Academic Journal of Applied Mathematical Sciences
ISSN(e): 2415-2188, ISSN(p): 2415-5225
Vol. 5, Issue. 5, pp: 50-56, 2019
URL: https://arpgweb.com/journal/journal/17
DOI: https://doi.org/10.32861/ajams.55.50.56

Academic Research Publishing Group

Original Research Open Access

Choice of Confounding in the 2^k Factorial Design in 2^b Blocks

Francis C. Eze
Department of Statistics, Nnamdi-Azikiwe University, Awka, Anambra State, Nigeria

Abstract

In 2^k complete factorial experiment, the experiment must be carried out in a completely randomized design. When the numbers of factors increase, the number of treatment combinations increase and it is not possible to accommodate all these treatment combinations in one homogeneous block. In this case, confounding in more than one incomplete block becomes necessary. In this paper, we considered the choice of confounding when k > 2. Our findings show that the choice of confounding depends on the number of factors, the number of blocks and their sizes. When two more interactions are to be confounded, their product module 2 should be considered and thereafter, a linear combination equation should be used in allocating the treatment effects in the principal block. Other contents in other blocks are generated by multiplication module 2 of the effects not in the principal block. Partial confounding is recommended for the interactions that cannot be confounded.

Keywords: Confounding; Partial confounding; Principal block; Module.

1. Introduction

It is sometimes impossible to run all the observations in a 2^k factorial design under homogenous condition [1]. This means that, there must be available for experimentation 2^k homogenous blocks to accommodate the treatments. In this case, the experiment must be performed in more than one incomplete block and such situation is confounding in 2^k factorial experiment.

Confounding variables or confounders are often defined as the variables correlate (positively or negatively) with both the dependent variable and the independent variable. A Confounder is an extraneous variable whose presence affects the variables being studied so that the results do not reflect the actual relationship between the variables under study [2].

2. Methodology

2.1. Construction of 2^k Factorial Design Confounded in 2^b Blocks

Basically, we confound to reduce the experimental error and the block size and it is unavoidable when the treatment is greater than the block size.

Information on the confounded effect is lost and as such we do not confound the main effect. We choose for confounding those interactions whose effects are expected to be small or unimportant. The block which contains the treatment (I) is called the principal block. The 2^k factorial effects are distributed among the blocks as follows:

i. To the principle block, assign all the treatments which have zero or even number of letters in common with the confounded interactions if the confounded interaction is even.

ii. The contents of the other block is generated by taking the product module 2 between the treatment in the principal block and any other treatment not previously assigned as in (i).

iii. In general, if 2^k treatment are to be observed in 2^b blocks, then we must confound or select b independently chosen interactions and thereafter we notice the 2^b-b-1 additional interaction are also confounded.

Experiments with larger numbers of factors can be divided into a larger number of blocks by identifying the subgroup and cosets of the treatment group associated with particular contrast subgroups [3]. This is the even or odd rule. In odd or even method of confounding, the key block or principal block will contain the even number of treatments while the other block will contain odd number of treatments [4].

There are some methods of identifying confounded effects. Some of them are:

a. The even versus odd method
b. Expansion of products. In this method, the factorial effect whose estimate is desired, takes the value -1 and others take +1. The product gives the effect. The effect of AB in 2^2 factorial experiment will be AB: (a-1)(b-1)-ab-a-b+(1).
c. Defining Contrast Method. This method involves constructing the defining contrasts using the general expression such as $L_1 = \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_k x_k$ Where, x_i is the ith factor appearing in a particular treatment combination ; a_i is the exponent appearing on the ith factor in the effect to be confounded for the 2^k system, we have a_i = 0 or 1 and x_i = 0 or 1. Treatment combinations that produce the same block value of L (Mod 2) will be placed in the same block.

The problem is how to choose interactions to be confounded in 2^k factorial experiment when k > 2.

2.2. Confounding Compared to Balanced Incomplete Block Design (BIBD)

In order to understand the relationship between confounding and BIBD we consider 2^4 factorial experiments. The treatment combinations are (1), a, b, ab, c, ac, abc, d, ad, bd, abd, cd, acd, bcd, abcd.

Suppose this 2^4 is to be conducted in a randomized block design, the corresponding model is

$$E(y_{ij}) = \mu + \tau_i + \beta_j \qquad \begin{cases} i = 1,2,\ldots,t \\ j = 1,2,\ldots,b \end{cases} \tag{1}$$

Where

μ is a constant

τ_i is the treatment effect.

β_j is the block effect and

Now using the even and odd method, the factorial effects are

$$A = \frac{1}{2^k r}\left[abcd - bcd + acd - cd + abd - bd + ad - d + abc - bc + ac - c + ab - b + a - (1)\right]$$

$$B = \frac{1}{2^k r}\left[abcd + bcd - acd - cd + abd + bd + ad - d + abc + bc - ac - c + ab + b - a - (1)\right]$$

$$AB = \frac{1}{2^k r}\left[abcd - bcd - acd + cd + abd - bd - ad + d + abc - bc - ac + c + ab - b - a + (1)\right]$$

$$C = \frac{1}{2^k r}\left[abcd + bcd + acd + cd - abd - bd - ad - d + abc + bc + ac + c - ab - b - a - (1)\right]$$

$$AC = \frac{1}{2^k r}\left[abcd - bcd + acd - cd - abd + bd - ad + d + abc - bc + ac - c - ab + b - a + (1)\right]$$

$$BC = \frac{1}{2^k r}\left[abcd + bcd - acd - cd - abd - bd + ad + d + abc + bc - ac - c - ab - b + a + (1)\right]$$

$$ABC = \frac{1}{2^k r}\left[abcd - bcd - acd + cd - abd + bd + ad - d + abc - bc - ac + c - ab + b + a - (1)\right]$$

$$D = \frac{1}{2^k r}\left[abcd + bcd + acd + cd + abd + bd + ad + d - abc - bc - ac - c - ab - b - a - (1)\right]$$

$$AD = \frac{1}{2^k r}\left[abcd - bcd + acd - cd + abd - bd + ad - d - abc + bc - ac + c - ab + b - a + (1)\right]$$

$$BD = \frac{1}{2^k r}\left[abcd + bcd - acd - cd + abd + bd - ad - d - abc - bc + ac + c - ab - b + a + (1)\right]$$

$$ABD = \frac{1}{2^k r}\left[abcd - bcd - acd + cd + abd - bd - ad + d - abc + bc + ac - c - ab + b + a - (1)\right]$$

$$CD = \frac{1}{2^k r}\left[abcd + bcd + acd + cd - abd - bd - ad - d - abc - bc - ac - c + ab + b + a + (1)\right]$$

$$ACD = \frac{1}{2^k r}\left[abcd - bcd + acd - cd - abd + bd - ad + d - abc + bc - ac + c + ab - b + a - (1)\right]$$

$$BCD = \frac{1}{2^k r}\left[abcd + bcd - acd - cd - abd - bd + ad + d - abc - bc + ac + c + ab + b - a - (1)\right]$$

$$ABCD = \frac{1}{2^k r}\left[abcd - bcd - acd + cd - abd + bd + ad - d - abc + bc + ac - c + ab - b - a + (1)\right]$$

r is the number of replication.

The methods described in section 2.1 may be used to construct a 2^k factorial design confounded in 2^b blocks with the block size exactly 2^{k-b} runs.

Preferably, only higher order interactions, that is, interactions with three or more factors are confounded, because their loss is immaterial [5]. An experimenter is generally interested in main effects and two factor interactions and these should not be confounded as far as possible.

In 2^4 factorial experiments in 2^1 blocks, let the confounded interaction effects be abcd. The block size will be 2^{4-1}= 8. Using the even rule method, the principal block will contain all the treatments that have even number of letters in common with the confounded effect. The contents of the other block are generated by multiplication module 2 of the effects not in the principal block.

$$Block\ 1 \qquad\qquad\qquad\qquad Block\ 2$$

$$\begin{bmatrix} (1) \\ ab \\ ac \\ bc \\ ad \\ bd \\ cd \\ abcd \end{bmatrix} \qquad\qquad\qquad\qquad \begin{bmatrix} a \\ b \\ c \\ abc \\ d \\ abd \\ acd \\ bcd \end{bmatrix}$$

The block effects of blocks 1 and 2 are β_1 and β_2, respectively. Then the average responses corresponding to treatment combinations (1), a, b, ab, c, ac, abc, d, ad, bd, abd, cd, acd, bcd, abcd are

$$E[y(a)] = \mu + \beta_2 + \tau(a)$$
$$E[y(b)] = \mu + \beta_2 + \tau(b)$$
$$E[y(c)] = \mu + \beta_2 + \tau(c)$$
$$E[y(abc)] = \mu + \beta_2 + \tau(abc)$$
$$E[y(d)] = \mu + \beta_2 + \tau(d)$$
$$E[y(abd)] = \mu + \beta_2 + \tau(abd)$$
$$E[y(acd)] = \mu + \beta_2 + \tau(acd)$$
$$E[y(bcd)] = \mu + \beta_2 + \tau(bcd)$$
$$E[y(abcd)] = \mu + \beta_1 + \tau(abcd)$$
$$E[y(cd)] = \mu + \beta_1 + \tau(cd)$$
$$E[y(bd)] = \mu + \beta_1 + \tau(bd)$$
$$E[y(ad)] = \mu + \beta_1 + \tau(ad)$$
$$E[y(bc)] = \mu + \beta_1 + \tau(bc)$$
$$E[y(ac)] = \mu + \beta_1 + \tau(ac)$$
$$E[y(ab)] = \mu + \beta_1 + \tau(ab)$$
$$E[y(1)] = \mu + \beta_1 + \tau(1)$$

The $y(a), y(b), ..., y(1)$ and $\tau(a), \tau(b), ..., \tau(1)$ denote the responses and treatments to a, b,…, (1) respectively.

Ignoring, the factor $\frac{1}{2^k r}$ which is common in A, B,…,ABCD and using $E[y(a)], E[y(b)], ..., E[y(abcd)]$, the effect A can be expressed as follows

$$A = \tau(abcd) - \tau(bcd) + \tau(acd) - \tau(cd) + \tau(abd) - \tau(bd) + \tau(ad) - \tau(d) + \tau(abc)$$
$$-\tau(bc) + \tau(ac) - \tau(c) + \tau(ab) - \tau(b) + \tau(a) - \tau(1) \tag{2}$$

From equation (2), the block effect is not present in A and it is not mixed up with the treatment effects. Here, we say that the main effect is not confounded.

Similarly, for the main effect, *B*

$$B = \tau(abcd) + \tau(bcd) - \tau(acd) - \tau(cd) + \tau(abd) + \tau(bd) - \tau(ad) - \tau(d) + \tau(abc)$$
$$+\tau(bc) - \tau(ac) - \tau(c) + \tau(ab) + \tau(b) - \tau(a) - \tau(1) \tag{3}$$

Similarly, B is not confounded.

Others are

$$C = \tau(abcd) + \tau(bcd) + \tau(acd) + \tau(cd) - \tau(abd) - \tau(bd) - \tau(ad) - \tau(d) + \tau(abc)$$
$$+\tau(bc) + \tau(ac) + \tau(c) - \tau(ab) - \tau(b) - \tau(a) - \tau(1)$$

C is also not confounded.

$$D = \tau(abcd) + \tau(bcd) + \tau(acd) + \tau(cd) + \tau(abd) + \tau(bd) + \tau(ad) + \tau(d) - \tau(abc)$$
$$-\tau(bc) - \tau(ac) - \tau(c) - \tau(ab) - \tau(b) - \tau(a) - \tau(1)$$

D is not confounded.

$$AB = \tau(abcd) - \tau(bcd) - \tau(acd) + \tau(cd) + \tau(abd) - \tau(bd) - \tau(ad) + \tau(d) + \tau(abc)$$
$$-\tau(bc) - \tau(ac) + \tau(c) + \tau(ab) - \tau(b) - \tau(a) + \tau(1)$$

AB interaction is not confounded.

$$AC = \tau(abcd) - \tau(bcd) + \tau(acd) - \tau(cd) - \tau(abd) + \tau(bd) - \tau(ad) + \tau(d) + \tau(abc)$$
$$-\tau(bc) + \tau(ac) - \tau(c) - \tau(ab) + \tau(b) - \tau(a) + \tau(1)$$

AC interaction not confounded.

$$AD = \tau(abcd) - \tau(bcd) + \tau(acd) - \tau(cd) + \tau(abd) - \tau(bd) + \tau(ad) - \tau(d) - \tau(abc)$$
$$+\tau(bc) - \tau(ac) + \tau(c) - \tau(ab) + \tau(b) - \tau(a) + \tau(1)$$

AD interaction not confounded.

$$BC = \tau(abcd) + \tau(bcd) - \tau(acd) - \tau(cd) - \tau(abd) - \tau(bd) + \tau(ad) + \tau(d) + \tau(abc)$$
$$+\tau(bc) - \tau(ac) - \tau(c) - \tau(ab) - \tau(b) + \tau(a) + \tau(1)$$

BC interaction not confounded.

$$BD = \tau(abcd) + \tau(bcd) - \tau(acd) - \tau(cd) + \tau(abd) + \tau(bd) - \tau(ad) - \tau(d) - \tau(abc)$$
$$-\tau(bc) + \tau(ac) + \tau(c) - \tau(ab) - \tau(b) + \tau(a) + \tau(1)$$

$$ABC = \tau(abcd) - \tau(bcd) - \tau(acd) + \tau(cd) - \tau(abd) + \tau(bd) + \tau(ad) - \tau(d) + \tau(abc)$$
$$-\tau(bc) - \tau(ac) + \tau(c) - \tau(ab) + \tau(b) + \tau(a) - \tau(1)$$

ABC interaction is not confounded.

$$ACD = \tau(abcd) - \tau(bcd) + \tau(acd) - \tau(cd) - \tau(abd) + \tau(bd) - \tau(ad) + \tau(d) - \tau(abc)$$
$$+\tau(bc) - \tau(ac) + \tau(c) + \tau(ab) - \tau(b) + \tau(a) - \tau(1)$$

ACD interaction is not confounded.

In summary, the main effects, two factors and three factors interactions are not confounded.

The effects of $ABCD$ is

$$ABCD = 8(\beta_1 - \beta_2) + \tau(abcd) - \tau(bcd) - \tau(acd) + \tau(cd) - \tau(abd) + \tau(bd) + \tau(ad) - \tau(d) - \tau(abc)$$
$$+\tau(bc) + \tau(ac) - \tau(c) + \tau(ab) - \tau(b) - \tau(a) + \tau(1)$$

Here the block effects are present in $ABCD$. In fact, the block effects are β_1 and β_2 and are mixed up with the treatment effects and cannot be separated individually from the treatment effects in $ABCD$. So $ABCD$ is said to be confounded (or mixed up) with the blocks.

Now suppose we decide to have 2^2 blocks with block size of $2^{4-2} = 4$. In this case, we have 4 blocks and as such 3 interactions will be confounded in blocks. 2 interactions will be chosen independently and thereafter 1 additional interaction will be confounded. In general, if 2^k treatments are to be observed in 2^b blocks, then we must confound or select b independently chosen interactions and thereafter we notice that $2^b - b - 1$ additional interactions are also confounded [6].

The confoundable three factor interactions are ABC, ABD, ACD, and BCD.

The generalized interactions of these three factors interactions are:

$$(ABC)(ABD) = A^2B^2CD = CD \tag{4}$$

$$(ABC)(ACD) = A^2BC^2D = BD \tag{5}$$

$$(ABC)(BCD) = AB^2C^2D = AD \tag{6}$$

$$(ACD)(ABD) = A^2BCD^2 = BC \tag{7}$$

$$(ACD)(BCD) = ABC^2D^2 = AB \tag{8}$$

$$(ABD)(BCD) = AB^2D^2 = A \tag{9}$$

In equation (9) interactions ABD and BCD will not be considered for confounding because; their generalized interaction gives the main effect A which cannot be confounded.

Using the method of linear combination

$$L_1 = \alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_k x_k$$

for interactions ABC, ABD and CD we have

$$L_1 = X_1 + X_2 + X_3 = 0 \mod 2$$
$$L_2 = X_1 + X_2 + X_4 = 0 \mod 2$$
$$L_3 = X_3 + X_4 = 0 \mod 2$$

The equal block contents for ABC, ABD and CD cannot be obtained.

However, using ABC, ACD and BD we have

$$L_1 = X_1 + X_2 + X_3 = 0 \mod 2$$
$$L_2 = X_1 + X_3 + X_4 = 0 \mod 2$$
$$L_3 = X_2 + X_4 = 0 \mod 2$$

The block contents are

Block 1	Block 2	Block 3	Block 4
$\begin{bmatrix} 1 \\ abd \\ ac \\ bcd \end{bmatrix}$	$\begin{bmatrix} abcd \\ c \\ bd \\ a \end{bmatrix}$	$\begin{bmatrix} acd \\ bc \\ d \\ ab \end{bmatrix}$	$\begin{bmatrix} cd \\ abc \\ ad \\ b \end{bmatrix}$

Block 1 satisfies the equation $L_1 = L_2 = L_3 = 0 \mod 2$ for interactions ABC, ACD and BD.

The contents of other blocks are obtained by multiplication module 2 of other effects not in block 1.

The block effects for ABC are:

$$ABC = 4(-\beta_1 + \beta_2 - \beta_3 + \beta_4) + \tau(abcd) - \tau(bcd) - \tau(acd) + \tau(cd) - \tau(abd) + \tau(bd) + \tau(ad) - \tau(d) + \tau(abc) - \tau(bc) - \tau(ac) + \tau(c) - \tau(ab) + \tau(b) + \tau(a) - \tau(1)$$

Similarly, the block effects for ACD and BD confounded interactions are

$$ACD = 4(-\beta_1 + \beta_2 + \beta_3 - \beta_4) + \tau(abcd) - \tau(bcd) + \tau(acd) - \tau(cd) - \tau(abd) + \tau(bd) - \tau(ad) + \tau(d) - \tau(abc) + \tau(bc) - \tau(ac) + \tau(c) + \tau(ab) - \tau(b) + \tau(a) - \tau(1)$$

$$BD = 4(\beta_1 + \beta_2 - \beta_3 - \beta_4) + \tau(abcd) + \tau(bcd) - \tau(acd) - \tau(cd) + \tau(abd) + \tau(bd) - \tau(ad) - \tau(d) - \tau(abc) - \tau(bc) + \tau(ac) + \tau(c) - \tau(ab) - \tau(b) + \tau(a) + \tau(1)$$

We then conclude that are ABC, ACD and BD confounded.

Other confoundable interactions are:

$$ABC, BCD \text{ and AD}$$
$$ACD, ABD \text{ and } BC$$
$$ACD, BCD \text{ and } AB$$

and their corresponding linear combinations are :

$$L_1 = X_1 + X_2 + X_3 = 0 \mod 2$$
$$L_2 = X_2 + X_3 + X_4 = 0 \mod 2$$
$$L_3 = X_1 + X_4 = 0 \mod 2$$

$$L_1 = X_1 + X_3 + X_4 = 0 \mod 2$$
$$L_2 = X_1 + X_2 + X_4 = 0 \mod 2$$
$$L_3 = X_2 + X_3 = 0 \mod 2$$
$$L_1 = X_1 + X_3 + X_4 = 0 \mod 2$$
$$L_2 = X_2 + X_3 + X_4 = 0 \mod 2$$
$$L_3 = X_1 + X_2 = 0 \mod 2$$

The block contents for the above three sets of linear combination sets cannot have equal allocation of the treatment effects. We therefore conclude that among the seven confoundable interactions, only ABC, ACD and BD can be confounded.

2.3. Partial Confounding

When a factorial experiment is to be conducted or carried out in more than one replication, it is advisable to confound different interaction in different replication.

There are two basic choices when replicating a confounded 2^k. Complete confounding means that you confound the same effects in every replication. Partial confounding means that you confound different effects in every replication. The efficiency of the estimate is the fraction of replicates where the effect is not confounded. E.g., three replications and only confounded in one is 2/3 efficiency. With partial confounding you can get non-zero efficiency for more effects. Complete confounding gives you zero efficiency for the completely confounded effects [7].

The following interactions can be confounded by partial confounding in different replications.

$$ABC, BCD \text{ and AD}$$
$$ACD, ABD \text{ and } BC$$
$$ACD, BCD \text{ and } AB$$

For ABC, BCD, and AD we have;

Rep 1 $(L_1 = X_1 + X_2 + X_3)$		Rep 2 $(L_1 = X_2 + X_3 + X_4)$		Rep 3 $(L_1 = X_1 + X_4)$	
Block 1	*Block* 2	*Block* 1	*Block* 2	*Block* 1	*Block* 2
(1)	*abcd*	(1)	*abcd*	(1)	*bcd*
bcd	*a*	*acd*	*b*	*acd*	*ab*
acd	*b*	*cd*	*ab*	*abd*	*ac*
abd	*c*	*abd*	*c*	*ad*	*abc*
bc	*ad*	*bd*	*ac*	*abcd*	*a*
ac	*bd*	*abc*	*d*	*bc*	*d*
ab	*cd*	*bc*	*ad*	*c*	*bd*
d	*abc*	*abc*	*d*	*b*	*c*
ABC confounded		*BCD* confounded		*AD* confounded	

Similarly, for ACD, ABD and BC we have;

Rep 1 $(L_1 = X_1 + X_3 + X_4)$		Rep 2 $(L_1 = X_1 + X_2 + X_4)$		Rep 3 $(L_1 = X_2 + X_3)$	
Block 1	*Block* 2	*Block* 1	*Block* 2	*Block* 1	*Block* 2
(1)	*acd*	(1)	*cd*	(1)	*acd*
bcd	*b*	*bcd*	*b*	*abcd*	*b*
cd	*a*	*acd*	*a*	*bcd*	*ab*
abd	*bc*	*bd*	*bc*	*ad*	*c*
ad	*c*	*ad*	*ac*	*d*	*ac*
abc	*bd*	*abc*	*abd*	*abc*	*ad*
ac	*d*	*c*	*d*	*bc*	*abd*
b	*abcd*	*ab*	*abcd*	*a*	*cd*
ACD confounded		*ABD* confounded		*BC* confounded	

Finally for ACD, BCD and AB we have;

Rep 1 $(L_1 = X_1 + X_3 + X_4)$

$$\begin{array}{cc} Block\ 1 & Block\ 2 \\ \begin{bmatrix} (1) \\ bcd \\ cd \\ abd \\ ad \\ abc \\ ac \\ b \end{bmatrix} & \begin{bmatrix} acd \\ b \\ a \\ bc \\ c \\ bd \\ d \\ abcd \end{bmatrix} \end{array}$$

ACD confounded

Rep 2 $(L_1 = X_2 + X_3 + X_4)$

$$\begin{array}{cc} Block\ 1 & Block\ 2 \\ \begin{bmatrix} (1) \\ acd \\ cd \\ abd \\ bd \\ abc \\ bc \\ a \end{bmatrix} & \begin{bmatrix} abcd \\ b \\ ab \\ c \\ ac \\ d \\ ad \\ bcd \end{bmatrix} \end{array}$$

BCD confounded

Rep 3 $(L_1 = X_1 + X_2)$

$$\begin{array}{cc} Block1 & Block\ 2 \\ \begin{bmatrix} (1) \\ abcd \\ cd \\ abd \\ d \\ abc \\ c \\ ab \end{bmatrix} & \begin{bmatrix} bcd \\ a \\ b \\ ac \\ bc \\ abd \\ bd \\ acd \end{bmatrix} \end{array}$$

AB confounded

2.4. Discussion of Findings

In 2^k factorial design in 2^b blocks of size 2^{k-b} with $k > 2$, the highest order interactions are always confounded. Other confoundable interactions should also be considered depending on the number of blocks and the sizes. When two or more of the interactions are to be considered, it is noticed that extra $2^b - b - 1$ additional interactions are also confounded by noticing their product module 2.

In choosing the set of interactions to be confounded, a linear combination of the form $L_1 = \alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_k x_k = 0 \bmod 2$ should be considered in allocation treatment effects in the principal block. When equal numbers of treatments are allocated in 2^b blocks of size 2^{k-b} such interactions are confounded, otherwise they are not. However, if the experimenter wishes to confound the interactions which cannot be confounded with equal allocation of treatment effects, partial confounding becomes the only option.

3. Conclusions

Confounding in 2^k factorial experiments depend on the size of k, the number of blocks and their sizes. If k is greater than 3, all possible higher order interactions should be considered for confounding. The method of linear combinations should then be used to decide which interactions to be confounded.

References

[1] Montgomery, D. C. and Runger, G. C., 1994. *Applied statistics and probability for engineers*. John Wiley and Sons, Inc.

[2] Mohamad, A. P., Ahmad, R. B., and Mohsen, V., 2012. "How to control confounding effects by statistical analysis." *Gastroenterol Hepatol Bed Bench. Spring*, vol. 5, pp. 79–83.

[3] Cox, D. R. and Reid, N., 2000. *The theory of the design of experiments*: Chapman and Hall/CRC.

[4] Jaisankar, R. and Pachamuthu, M., 2012. "Methods for identification of confounded effects in factorial experiments." *Int. J. of Mathematical Sciences and Applications*, vol. 2, pp. 751-758.

[5] Jaggi, S. and Batra, P. K., 2018. *Confounding in factorial experiment and fractional factorials*. Library Avenue, NewDelhi-110012: Indian Agricultural Statistics Research Institute. pp. 1-14.

[6] Montgomery, D. C., 2001. *Design and analysis of experiments*. New York: John Willey and Sons

[7] Oehlert, G. W., 2014. *Confounding two-series factorials*. School of Statistics, University of Minnesota.