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Finite-Time Stabilization of Switched Systems with Time-Varying Delay

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Abstract

This paper studies the problem of finite-time stabilization of a class of switched linear time-varying delay systems. An event-triggered sampling mechanism and an event-triggered state feedback control are proposed. Based on Lyapunov-like function method, linear matrix inequality technique and averaged dwell time method, sufficient conditions for switched delay systems under event-triggered state feedback control are given to ensure the finite-time stabilization of the switched delay systems. Finally, a numerical example is given to verify the validity of the proposed results.

Keywords: Switched delay system; Event-triggered mechanism; Finite-time stable; Average dwell time.

1. Introduction

Switched systems are widely used in the practical engineering and have important research significance. The stability of the switched system is a fundamental problem in the theoretical study of the switching system [1-3]. Due to the existence of external disturbance, time delay, and uncertainty in the actual physical system, it is worth paying attention to study the control problem of switch system with delay. In Phat and Ratchagit [4], Stability and stabilization of switched linear discrete-time systems with interval time-varying delay was studied by Wang, *et al.* [5] analyzed the stability of switched delay systems with all subsystems unstable. We have noted that most of the previous work on the stability of systems was on the Lyapunov stability in infinite time intervals. However the behavior of some systems can only be defined within a limited time interval. In this case, it is necessary to study the finite time stability of the system [6-8]. In Yang, *et al.* [8], Yang at al. considered finite-time boundedness and stabilization of uncertain switched delayed neural networks of neutral type. In Xiang and Xiao [7], Xiang at al. dealt with finite-time stability and stabilization for switched linear systems. In Wang, *et al.* [6], finite-time stability for continuous-time switched systems in the presence of impulse effects was concerned.

In sampling control system, generally adopt time-triggered mechanism, that is, periodic sampling controller is used to control the system. This traditional time-triggered mechanism is helpful to simplify the system performance analysis, but its preset sampling period may cause a waste of system resources. Therefore, in order to reduce the sampling update and network communication frequency of the controller, an event triggering strategy different from time triggering is proposed [9, 10]. In [9], Tallapragada at al. investigated on event triggered tracking for nonlinear systems. The work of Liu and Jiang [10] studied event-triggered control of nonlinear systems with state quantization. So far, the event triggering mechanism has made some theoretical achievements in the study of the stability of nonswitched systems. However, the problem of event triggering control for the switched delay system has yet to be solved. Therefore, in this study, we focus on finite-time stabilization for a class of switched systems with time-varying delay. The main contributions of this paper lie in: (i) Develop event-triggered mechanism and design a controller. (ii) Sufficient conditions for unforced switched system with time-varying delay are presented. (iii) The criterion of finite-time stabilization for switched systems under the event-triggered control is given.

The paper is organized as follows. In Section 2, a description of switched systems, important definitions, event-triggered condition and some necessary lemmas are given. Section 3 analyzes the finite-time stabilization of the switched system with time-varying delay. A numerical example is shown in Section 4 to illustrate the results. Section 5 gives the conclusion of this paper.

Notation. □ represents the set of nonnegative integers. R^n and $R^{n\times m}$ represent the n dimensional Euclidean space and the set of all $n\times m$ real matrices respectively. $X > 0 (X \geq 0)$ is a real symmetric positive definite matrix (positive semi-definite matrix). $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of matrix A, respectively. * represents the symmetric blocks in a matrix.

2. Preliminaries and System Specification

Consider a class of linear switched systems with time-varying delay:

$$\begin{cases}\dot{x}(t)=A_{\sigma(t)}x(t)+A_{d\sigma(t)}x(t-\tau(t))+B_{\sigma(t)}u(t),\\ x(t)=\phi(t)\quad t\in[-h,0],\end{cases} \tag{1}$$

where $x(t)\in R^{n}$ denotes the state vector, $u(t)\in R^{m}$ is the control input, $\phi(t)$ is a continuous initial function on $[-h,0]$. $\tau(t)$ represents the time-varying delay and satisfies

$$0<\tau(t)<h,\quad 0<\dot{\tau}(t)<\hat{h}<1.$$

where *h* and $\hat{h}$ are positive constants. The switching signal is define as $\sigma(t):[0,\infty)\rightarrow M=\{1,2,\cdots,N\}$ which is a piecewise and right continuous constant function. *N* represents the number of subsystems. The corresponding switching sequence is

$$\Sigma=\{x_0;(i_0,t_0),\cdots,(i_k,t_k),\cdots,|\,i_k\in M,k=0,1,\cdots\}.$$

When $t\in[t_k,t_{k+1}),\ i_k th$ subsystem is activated. A_i,A_{di},B_i are known constant matrices with appropriate dimensions. B_i has full column rank.

In order to get the main results, we give the following definitions and lemmas.

Lemma 1. [11]. For a given matrix $B\in R^{p\times m}$ with rank (*B*) = *p*, assume that $X\in R^{m\times m}$ is a symmetric matrix, then there exists a matrix $\hat{X}\in R^{p\times p}$ such that $BX=\hat{X}B$, if and only if

$$X=V\begin{bmatrix}\hat{X}_{11} & \\ & \hat{X}_{22}\end{bmatrix}V^{T}, \tag{2}$$

where $\hat{X}_{11}\in R^{p\times p}$ and $\hat{X}_{22}\in R^{(m-p)\times(m-p)}$.

Lemma 2. (Jensen's Inequality) For any matrix $M\in R^{n\times n},M=M^{T}>0,$ scalars a and $b:a<b,$ vector $x:[a,b]\mapsto R$ such that the integration concerned are well defined, then:

$$\left(\int_a^b x(s)ds\right)^{T}M\left(\int_a^b x(s)ds\right)\leq(b-a)\int_a^b x^{T}(s)Mx(s)ds.$$

Lemma 3. Liu, *et al.* [11]. For any real vectors *u*,*v* and a symmetric positive matrix Q with compatible dimension, the following inequality holds:

$$u^{T}v+v^{T}u\leq u^{T}Qu+v^{T}Q^{-1}v. \tag{3}$$

Lemma 4. For the given matrix

$$S=\begin{pmatrix}S_{11} & S_{12}\\ S_{12}^{T} & S_{22}\end{pmatrix}<0,$$

where $S_{11}=S_{11}^{T},S_{22}=S_{22}^{T},$ the followings are equivalent:

(1) $S_{11}<0,S_{22}-S_{12}^{T}S_{11}^{-1}S_{12}<0;$

(2) $S_{22}<0,S_{11}-S_{12}S_{22}^{-1}S_{12}^{T}<0.$

Definition 1. (Average dwell time [13]). For any switching signal $\sigma(t)$ and $t_2>t_1\geq 0,$ let $N_{\sigma}(t_1,t_2)$ indicate the switching number of $\sigma(t)$ over (t_1,t_2). If

$$N_{\sigma}(t_1,t_2)\leq N_0+(t_2-t_1)/\tau_a, \tag{4}$$

holds for constants $N_0\geq 0,\tau_a\geq 0,$ then the positive constant τ_a is called an average dwell time and N_0 is the chattering bound. Without loss of generality, we choose $N_0=0.$

Definition 2. Lin, *et al.* [12]. Given three positive constants c_1,c_2,T with $c_2>c_1,$ a positive definite matrix *R* and a switching signal $\sigma(t).$ The switched linear system (1) with $u(t)=0$ is said to be finite-time stable with respect to $(c_1,c_2,T,R,\sigma),$ if

$$\sup_{-h\leq\theta\leq 0}\{x^{T}(\theta)Rx(\theta)\}\leq c_1\Rightarrow x^{T}(t)Rx(t)<c_2,\forall t\in[0,T]. \tag{5}$$

Definition 3. Lin, *et al.* [12]. Given three positive constants c_1,c_2,T with $c_2>c_1,$ and a positive definite matrix *R*. The switched system (1) with $u(t)=0$ is said to be uniformly finite-time stable with respect to $(c_1,c_2,T,R),$ if condition (5) holds for any switching signal $\sigma(t).$

In this section, we aim to develop an event-triggered mechanism and construct a controller which can guarantee the finite-time stabilization of system (1).

First, we develop the triggering condition based on the system state as follows:

$$\|e(t)\|^2 \ge \rho\|x(t)\|^2,$$

where $e(t) = x(\bar{t}_s) - x(t),$ is the error signal of the latest sampling state and the current state of the system. $0 < \rho < 1$ is a given positive threshold.

Because of the event triggering mechanism is used in the switched system, the system state $x(t)$ is first transmitted to the event triggering mechanism, through the designed triggering mechanism, we can obtain that the state $x(t)$ of the sampling system at the triggering time $\{\bar{t}_s\}_{k=0}^{\infty}$ is $x(\bar{t}_s).$ Furthermore, the control signal is updated by calculation, and the discrete signal is converted into continuous signal by the zero order holder, which is implemented in the subsystem by the actuator. Assumed that there is no transmission delay in the feedback channel, that is, the triggering sampling, controller signal updating and control signal application are synchronous. When an event happens, the controller updates the latest state and switching information and holds the information until the next event happens. We have the following event-trigger instant sequence: $\{\bar{t}_s\}_{k=0}^{\infty},$ with $\bar{t}_s < \bar{t}_{s+1},$ the next sampling instant $\bar{t}_{s+1}$ can be determined by

$$\bar{t}_{s+1} = \inf\{t > \bar{t}_s \mid \|e(t)\|^2 \ge \rho\|x(t)\|^2\}. \tag{6}$$

Let $\bar{t}_0 = t_0,$ without loss of generality, we assume that there is no Zeno behavior in this paper. Then $\forall t \in [t_k, t_{k+1}),$ the state feedback controller is set to

$$u(t) = K_{\sigma(t)} x(\bar{t}_S), \tag{7}$$

where $K_{\sigma(t)}$ is the controller gain. On the continuous sampling interval, the controller only updates the information of sampling time. Therefore, applying the state feedback controller (7) to the linear switched system (1), the closed-loop system can be obtained as follows:

$$\dot{x}(t) = \left(A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)}\right) x(t) + A_{d\sigma(t)} x\left(t - \tau(t)\right) + B_{\sigma(t)} K_{\sigma(t)} e(t) \tag{8}$$

3. Main Results

Consider an unforced switched system with time-varying delay

$$\dot{x}(t) = A_i x(t) + A_{di} x(t - \tau(t)) \tag{9}$$

In this subsection, we will give some sufficient conditions for finite-time stability of systems (9).

Let

$$\bar{P}_i = R^{-\frac{1}{2}} P_i R^{-\frac{1}{2}}, \quad \bar{Q}_i = R^{-\frac{1}{2}} Q_i R^{-\frac{1}{2}}, \quad \bar{R}_i = R^{-\frac{1}{2}} R_i R^{-\frac{1}{2}},$$
$$\lambda_1 = \lambda_{min}\left(\bar{P}_i\right), \quad \lambda_2 = \lambda_{max}\left(\bar{P}_i\right), \quad \lambda_3 = \lambda_{max}\left(\bar{R}_i\right), \quad \lambda_4 = \lambda_{max}\left(\bar{Q}_i\right). \tag{10}$$

Theorem 1. For given a matrix $R > 0,$ and positive scalars $c_1 \le c_2, T,$ if there exist positive definite symmetric matrices $P_i, R_i, Q_i,$ with appropriate dimensions for each $i \in M,$ and positive scalars $\alpha, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \mu \ge 1,$ such that

$$\Sigma_i = \begin{bmatrix} P_i A_i + A_i^T P_i + R_i + Q_i - \alpha P_i & P_i A_{di} & 0 & 0 & 0 \\ * & -(1-\hat{h})Q_i & 0 & 0 & 0 \\ * & * & -R_i & 0 & 0 \\ * & * & * & -\frac{\alpha}{h} R_i & 0 \\ * & * & * & * & -\frac{\alpha}{h} Q_i \end{bmatrix} < 0, \tag{11}$$

$$P_i \le \mu P_j,\ R_i \le \mu R_j,\ Q_i \le \mu Q_j,\ \forall i,\ j \in M, \tag{12}$$

$$c_1(\lambda_2 + h\lambda_3 + h\lambda_4) < \lambda_1 c_2 e^{-\alpha T}, \tag{13}$$

then, the system (9) is finite-time stable with respect to (c_1, c_2, T, R, σ) for any switching signal $\sigma(t)$ with average dwell time τ_a satisfying

$$\tau_a > \tau_a^* = \frac{T\, ln\, \mu}{ln(\lambda_1 c_2) - ln(c_1(\lambda_2 + h\lambda_3 + h\lambda_4)) - \alpha T}. \tag{14}$$

Proof: Construct Lyapunov like function as follows:

$$V_i(t) = x^T(t)P_i x(t) + \int_{t-h}^{t} x^T(s)R_i x(s)ds + \int_{t-\tau(t)}^{t} x^T(s)Q_i x(s)ds. \tag{15}$$

Taking the time derivative of (15) along solutions of system (9) gives

$$\begin{aligned}\dot{V}_i(t) - \alpha V_i(t) = {} & x^T(t)\left(P_iA_i + A_i^TP_i\right)x(t) + x^T(t)P_iA_{di}x\left(t-\tau(t)\right) + x^T\left(t-\tau(t)\right)A_{di}^TP_ix(t)\\ & + x^T(t)R_ix(t) - x^T(t-h)R_ix(t-h) + x^T(t)Q_ix(t) - (1-\hat{h})x^T\left(t-\tau(t)\right)Q_ix\left(t-\tau(t)\right)\\ & - \alpha x^T(t)P_ix(t) - \alpha\int_{t-h}^{t} x^T(s)R_ix(s)ds - \alpha\int_{t-\tau(t)}^{t} x^T(s)Q_ix(s)ds.\end{aligned}$$

By Jensen's Inequality, one has

$$-\alpha\int_{t-h}^{t} x^T(s)R_ix(s)ds \le -\frac{\alpha}{h}\left(\int_{t-h}^{t} x(s)ds\right)^T R_i\left(\int_{t-h}^{t} x(s)ds\right),$$

$$-\alpha\int_{t-\tau(t)}^{t} x^T(s)Q_ix(s)ds \le -\frac{\alpha}{h}\left(\int_{t-\tau(t)}^{t} x(s)ds\right)^T Q_i\left(\int_{t-\tau(t)}^{t} x(s)ds\right).$$

So, we have

$$\begin{aligned}\dot{V}_i(t) - \alpha V_i(t) \le {} & x^T(t)(P_iA_i + A_i^TP_i)x(t) + x^T(t)P_iA_{di}x(t-\tau(t))\\ & + x^T(t-\tau(t))A_{di}^TP_ix(t) + x^T(t)R_ix(t)\\ & - x^T(t-h)R_ix(t-h) + x^T(t)Q_ix(t) - (1-\hat{h})x^T(t-\tau(t))Q_ix(t-\tau(t))\\ & - \alpha x^T(t)P_ix(t) - \frac{\alpha}{h}\int_{t-h}^{t} x^T(s)dsR_i\int_{t-h}^{t} x(s)ds - \frac{\alpha}{h}\int_{t-\tau(t)}^{t} x^T(s)dsQ_i\int_{t-\tau(t)}^{t} x(s)ds\\ = {} & x^T(t)(P_iA_i + A_i^TP_i + R_i + Q_i - \alpha P_i)x(t) + x^T(t)P_iA_{di}x(t-\tau(t))\\ & + x^T(t-\tau(t))A_{di}^TP_ix(t) - x^T(t-h)R_ix(t-h)\\ & - x^T(t-\tau(t))(1-\hat{h})Q_ix(t-\tau(t)) - \frac{\alpha}{h}\int_{t-\tau(t)}^{t} x^T(s)dsQ_i\int_{t-\tau(t)}^{t} x(s)ds\\ & - \frac{\alpha}{h}\int_{t-h}^{t} x^T(s)dsR_i\int_{t-h}^{t} x(s)ds\\ = {} & \xi^T(t)\Sigma_i\xi(t),\end{aligned} \tag{16}$$

where

$$\xi(t) = [x^T(t), x^T(t-\tau(t)), x^T(t-h), \int_{t-h}^{t} x^T(s)ds, \int_{t-\tau(t)}^{t} x^T(s)ds]^T,$$

$$\Sigma_i = \begin{bmatrix} P_iA_i + A_i^TP_i + R_i + Q_i - \alpha P_i & P_iA_{di} & 0 & 0 & 0\\ * & -(1-\hat{h})Q_i & 0 & 0 & 0\\ * & * & -R_i & 0 & 0\\ * & * & * & -\frac{\alpha}{h}R_i & 0\\ * & * & * & * & -\frac{\alpha}{h}Q_i \end{bmatrix}.$$

From (11), we can obtain

$$\dot{V}_i(t) - \alpha V_i(t) < 0.$$

According to (12), (13) and (15), assume that $\sigma(t_k) = i, \sigma(t_k^-) = j,$ we have

$$V_i(t_k) \le \mu V_j(t_k^-), \tag{17}$$

For any $t \in (0,T),$ let N indicate the switching number of $\sigma(t)$ over $(0,T).$ By iterative computation, we can further obtain

$$\begin{aligned}V(t) & < e^{\alpha(t-t_k)}V(t_k)\\ & \le \mu e^{\alpha(t-t_k)}V(t_k^-)\\ & \le \cdots\\ & \le \mu^N e^{\alpha T}V(0).\end{aligned}$$

Recalling that $N \le T/\tau_a,$ so we have

$$V(t) < \mu^{\frac{T}{\tau_a}}e^{\alpha T}V(0). \tag{18}$$

On the other hand,

$$V(t) \ge \lambda_{\min}(\overline{P}_i)x^T(t)Rx(t) = \lambda_1 x^T(t)Rx(t), \tag{19}$$

$$\begin{aligned} V(0) &= x^T(0)P_i x(0) + \int_{-h}^{0} x^T(s)R_i x(s)ds + \int_{-\tau(0)}^{0} x^T(s)Q_i x(s)ds \\ &\le \lambda_{\max}(\overline{P}_i)x^T(0)Rx(0) + h\lambda_{\max}(\overline{R}_i)\sup_{-h\le\theta\le 0}\left\{x^T(\theta)Rx(\theta)\right\} + h\lambda_{\max}(\overline{Q}_i)\sup_{-h\le\theta\le 0}\left\{x^T(\theta)Rx(\theta)\right\} \\ &\le (\lambda_2 + h\lambda_3 + h\lambda_4)c_1, \end{aligned} \tag{20}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4,$ satisfied (10). Combine (18), (19) with (20), we can get

$$\begin{aligned} x^T(t)Rx(t) &\le \frac{V(t)}{\lambda_1} \\ &< \frac{\mu^{\frac{T}{\tau_a}} e^{\alpha T} V(0)}{\lambda_1} \\ &< \frac{\mu^{\frac{T}{\tau_a}} e^{\alpha T}(\lambda_2 + h\lambda_3 + h\lambda_4)c_1}{\lambda_1}. \end{aligned} \tag{21}$$

If $\mu = 1,$ according (13)

$$\begin{aligned} x^T(t)Rx(t) &\le \frac{V(t)}{\lambda_1} \\ &< \frac{e^{\alpha T}(\lambda_2 + h\lambda_3 + h\lambda_4)c_1}{\lambda_1} \\ &< c_2. \end{aligned} \tag{22}$$

If $\mu > 1,$ according (14)

$$\frac{T}{\tau_a} < \frac{\ln(\lambda_1 c_2) - \ln((\lambda_2 + h\lambda_3 + h\lambda_4)c_1) - \alpha T}{\ln \mu} \tag{23}$$

Substituting (23) into (21) yields

$$x^T(t)Rx(t) < \frac{(\lambda_2 + h\lambda_3 + h\lambda_4)c_1}{\lambda_1} e^{\alpha T} \frac{\lambda_1 c_2}{(\lambda_2 + h\lambda_3 + h\lambda_4 c_1)} e^{-\alpha T} = c_2 \tag{24}$$

According to Definition 2, the switched delay system (9) is finite-time stable with respect to (c_1, c_2, T, R, σ). The proof is completed here.

Theorem 2. For given a matrix $R > 0,$ and positive scalars $c_1 \le c_2, T,$ the system (8) is finite-time stable with respect to (c_1, c_2, T, R, σ) for any switching signal $\sigma(t)$ with average dwell time τ_a satisfying (14), if there exist positive definite symmetric matrices $P_i, R_i, Q_i,$ with appropriate dimensions for each $i \in M,$ and positive scalars $\alpha, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \mu \ge 1,$ such that (12), (13) and the following inequality hold:

$$\Gamma_i = \begin{bmatrix} \Gamma_i^{11} & P_i A_{di} & 0 & 0 & 0 & B_i Y_i \\ * & -(1-\hat{h})Q_i & 0 & 0 & 0 & 0 \\ * & * & -R_i & 0 & 0 & 0 \\ * & * & * & -\frac{\alpha}{h}R_i & 0 & 0 \\ * & * & * & * & -\frac{\alpha}{h}Q_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{25}$$

where

$$\Gamma_i^{11} = P_i A_i + A_i^T P_i + R_i + Q_i - \alpha P_i + B_i Y_i + Y_i^T B_i^T + \rho I$$

$$P_i = V_i^T \begin{bmatrix} \hat{P}_{11i} & \\ & \hat{P}_{22i} \end{bmatrix} V_i, \quad P_i B_i = B_i \hat{P}_i.$$

Furthermore, the controller gains are given by $K_i = \hat{P}_i^{-1} Y_i, \quad \forall i \in M.$

Proof: Consider Lyapunov like function (15). Taking the time derivative of (15) along solutions of system (8) gives

$$\dot{V}_i(t) - \alpha V_i(t) \le \xi^T(t)\Sigma_i \xi(t) + x^T(t)\left(P_i B_i K_i + K_i^T B_i^T P_i^T\right)x(t) + x^T(t)P_i B_i K_i e(t) + e^T(t)K_i^T B_i^T P_i x(t), \tag{26}$$

where Σ_i is give by (11).

According event-triggered condition (6), and Lemma 3, we can get

$$\begin{aligned}\dot{V}_i(t)-\alpha V_i(t) &\le \xi^T(t)\Sigma_i\xi(t)+x^T(t)\left(P_iB_iK_i+K_i^TB_i^TP_i^T+P_iB_iK_iK_i^TB_i^TP_i+\rho I\right)x(t)\\ &=\xi^T(t)\bar{\Sigma}_i\xi(t),\end{aligned} \tag{27}$$

where

$$\bar{\Sigma}_i=\begin{bmatrix}\bar{\Sigma}_i^{11} & P_iA_{di} & 0 & 0 & 0\\ 0 & -(1-\hat{h})Q_i & 0 & 0 & 0\\ 0 & 0 & -R_i & 0 & 0\\ 0 & 0 & 0 & -\dfrac{\alpha}{h}R_i & 0\\ 0 & 0 & 0 & 0 & -\dfrac{\alpha}{h}Q_i\end{bmatrix}, \tag{28}$$

$$\bar{\Sigma}_i^{11}=P_iA_i+A_i^TP_i+R_i+Q_i-\alpha P_i+P_iB_iK_i+K_i^TB_i^TP_i+P_iB_iK_iK_i^TB_i^TP_i+\rho I,$$

Let $Y_i=\hat{P}_iK_i$. Using Lemma 4, $P_iB_i=B_i\hat{P}_i$, and (25) , we get

$\bar{\Sigma}_i<0.$

Other proofs are similar to those of Theorem 1, which is omitted here.

4. Numerical Example

In this section, a numerical example is given to illustrate the effectiveness of proposed Theorem.

Consider system (1) with two subsystems, and system matrix parameters are

$$A_1=\begin{bmatrix}-0.02 & 0.03\\ -0.04 & 0.02\end{bmatrix}, A_{d1}=\begin{bmatrix}0.05 & 0.06\\ -0.02 & -0.04\end{bmatrix}, B_1=\begin{bmatrix}0.01 & 0\\ 0 & 0.04\end{bmatrix},$$

$$A_2=\begin{bmatrix}-0.04 & 0.02\\ -0.2 & 0.03\end{bmatrix}, A_{d2}=\begin{bmatrix}0.02 & 0.05\\ -0.04 & -0.03\end{bmatrix}, B_2=\begin{bmatrix}0.02 & 0\\ 0 & 0.03\end{bmatrix},$$

The values of other parameters are given as follows:

$c_1=0.1, c_2=30, T=10, R=I, \alpha=0.5, h=0.5, \hat{h}=0.2, \rho=0.1, \mu=1.02, \tau(t)=\left|0.1\sin t\right|.$

Solving inequalities (12), (13) and (25), we obtain the following controller gains:

$$K_1=\begin{bmatrix}-42.0815 & 2.7827\\ 0.6618 & -11.5558\end{bmatrix}, K_2=\begin{bmatrix}-18.8503 & 6.7602\\ 4.6727 & -18.7314\end{bmatrix}.$$

Then, according to condition (14), $\tau_a>\tau_a^*=1.9011.$ We chose $\tau_a=2.$ According Theorem 2, the system (8) is finite-time stable with respect to $(c_1,c_2,T,R,\sigma).$ The switching signals of controlled system is shown in Fig. 1. Fig. 2. depicts triggered instants. The system state is shown in Fig. 3.

Fig-1. Switching signals

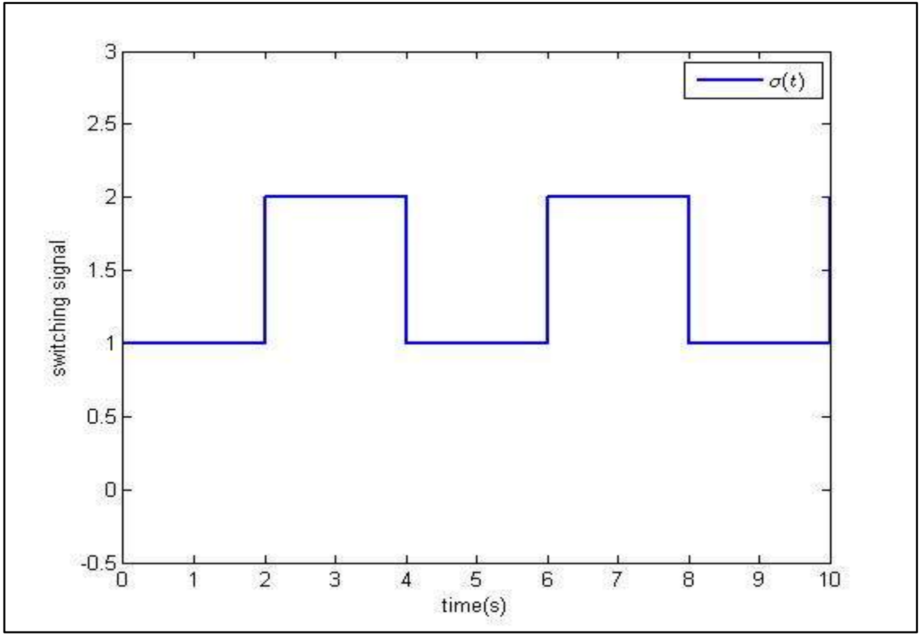


Fig-2. Event-triggered instants

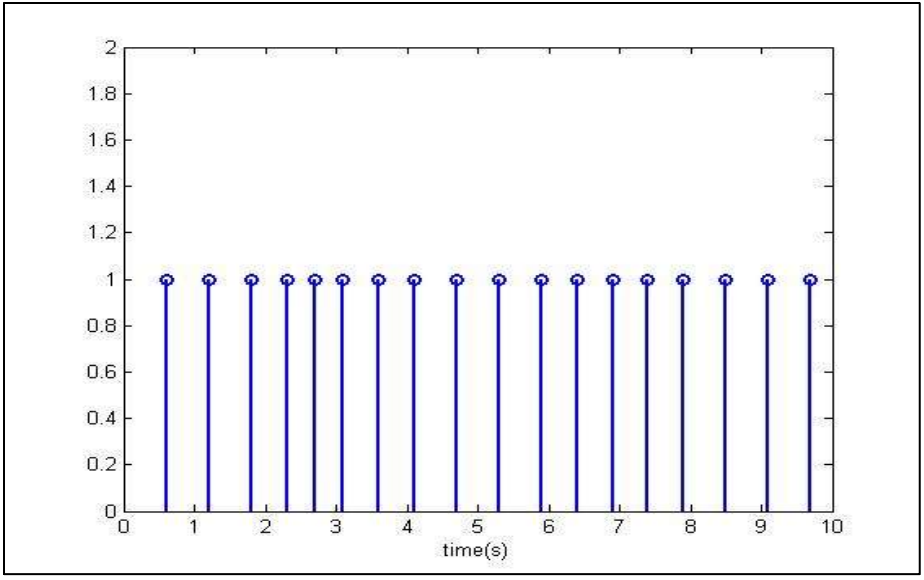
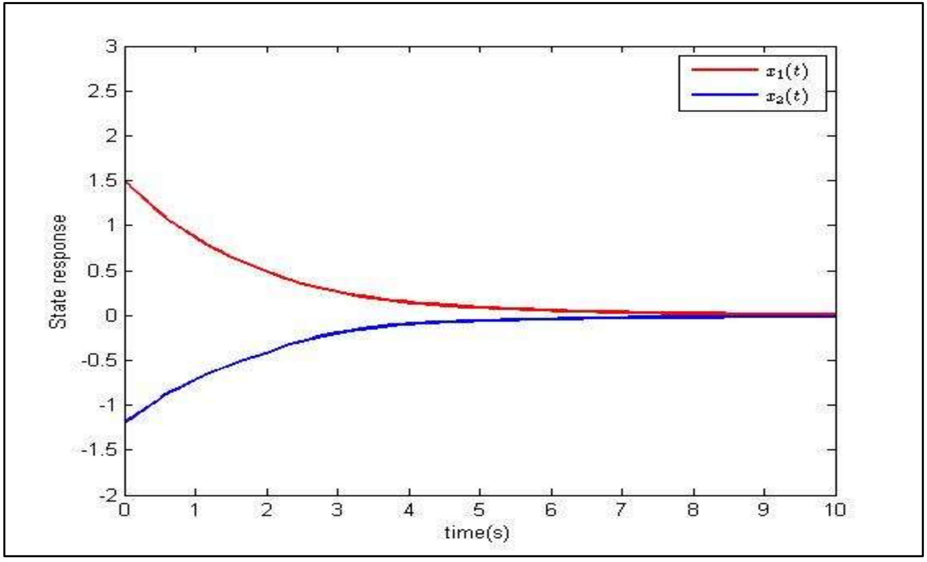


Fig-3. System state responses



5. Conclusion

In this paper, we propose an event-triggered sampling mechanism and a state feedback control for switched linear time-varying delay systems. Different from time-triggered control systems, event-triggered control systems will not be updated until some error signal exceeds a well-set threshold. Sufficient conditions have been formed to guarantee the finite-time stabilization of the switched delay systems. Finally, a numerical example has been given to verify the effectiveness of proposed Theorem.

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References

[1] Kundu, A. and Chatterjee, D., 2015. "Stabilizing switching signals for switched systems." *IEEE Trans. Autom, Control*, vol. 60, pp. 882-888.

[2] Sun, X. M., Fu, J., Sun, H. F., and Zhao, J., 2005. "Stability of linear switched neutral delay systems." *Proc. Chin. Soc. Elect. Eng.*, vol. 25, pp. 42–46.

[3] Wang, Zhao, J., and Jiang, B., 2013. "Stabilization of a class of switched linear neutral systems under asynchronous switching." *IEEE Trans. Autom. Control.*, vol. 58, pp. 2114-2119.

[4] Phat, V. N. and Ratchagit, K., 2011. "Stability and stabilization of switched linear discrete-time systems with interval time-varying delay." *Nonlinear Anal., Hybrid Syst.*, vol. 5, pp. 605-612.

[5] Wang, Sun, H., and Zong, G., 2016. "Stability analysis of switched delay systems with all subsystems unstable." *Int. J. Control Autom. Syst.*, vol. 14, pp. 1262-1269.

[6] Wang, Shi, X., Wang, G., and Zuo, Z., 2012. "Finite-time stability for continuous-time switched systems in the presence of impulse effects." *IET Control Theory Appl.*, vol. 6, pp. 1741-1744.

[7] Xiang, W. and Xiao, J., 2013. "Finite-time stability and stabilisation for switched linear systems." *International Journal of Systems Science*, vol. 44, pp. 384-400.

[8] Yang, X. Y., Tian, Y. J., and Li, X. D., 2018. "Finite-time boundedness and stabilization of uncertain switched delayed neural networks of neutral type." *Neurocomputing*, vol. 314, pp. 468-478.

[9] Tallapragada, P. and Chopra, N., 2013. "On event triggered tracking for nonlinear systems." *IEEE Trans. Autom. Control*, vol. 58, pp. 2343-2348.

[10] Liu and Jiang, Z. P., 2019. "Event-triggered control of nonlinear systems with state quantization." *IEEE Trans. Autom. Control*, vol. 64, pp. 797-803.

[11] Liu, Yu, X. H., Ma, G. Q., and Xi, H. S., 2016. "On sliding mode control for networked control systems with semi-Markovian switching and random sensor delays." *Inf. Sci.*, pp. 44-58.

[12] Lin, X., Du, H., and Li, S., 2013. "Finite-time stability and finite-time weighted L2-gain analysis for switched systems with time-varying delay." *IET Control Theory Appl.*, vol. 7, pp. 1058-1069.