right)\right)\right.$$
$$\left(c_2,\left(\left\{\frac{h_1}{(0.8,0)},\frac{h_2}{(0.9,0.1)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(0.8,0.1)},\frac{c_2}{(0.8,0)}\right\},\left\{\frac{v_1}{(0.7,0)},\frac{v_2}{(0.6,0)}\right\}\right)\right)$$
$$\left.\left(c_3,\left(\left\{\frac{h_1}{(1,0)},\frac{h_2}{(0.8,0)},\frac{h_3}{(0.7,0.1)}\right\},\left\{\frac{c_1}{(0.7,0)},\frac{c_2}{(0.8,0.1)}\right\},\left\{\frac{v_1}{(0.5,0)},\frac{v_2}{(0.4,0.1)}\right\}\right)\right)\right\}$$

$$\left(c_4,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.6,0.2)}\right\},\left\{\frac{v_1}{(0.9,0)},\frac{v_2}{(0.7,0.3)}\right\}\right)\right)$$

$$\left(c_5,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0.5,0.3)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.8,0.1)}\right\}\right)\right)\}$$

**Remark 4.3.** The Atanassov union can be replaced by any *S-norm* which is a general intuitionistic fuzzy union (see [12].

**Proposition 4.4.** *If* $(F,A)$ , $(G,B)$ *and* $(H,C)$ *are three intuitionistic fuzzy soft multisets over* $U$ , *by using the basic* intuitionistic *fuzzy union ,then*

i. $(F,A)\bigcup((G,B)\bigcup(H,C)) = ((F,A)\bigcup(G,B))\bigcup(H,C),$

ii. $(F,A)\bigcup(F,A) = (F,A),$

iii. $(F,A)\bigcup(G,A)_{\approx 0_i} = (R,A),$

iv. $(F,A)\bigcup(G,A)_0 = (F,A),$

v. $(F,A)\bigcup(G,B)_{\approx 0_i} = (R,D),$

vi. $(F,A)\bigcup(G,B)_0 = \begin{cases}(F,A) & if\ \ A=B,\\ (R,D) & otherwise\end{cases}$ , where $D = A\bigcup B,$

vii. $(F,A)\bigcup(G,A)_{\approx 1_i} = (R,A)_{\approx 1_i}$ ,

viii. $(F,A)\bigcup(G,A)_1 = (G,A)_1,$

ix. $(F,A)\bigcup(G,B)_{\approx 1_i} = \begin{cases}(R,D)_{\approx 1_i} & if\ \ A=B,\\ (R,D) & otherwise\end{cases}$ , where $D = A\bigcup B,$

x. $(F,A)\bigcup(G,B)_1 = \begin{cases}(G,B)_1 & if\ \ A\subseteq B,\\ (R,D) & \text{otherwise}\end{cases}$ ,where $D = A\bigcup B.$

**Proof:** The proof is straightforward .

**Definition 4.5.** The *intersection* of two intuitionistic fuzzy soft multisets $(F,A)$ and $(G,B)$ over $U$ denoted by $(F,A)\bigcap(G,B)$ is the intuitionistic fuzzy soft multiset $(H,C)$ where $C = A\bigcup B,\ \forall\varepsilon\subset C,$ and

$$H(\varepsilon)=\begin{cases}F(\varepsilon) & if\ \ \varepsilon\in A-B,\\ G(\varepsilon) & if\ \ \varepsilon\in B-A,\\ \bigcap\limits_{Atan}(F(\varepsilon),G(\varepsilon)) & if\ \ \varepsilon\in A\bigcap B,\end{cases}$$

where $\bigcap\limits_{Atan}$ is Atanassov intersection.

**Example 4.6.** Consider Example 3.10. By using the Atanassov intersection which is the basic intuitionistic fuzzy intersection we have

$(F,A)\bigcup(G,B) = (H,C) =$

$$\left\{\left(c_1,\left(\left\{\frac{h_1}{(0.2,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.8,0)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.5,0.2)}\right\},\left\{\frac{v_1}{(0.8,0)},\frac{v_2}{(0.7,0.3)}\right\}\right)\right)\right.$$

$$\left(c_2,\left(\left\{\frac{h_1}{(0.7,0)},\frac{h_2}{(0.7,0.2)},\frac{h_3}{(1,0)}\right\},\left\{\frac{c_1}{(0.8,0.1)},\frac{c_2}{(0.6,0.1)}\right\},\left\{\frac{v_1}{(0.5,0.3)},\frac{v_2}{(0.4,0.2)}\right\}\right)\right)$$

$$\left.\left(c_3,\left(\left\{\frac{h_1}{(1,0)},\frac{h_2}{(0.8,0)},\frac{h_3}{(0.7,0.1)}\right\},\left\{\frac{c_1}{(0.7,0)},\frac{c_2}{(0.8,0.1)}\right\},\left\{\frac{v_1}{(0.5,0)},\frac{v_2}{(0.4,0.1)}\right\}\right)\right)\right\}$$

$$\left(c_4,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0.8,0)},\frac{c_2}{(0.6,0.2)}\right\},\left\{\frac{v_1}{(0.9,0)},\frac{v_2}{(0.7,0.3)}\right\}\right)\right)$$

$$\left.\left(c_5,\left(\left\{\frac{h_1}{(0.3,0)},\frac{h_2}{(0.4,0.1)},\frac{h_3}{(0.9,0)}\right\},\left\{\frac{c_1}{(0,1)},\frac{c_2}{(0.5,0.3)}\right\},\left\{\frac{v_1}{(0.8,0.1)},\frac{v_2}{(0.8,0.1)}\right\}\right)\right)\right\}$$

**Remark 4.7.** The Atanassov intersection can be replaced by any *T-norm* which is a general intuitionistic fuzzy intersection (see [12].

**Proposition 4.8.** *If* $(F,A)$ *,* $(G,B)$ *and* $(H,C)$ *are three intuitionistic fuzzy soft multisetss over* $U$ *, by using the basic intuitionistic fuzzy intersection ,then*

i. $(F,A)\cap\big((G,B)\cap(H,C)\big)=\big((F,A)\cap(G,B)\big)\cap(H,C),$

ii. $(F,A)\cap(F,A)\;=(F,A),$

iii. $(F,A)\cap(G,A)_{\approx 0_i}\;=(R,A)_{\approx 0_i}\,,$

iv. $(F,A)\cap(G,A)_{0_i}\;=(R,A)_{0_i}\,,$

v. $(F,A)\cap(G,B)_{\approx 0_i}\;=\begin{cases}(R,D)_{\approx 0_i} & if\;\; A\subseteq B\\ (R,D) & \text{otherwise}\end{cases}$, where $D=A\cup B$,

vi. $(F,A)\cap(G,B)_0\;=\begin{cases}(R,D)_0 & if\;\; A\subseteq B\\ (R,D) & \text{otherwise}\end{cases}$, where $D=A\cup B$,

vii. $(F,A)\cap(G,A)_{\approx 1_i}\;=(R,D),$

viii. $(F,A)\cap(G,A)_1=(F,A),$

ix. $(F,A)\cap(G,B)_{\approx 1_i}\;=(R,D),$

x. $(F,A)\cap(G,B)_1=\begin{cases}(F,A) & \text{if A}\supseteq\text{B}\\ (R,D) & \text{otherwise}\end{cases}.$

## 5. Intuitionistic Fuzzy Soft set Based Decision Making

We begin this section with a novel algorithm designed for solving intuitionistic fuzzy soft set based decision making problems, which was presented by Y. Jiang Y. Tang and Q. Chen which was presented in Jiang, *et al.* [13].

### 5.1. Y. Jiang ,Y. Tang and Q. Chen's Algorithm 1.

a. Input the intuitionistic fuzzy soft set $\overline{w}=\langle F,A\rangle$.

b. Input a threshold intuitionistic fuzzy set $\lambda : A \to [0,] \times [0,]$ or choose the mid-level decision rule; or choose the top-bottom-level decision rule or the top-top or bottom-bottom-level decision rule for decision making.

c. Compute the level soft set $L(\overline{w};\lambda)$ or the $(s,t)-$ level soft set $L(\overline{w},s,t)$; or the mid, top-bottom, top-top or bottom-bottom-level soft set.

d. Present the level soft set $L(\overline{w};\lambda)$.

e. The optimal decision is to select $o_k$ if $c_k = \max_i c_i$.

f. If $k$ has more than one value then any one of $o_k$ may be chosen.

## 5.2. Intuitionistic Fuzzy Soft Multiset Theoretic Approach to Decision Making Problems.

In this section we suggest the following algorithm to solving intuitionistic fuzzy soft multisets based decision making problems. We note here that we will use the name $(AA)$ for Y. Jiang ,Y. Tang and Q. Chen's algorithm 1 (see [13].

**Algorithm 1:**

a. Input the intuitionistic fuzzy soft multisets $(F,A)$.

b. Apply $(AA)$ to the first intuitionistic fuzzy soft multisets part in $(F,A)$ to get the decision $S_{k_1}$.

c. Redefine the intuitionistic fuzzy soft multisets $(F,A)$ by keeping all values in each row where $S_{k_1}$ is maximum and replacing the membership values in the other rows by zero to get $(F,A)_{new1}$.

d. Apply $(AA)$ to the second intuitionistic fuzzy soft multisets part in $(F,A)_{new1}$ to get the decision $S_{k_2}$.

e. Redefine the intuitionistic fuzzy soft multisets $(F,A)_{new1}$ by keeping the first and second parts and apply the method in step 3 for the third part to get $(F,A)_{new2}$.

f. Apply $(AA)$ for the third intuitionistic fuzzy soft multiests part in $(F,A)_{new2}$ to get the decision $S_{k_3}$.

g. The decision $(S_{k_1},S_{k_2},S_{k_3})$. Where $S_{k_1},S_{k_2}$ and $S_{k_3}$ the decision come from step 2, 4 and 6 respectively.

## 5.3. Application in a Decision Making Problem:

Consider Example 3.1.Which is represent the intuitionistic fuzzy soft multisets theory.

**Table-1.** Tabular representation the first intuitionistic fuzzy soft multisets part.

| $U$ | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\xi_5$ |
|---|---|---|---|---|---|
| $h_1$ | $(0.2,0.4)$ | $(0.2,0.4)$ | $(0.7,0.2)$ | $(0.9,0)$ | $(1,0)$ |
| $h_2$ | $(0.4,0.1)$ | $(0.4,0.1)$ | $(0.7,0.1)$ | $(0.5,0.3)$ | $(0,0.1)$ |
| $h_3$ | $(0.8,0.1)$ | $(0.8,0.1)$ | $(1,0)$ | $(0.5,0.5)$ | $(0.7,0.2)$ |

Now we apply $(AA)$ to the first intuitionistic fuzzy soft multisets part in $(F,A)$ by using the Mid-level decision rule in table (2).

**Table-2.** Tabular representation of the mid-level decision rule with choice value.

| $U_1$ | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\xi_5$ | **Choice value** |
|---|---|---|---|---|---|---|
| $h_1$ | 0 | 0 | 0 | 1 | 1 | $c_1 = 2$ |
| $h_2$ | 0 | 0 | 0 | 0 | 0 | $c_2 = 0$ |
| $h_3$ | 1 | 1 | 1 | 0 | 0 | $c_3 = 3$ |

From table (2) it follows that the maximum choice value is $c_3 = 3$ so the optimal decision is to select $h_3$.

Now we redefine the intuitionistic fuzzy soft multiset $(F, A)$ by keeping all values in each row where $h_3$ is the maximum and replacing membership value in other rows by zero to get $(F, A)_{new1}$ .

**Table-3.** Tabular representation of $(F, A)_{new1}$.

| $U_2$ | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\xi_5$ |
|---|---|---|---|---|---|
| $c_1$ | $(0.8, 0)$ | $(1, 0)$ | $(0.8, 0.1)$ | $(0,1)$ | $(0,1)$ |
| $c_2$ | $(0.5, 0.5)$ | $(0.8, 0.1)$ | $(0.6, 0.3)$ | $(0,1)$ | $(0,1)$ |

Now we apply $(AA)$ for the second intuitionistic fuzzy soft multisets part in $(F, A)_{new1}$ to take the decision form the availability $U_2$.

The tabular representation of resultant second intuitionistic fuzzy soft multiset part will be as in table (4).

**Table-4.** Tabular representation of $U_2$ - intuitionistic fuzzy soft multisets part.

| $U_2$ | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\xi_5$ | Choice value |
|---|---|---|---|---|---|---|
| $c_1$ | 1 | 1 | 1 | 1 | 1 | $c_1 = 5$ |
| $c_2$ | 0 | 0 | 0 | 1 | 1 | $c_2 = 2$ |

From this table it follows the maximum choice value is $c_1 = 5$, so the optimal decision is to select $c_1$.

Next we define the intuitionistic fuzzy soft multiset $(F, A)_{new1}$ by keeping all values in each row where $c_1$ is the maximum and replacing the membership value in other rows by zero to get $(F, A)_{new2}$.

**Table-5.** Tabular representation of $(F, A)_{new2}$.

| $U_3$ | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\xi_5$ |
|---|---|---|---|---|---|
| $v_1$ | $(0.8, 0.1)$ | $(0.6, 0)$ | $(0.5, 0.3)$ | $(0,1)$ | $(0,1)$ |
| $v_2$ | $(0.7, 0.2)$ | $(0.8, 0.1)$ | $(0.4, 0.4)$ | $(0,1)$ | $(0,1)$ |

Now we apply $(AA)$ to the third intuitionistic fuzzy soft multiset part in $(F, A)_{new2}$ to take the decision form the availability $U_3$. The tabular representation of resultant third intuitionistic fuzzy soft multiset part will be as in table (6)

**Table-6.** Tabular representation of $U_3$ - intuitionistic fuzzy soft multisets part.

| $U_3$ | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\xi_5$ | Choice value |
|---|---|---|---|---|---|---|
| $v_1$ | 1 | 1 | 1 | 1 | 1 | $c_1 = 5$ |
| $v_2$ | 0 | 0 | 0 | 1 | 1 | $c_2 = 2$ |

From the above table, it is clear that the maximum choice value is $c_1 = 5$ so the optimal decision is to select $v_1$.

Then from the above results the decision is $(h_3, c_1, v_1)$.

### 5.4. Y. Jiang , Y. Tang and Q. Chen's algorithm 2.

**Algorithm 2:**

a. Input a weighted intuitionistic fuzzy soft set $\xi = \langle F, A, w \rangle$.

b. Input a threshold intuitionistic fuzzy set $\lambda : A \to [0,1] \times [0,1]$ ( or give a threshold value pair $(s,t) \in [0,1] \times [0,1]$ ) or choose the mid-level decision rule; or choose the top-bottom-level decision rule; or choose the top-top-level decision rule; or choose the bottom-bottom-level decision rule) for decision making.

c. Compute the level soft set $L(\langle F, A \rangle; \lambda)$ of $\xi$ w.r.t. the threshold intuitionistic fuzzy set $\lambda$ ( or the set $(s,t)$ -level soft set $L(\langle F, A \rangle; s, t)$ ) or the mid-level soft set $L(\langle F, A \rangle; \text{mid})$ ; or
the top– bottom-level soft set $L(\langle F, A \rangle; \text{topbottom})$ or the top– top-level soft set $L(\langle F, A \rangle; \text{toptop})$ ; bottom– bottom-level soft set $L(\langle F, A \rangle; \text{bottombottom})$).

d. Present the level soft set $L(\langle F, A \rangle; \lambda)$ or $L(\langle F, A \rangle; s, t)$ ; or $L(\langle F, A \rangle; \text{mid})$ or $L(\langle F, A \rangle; \text{toptop})$ or $L(\langle F, A \rangle; \text{topbottom})$ or $L(\langle F, A \rangle; \text{bottombottom})$. In tabular form and compute the weighted choice value $c_i'$ of $o_i$ for all $i$.

e. The optimal decision is to select $o_i$ if $c_k' = \max_i c_i'$.

f. If $k$ has more than one value then any one of $o_k$ may be chosen.

### 5.5. Weighted Intuitionistic Fuzzy Soft Multisets Based Decision Making

In this section we suggest the following algorithm to solving intuitionistic fuzzy soft multisets based decision making problems. We note here that we will use the name $(AA)_2$ for Yuncheng Jiang ,Yong Tang and Qimai Chen's algorithm 2.

**Algorithm 2:**

a. Input the weighted intuitionistic fuzzy soft multisets $(F, A)$.

b. Apply $(AA)_2$ to the first weighted intuitionistic fuzzy soft multisets part in $(F, A)$ to get the decision $S_{k_1}$.

c. Redefine the weighted intuitionistic fuzzy soft multisets $(F, A)$ by keeping all values in each row where $S_{k_1}$ is maximum and replacing the membership values in other rows by zero to get $(F, A)_{new1}$.

d. Apply $(AA)$ to the second weighted intuitionistic fuzzy soft multisets part in $(F, A)_{new1}$ to get the decision $S_{k_2}$.

e. Redefine the weighted intuitionistic fuzzy soft $(F, A)_{new1}$ by keeping the first and second parts and apply the method in number 3 for the third part to get $(F, A)_{new2}$.

f. Apply $(AA)$ to the third weighted intuitionistic fuzzy soft multiests part in $(F, A)_{new2}$ to get the decision $S_{k_3}$.

g. The decision $(S_{k_1}, S_{k_2}, S_{k_3})$. Where $S_{k_1}, S_{k_2}$ and $S_{k_3}$ the decision come from number 2, 4 and 6 respectively.

**Example 5.6.** that Mr. $X$ has imposed the following weights for the parameters that describes the ''attractiveness of houses '' in Example 3.1. For the parameter ''costely '' , $w_1 = 0.8$; for the parameter ''very costely '' , $w_2 = 0.6;$ for the parameter''cheap '', $w_3 = 0.9;$ for the parameter ''wooden '' , $w_4 = 0.7;$ and for the parameter ''in green surrounding '' , $w_5 = 0.5.$ Thuse we have a weight function $w : A \to [0,1]$ and the intuitionistic fuzzy soft multiset as in Example 3.1 is a changed into a weighted intuitionistic fuzzy soft multiset.

**Table-7.** Tabular representation of first part of the weighted intuitionistic fuzzy soft multiset.

| $U$ | $\xi_1, w_1 = 0.8$ | $\xi_2, w_2 = 0.6$ | $\xi_3, w_3 = 0.9$ | $\xi_4, w_4 = 0.7$ | $\xi_5, w_5 = 0.5$ |
|---|---|---|---|---|---|
| $h_1$ | $(0.2, 0.4)$ | $(0.2, 0.4)$ | $(0.7, 0.2)$ | $(0.9, 0)$ | $(1, 0)$ |
| $h_2$ | $(0.4, 0.1)$ | $(0.4, 0.1)$ | $(0.7, 0.1)$ | $(0.5, 0.3)$ | $(0.8, 0.1)$ |
| $h_3$ | $(0.8, 0.1)$ | $(0.8, 0.1)$ | $(1, 0)$ | $(0.5, 0.5)$ | $(0.7, 0.2)$ |

Now we apply $(AA)_2$ for the first weighted intuitionistic fuzzy soft multisets part in $(F, A)$ by using the Mid-level decision rule in table (8).

**Table-8.** Tabular representation of the mid-level decision rule with choice value.

| $U_1$ | $\xi_1$ $w_1 = 0.8$ | $\xi_2$ $w_2 = 0.6$ | $\xi_3$ $w_3 = 0.9$ | $\xi_4$ $w_4 = 0.7$ | $\xi_5$ $w_5 = 0.5$ | **Weighted Choice value** |
|---|---|---|---|---|---|---|
| $h_1$ | 0 | 0 | 0 | 1 | 1 | $c_1 = 1.2$ |
| $h_2$ | 0 | 0 | 0 | 0 | 0 | $c_2 = 0$ |
| $h_3$ | 1 | 1 | 1 | 0 | 0 | $c_3 = 2.3$ |

From the above table, it is clear that the maximum weighted choice value is $c_3 = 2.3$. So the optimal decision is to select $h_3$.

Then for Mr. $X$ shoud buy $h_3$ as the best house.

Now we redefine the weighted intuitionistic fuzzy soft multisets for the second part by keeping all values in each rows where $h_3$ is the maximum and replacing the membership value in other rows by zero.

**Table-9.** Tabular representation of $(F, A)_{new1}$.

| $U_2$ | $\xi_1$ $w_1 = 0.8$ | $\xi_2$ $w_2 = 0.6$ | $\xi_3$ $w_3 = 0.9$ | $\xi_4$ $w_4 = 0.7$ | $\xi_5$ $w_5 = 0.5$ |
|---|---|---|---|---|---|
| $c_1$ | $(0.8, 0)$ | $(1, 0)$ | $(0.8, 0.1)$ | $(0, 1)$ | $(0, 1)$ |
| $c_2$ | $(0.5, 0.5)$ | $(0.8, 0.1)$ | $(0.6, 0.3)$ | $(0, 1)$ | $(0, 1)$ |

Now we apply $(AA)_2$ for the second weighted intuitionistic fuzzy soft multisets part in $(F, A)_{new1}$ to take the decision form the availability $U_2$.

The tabular representation of resultant second weighted intuitionistic fuzzy soft multiset part will be as in table (4).

**Table-10.** Tabular representation of $U_2$ - intuitionistic fuzzy soft multisets part.

| $U_2$ | $\xi_1$ $w_1 = 0.8$ | $\xi_2$ $w_2 = 0.6$ | $\xi_3$ $w_3 = 0.9$ | $\xi_4$ $w_4 = 0.7$ | $\xi_5$ $w_5 = 0.5$ | **Weighted Choice value** |
|---|---|---|---|---|---|---|
| $c_1$ | 1 | 1 | 1 | 1 | 1 | $c_1 = 5$ |
| $c_2$ | 0 | 0 | 0 | 1 | 1 | $c_2 = 2$ |

From this table, it is clear that the weighted maximum choice value is $c_1 = 5$ and so the optimal decision is to select $c_1$. Therefore Mr $X$ sould bay $c_1$ as the best car.

Now we redefine the intuitionistic fuzzy soft multiset for the third part by keeping all values in each row where $c_1$ is the maximum and replacing the membership value in other rows by zero as in table (11).

**Table-11.** Tabular representation of $(F,A)_{new2}$.

| $U_3$ | $\xi_1$ $w_1=0.8$ | $\xi_2$ $w_2=0.6$ | $\xi_3$ $w_3=0.9$ | $\xi_4$ $w_4=0.7$ | $\xi_5$ $w_5=0.5$ |
|---|---|---|---|---|---|
| $v_1$ | $(0.8,0.1)$ | $(0.6,0.2)$ | $(0.5,0.3)$ | $(0,1)$ | $(0,1)$ |
| $v_2$ | $(0.7,0.2)$ | $(0.5,0.3)$ | $(0.7,0.2)$ | $(0,1)$ | $(0,1)$ |

Now we applying $(AA)_2$ for the third weighted intuitionistic fuzzy soft multiset part in $(F,A)_{new2}$ to take the decision form the availability $U_3$. The tabular representation of resultant third weighted intuitionistic fuzzy soft multiset part will be as in table (12).

**Table-12.** Tabular representation of $U_3$ - intuitionistic fuzzy soft multisets part.

| $U_3$ | $\xi_1$ $w_1=0.8$ | $\xi_2$ $w_2=0.6$ | $\xi_3$ $w_3=0.9$ | $\xi_4$ $w_4=0.7$ | $\xi_5$ $w_5=0.5$ | **Weighted Choice value** |
|---|---|---|---|---|---|---|
| $v_1$ | 1 | 1 | 1 | 1 | 1 | $c_1=5$ |
| $v_2$ | 0 | 0 | 0 | 1 | 1 | $c_2=2$ |

From the above table, it is clear that the maximum choice value is $c_1=5$ so the optimal decision is to select $v_1$.Then from the above results the decision is $(h_3,c_1,v_1)$.

## 6. Conclusion

In this paper we have introduced the concept of intuitionistic fuzzy soft multiset and studied some of its properties. Application of this theory has been given to solve a decision making problem.

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