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Econometric Estimation of Production Function with Applications

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Abstract

This study focuses on Monte Carlo Methods in parameter estimation of production function. The ordinary least square (OLS) method is used to estimate the unknown parameters. The Monte Carlo simulation methods are used for the data generating process. The Cobb-Douglas production model with multiplicative error term is fitted to the data generated. From tables 1.1 to 1.3, the mean square error (MSE) of θ_1 are 0.007678, 0.001972 and 0.001253 respectively for sample sizes 20, 40 and 80. Our finding showed that the mean square error (MSE) value varies with the sum of the powers of the input variables.

Keywords: Cobb-douglas model; Production; Modeling; Parameter estimation; Capital.

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1. Introduction

For the past years, Production Function played an important role in modeling certain phenomena. This is due to increase computational powers and Monte Carlo Methodology. In economics, it is widely used in research works on production, cost function and demand. Econometric modeling requires the combination of an error term as well as the specification of its distribution. This study discusses the models meant for a task in statistical analysis of mathematical models- the estimation of unknown parameters and their distributions in models, based on generated data. A production function is a quantitative link between production inputs and outputs. Johnson and Samuel [1] said a production function establishes the functional relationship between the quantity of a specific product that can be produced within a time and a set of inputs used, given the existing technology in a socio-cultural environment. The traditional theory of production function of a firm expresses output as a function of two inputs, capital (k) and labour (L) in the form of the Cobb-Douglas function. The production function may be wrongly specified and the form of mis-specification at micro level may not exactly as it is at the macro level. For this reasons policies of government on production to individual firms may not be useful for the whole economy. Isaac [2]; observed that many useful microeconomic models, relations and concerts, such as potential output, technical change, the investment function and the demand for labour are based on the aggregation production. The Cobb-Douglas production function is the simplest production function widely used to represent the technological relationship between the amounts of two or more inputs, and the amount of output that can be produced by those inputs. It was used by Charles Cobb-Douglas and Paul Douglas in the study in which they modeled the growth of American Economy during the period (1899-1922). In spite of the important role played by the producing sectors or industries in Nigeria and other countries, they are faced with problems of estimation of parameters, measuring of returns to scale in production function. The Objectives of the study are:

i. Estimate Cobb-Douglas Production function

ii. Compute and Compare mean square errors for different returns to scale in Cobb –Douglas production model.

A research carried out by Hossain and Islam [3]; employed the nonlinear Cobb-Douglas production function to assess returns to scale and to estimate the level of productivity and allocative efficiency in manufacturing firms in the south west region of Bangladesh. The study found that fertilizer and seafood manufacturing firms have increasing reforms to scale, while textile, jute, and cement manufacturing firms have decreasing returns to scale, while textile, jute, and cement manufacturing firms have decreasing returns to scale. It was concluded that the average capital productivity is less than the capital productivity of all firms in the study. Md, *et al.* [4]; applied non-linear Cobb-Douglas (CD) production function with Autocorrelation problem to selected manufacturing industries in Bangladesh, and identified that Cobb-Douglas production function with additive errors was more suitable for some selected manufacturing industries. The man aim of the study was to detect the autocorrelation problem of Cobb-Douglas production model with additive errors [5], finally, the results showed that autocorrelation is present in some of the manufacturing industries. The autocorrelation problem was removed and the parameters of the Cobb-Douglas production function with additive errors were re-estimated. Dana and Jaromir [6]; affirmed that the practical application of the production function method requires making certain assumptions on the functional form of the production technology, returns to scale, and characteristics of the technological progress, as well as of the functioning of the markets. They argued that the functional form however, includes the assumption of a constant share of labour in output, which may be for a converging country. In this study, they tested whether this fact renders the application of Cobb-Douglas production unreliable for the Czech economy. They applied a more

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general form of production function and allowed labour share to develop according to the empirical data. Afzal and Manni [7]; investigated the nature and extent of productivity changes in the Cobb-Douglas production model components and the growth of the knowledge economy of selected countries, namely, Thailand, Malaysia, Singapore Philippines and south Korea which were analyzed over the period 2005 to 2010. In this study , the total factor productivity (TFP) index, individual country`s efficiency and productivity changes which took place within this period were estimated. The result obtained indicated that the Philippines and Singapore have highest increase in total factor productivity within the periods, and this growth in productivity was derived from both technical efficiency gain and technological progress. There was a remarkable growth in total factor productivity for Thailand and Philippines for the knowledge. Many studies have been done so far on Cobb-Douglas production model. For example, the study by Chowdhury and Islam [8]; also applied the nonlinear Cobb-Douglas (C-D) production functions in garments industries in Bangladesh which affirmed that both labour and capital contributed significantly to the total output in garment industries. The statistical Analytical system (SAS) was applied to estimate the Cobb-Douglas production function econometrically in this study. Ashfag and Muhammad [9]; estimated the nonlinear Cobb-Douglas production function to investigate the relationship between the production of cement and inputs labour and capital. The results of the estimates showed that there is a constant return to scale in the cement industry, moreover, the empirical results also showed that the capital contributes relatively less than the labour during the production process. From this study, it was concluded that there is a strong relationship between the input and output variables. Model is free from autocorrelation as its value (1.148) at 1% level of significance.

2. Materials and Methods

2.1. Estimation Method

This study considers Cobb-Douglas production function with Multiplication error term. The model is given by

$$Y = \Theta_0 K^{\Theta_1} L^{\Theta_2} e^u \quad \textbf{(1)}$$

Where Y = Crude oil production output

K = Capital invested in the production

L = Labour used in the production

Θ_0 = Positive constant or Technological constant.

Θ_1 and Θ_2 are positive parameters output elasticities of capital and Labour

U = Random or stochastic error

e = Base of natural logarithm

The model in (1) can be transformed to linear model by taking the natural logarithm of both sides of the equation to obtain a regression model of the form.

$$Lny = Ln\ \Theta_0 + \Theta_1 Ln\ K + \Theta_2\ Ln + u \quad \textbf{(2)}$$

The ordinary least square (OLS) estimation is used for the linear model to obtained the estimate $\Theta = (\Theta_0, \Theta_1, \Theta_2)$.

The choice of model parameters $(\Theta_0, \Theta_1, \Theta_2)$ is such that $\Theta_1 + \Theta_2 < 1$

$\Theta_1 + \Theta_2 = 1$ and $\Theta_1 + \Theta_2 > 1$, while the value of Θ_0 is arbitrary and kept constant at

$Ln\ \Theta_0 = 3$, $\Theta_0 = 20.09$

We use the following three sets of parameters:

$V_1 = (20.09, 0.35, 0.30)$, $V_2 = (20.09, 0.55, 0.45)$, $V_3 = (20.09, 0.75, 0.60)$

The input matrix is made of two variables K (Capital) and L (Labour) which are randomly generated and normally distributed independently.

2.2. Simulation

The Mont Carlo Study uses Sample size of 20, 40 and 80 with each experiment replicated 20 times under the following three conditions, varied one at a time while the others are kept: the sample size T and the parameters set $\Theta = (\Theta_0, \Theta_1, \Theta_2)$ used in the data generating process.

Mean square error is the most important criterion used to evaluate the performance of an estimator.

3. Results and Discussion

We have estimated a total of 180 equations. $\theta = (20.09, 0.35, 0.30)$ with sample sizes 20, 40, 80 and with 20 replications. In all the tables. N stands for the number of replication. The model (3) is a multiplicative error based model which is filled to the data generated. The strata software package is use to analysed the data. Monte Carlo results showing estimates with their bias, variance, mean square error (MSE), Standard deviation, are summarized and presented in Tables 1to11.

Table-1. Monte Carlo Estimate for variables for sample size N = 20, $(\theta_0, \theta_1, \theta_2)$ = (20.09, 0.35, 0.30) T = 2 lny = $\ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	15.964248	4. 125752	45.104058	62.12588	6.715955
θ_1	0.368320	0.018320	0.007342	0.007678	0.085688
θ_2	0.333575	0.033575	0.005586	0.006713	0.074739
$\theta_1+\theta_2$	0.701867	0.051867	0.018881	0.021571	0.137408

Table-2. Monte Carlo Estimate for variables for sample size T N = 20, $(\theta_0, \theta_1, \theta_2)$ = (20.09, 0.35, 0.30) T = 40 lny = $\ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	18.004218	2.085782	32.061799	36.412284	5.662314
θ_1	0.371508	0.021508	0.001510	0.001972	0.038855
θ_2	0.321911	0.021911	0.003444	0.003924	0.058683
$\theta_1+\theta_2$	0.697519	0.047519	0.004963	0.007221	0.070451

Table-3. Monte Carlo Estimate for variables for sample size T N = 20, $(\theta_0, \theta_1, \theta_2)$ = (20.09, 0.35, 0.30) T = 80 lny = $\ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	21.108742	1.018742	40.699586	41.737421	6.379623
θ_1	0.345666	0.004334	0.001234	0.001253	0.035131
θ_2	0.298823	0.001177	0.002032	0.002034	0.045081
$\theta_1+\theta_2$	0.644486	0.005514	0.002765	0.002796	0.052586

Table-4. Monte Carlo Estimates for variables for sample size T N = 20, $(\theta_0, \theta_1, \theta_2)$ = (20.09, 0.55, 0.45) T = 20 lny = $\ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	16.073577	4.016423	44.909952	61.041604	6.701489
θ_1	0.569329	0.019329	0.007371	0.007745	0.085855
θ_2	0.482515	0.032515	0.005562	0.006619	0.074576
$\theta_1+\theta_2$	1.051844	0.051844	0.018881	0.021569	0.137408

Table-5. Monte Carlo Estimates for variables for sample size T N = 20, $(\theta_0, \theta_1, \theta_2)$ = (20.09, 0.55, 0.45) T = 40 lny = $\ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	17.954216	2.135784	32.249889	36.811462	5.678899
θ_1	0.571497	0.021497	0.001511	0.001973	0.038871
θ_2	0.476909	0.026909	0.002925	0.003649	0.054079
$\theta_1+\theta_2$	1.046966	0.046996	0.004665	0.006873	0.068298

Table-6. Monte Carlo Estimates for variables for sample size T N = 20, $(\theta_0, \theta_1, \theta_2)$ = (20.09, 0.55, 0.45) T = 80 lny = $\ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

	Parameter Estimates				
Parameter	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	21.108737	1.018737	40.699522	41.776603	6.379618
θ_1	0.548666	0.001334	0.001241	0.001243	0.035226
θ_2	0.448823	0.001177	0.002032	0.002034	0.045081
$\theta_1+\theta_2$	0.994489	0.005511	0.002766	0.002796	0.052592

Table-7. Monte Carlo Estimates for variables for sample size T N = 20, $(\theta_0, \theta_1, \theta_2)$ = (20.09, 0.75, 0.60) T= 20 lny = $\ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	16.111710	3.978290	44.296391	60.123180	6.655553
θ_1	0.770279	0.020279	0.007360	0.007771	0.085790
θ_2	0.631881	0.031881	0.005410	0.006426	0.073554
$\theta_1+\theta_2$	1.402159	0.052159	0.018954	0.021675	0.137673

Table-8. Monte Carlo Estimates for variables for sample size T N = 20, (θ_0, θ_1, θ_2) = (20.09, 0.75, 0.60) T = 40 lny = lnθ_0 + $\theta_1$ln K+ $\theta_2$lnL + u

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	17.953678	2.136372	32.254912	36.818999	5.679341
θ_1	0.771508	0.021508	0.001510	0.003384	0.038855
θ_2	0.626930	0.026930	0.002922	0.003647	0.054052
θ_1+θ_2	1.398438	0.048438	0.004931	0.007277	0.070220

Table-9. Monte Carlo Estimates for variables for sample size T N = 20, (θ_0, θ_1, θ_2) = (20.09, 0.75, 0.60) T = 80 lny = lnθ_0 + $\theta_1$ln K+ $\theta_2$lnL + u

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(σ^2)	MSE	σ
θ_0	21.108740	1.018740	40.699587	41.737419	6.379623
θ_1	0.745666	0.004334	0.001234	0.001253	0.035131
θ_2	0.598823	0.001177	0.002032	0.002034	0.045081
θ_1+θ_2	1.344515	0.005485	0.002765	0.002795	0.052582

Table-10. Monte Carlo Values for variables. Sum θ_1 +θ_2 with sample size T and fixed N = 20 lny = lnθ_0 + $\theta_1$ln K+ $\theta_2$lnL + u

T	$\theta_1 + \theta_2$ = 0.65	$\theta_1 + \theta_2$ = 1.00	$\theta_1 + \theta_2$ = 1 .35
20	0.021571	0.021569	0.021675
40	0.007221	0.006873	0.007277
80	0.002796	0.002796	0.002795

Table-11. Monte Carlo Values for variables. Sample size T with sum θ_1 +θ_2 and fixed N = 20 lny = lnθ_0 + $\theta_1$ln K+ $\theta_2$lnL + u

Parameter	T = 20	T = 40	T = 80
$\theta_1 + \theta_2$ = 0 . 65	0.021571	0.007221	0.002796
$\theta_1 + \theta_2$ = 1.00	0.021569	0.006873	0.002796
$\theta_1 + \theta_2$ = 1.35	0.021675	0.007277	0.002795

From Tables 1 to 3, the mean square error (MSE) of θ_1 are 0.007678, 0.001972 and 0.001253 respectively for sample sizes 20, 40 and 80. The mean square error (MSE) for θ_2 are 0.006713, 0.003924 and 0.002034 respectively for sample sizes 20, 40 and 80, These results show that the value of the mean square error for sample size 20 is greater than the mean square error (MSE) for sample size 40, and also the value of the mean square error for sample size 40 is greater than the value of the mean square error (MSE) for sample size 80 and vice-verse . From the results, we observed that as the value of the mean square error (MSE) decreases as the sample size increases and vise-versa. In the same vein, we observed the same trend in Tables 4 to 9.

Table 10, shows Monte Carlo values for variables sum (θ_1 + θ_2) with sample size T and fixed N = 20. In this table, for instance, the mean square errors (MSE) for decrease return to scale (that is for θ_1 + θ_2 = 0.65) are 0.021571, 0.007221 and 0.002796 respectively, for sample sizes 20,40 and 80. These results indicate that the value of the mean square for the sample size 20 is greater than the value of the mean square error for sample size 40, and the value of the mean square error (MSE) for sample size 40 is greater than the value of the mean square error (MSE) for sample size 80. From Table 10, we observe that for each return to scale, the value of the mean square error decreases as the sample size increases, and vice-versa

4. Conclusion

The study has revealed a good number of interesting results which are useful in empirical studies in terms of both methodologies and practical relevance.

In economics, the sum of power of the input variables K and L is interpreted as a measure of returns to scale. The work has shown that the mean square errors (MSE) for θ_1 and θ_2 decrease as the sample size increases for various returns to scale.

From the findings of the study, we can conclude that Cobb-Douglas Production function is useful and powerful tool for the analysis and evaluation of the governmental structural policies in the context of crude oil producing sector of Nigeria.

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