Academic Journal of Applied Mathematical Sciences
ISSN(e): 2415-2188, ISSN(p): 2415-5225
Vol. 6, Issue. 6, pp: 58-65, 2020
URL: https://arpgweb.com/journal/journal/17
DOI: https://doi.org/10.32861/ajams.66.58.65

Academic Research Publishing Group

Original Research **Open Access**

Spectral Features of Systems With Chaotic Dynamics

Perevoznikov E. N.
Candidate of physical and mathematical sciences, docent, Mozhaisky Military Space Academy, Saint-Petersburg, Russia
Email: eperevoznikov@yandex.ru

Article History
Received: May 10, 2020
Revised: June 12, 2020
Accepted: June 17,2020
Published: June 20, 2020

Abstract
Using the Lorentz model and Hamiltonian systems without dissipation as an example, spectral methods for analyzing the dynamics of systems with chaotic behavior are considered. The insufficiency of the traditional approach to the study of perturbation dynamics based on an analysis of the roots of the classical spectral equation is discussed. It is proposed to study nonlinear systems using the method of constructing spectral equations with different eigenvalues, which allows one to take into account the randomness and multiplicity of states. The spectral features of instability and chaos for systems without dissipation are shown by the example of short-wave perturbations of a flow of a weakly ionized plasma gas.
Keywords: Systems with chaotic dynamics; Instabilities of non-dissipative systems; Spectral analysis methods.

1. Introduction
A significant number of publications devoted to nonlinear dynamical systems with complex chaotic dynamics and various transient processes in them speaks of the relentless interest in these systems and methods for describing them [1-7].

One of the main methods for analyzing the complex dynamics and conditions of chaos is the calculation of Lyapunov indicators [3, 4]. Determining Lyapunov indicators is not an easy task, mainly implemented by numerical methods. In this article, the estimated spectral methods — the classic one and proposed modification of it — the method of various eigenvalues (VE) are considered. The capabilities of the methods are compared on a well-known model with chaotic dynamics - the Lorentz model.

As a second example of a system with chaotic dynamics, author considers systems without dissipation, which are characterized by a high degree of nonequilibrium and in which instabilities and chaos are also associated with spectrum features. An example of such a system is the flow of a weakly ionized gas plasma in the approximation of high velocities and gradients, accompanied by various instabilities. Flows of weakly ionized plasma are formed upon the expiration of combustion products in the chambers of jet engines and were considered using various models in a number of sientific works [8-11].

2. Classic Spectral Stability Analysis Method
Let the equations (1), describing the system studied for stability be a totality of nonlinear autonomous equations

$$d_t a_\alpha \equiv \frac{da_\alpha}{dt} = F_\alpha\left(\{a_\alpha\}\right) \tag{1}$$

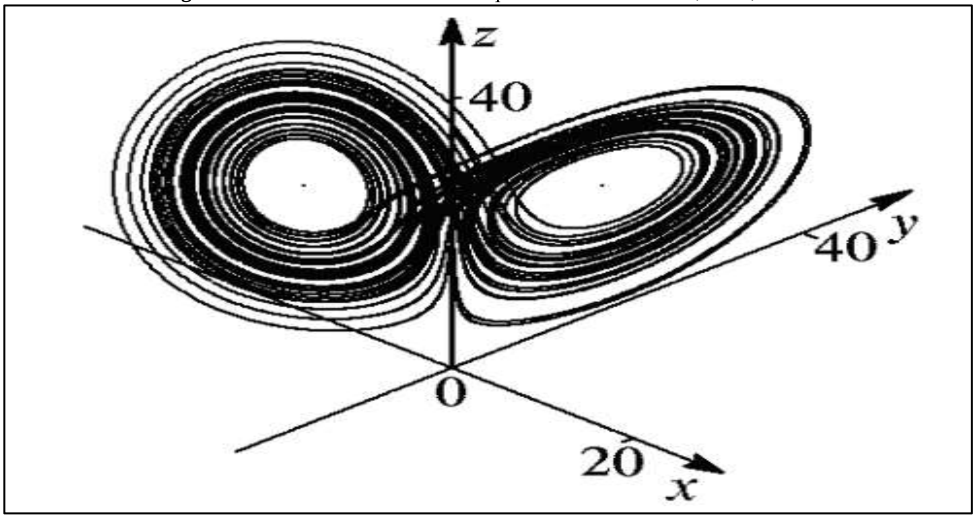
The dynamics of perturbations of system (1) in this case is described by the equations

$$d_t \delta a_\alpha = \sum_\beta \frac{\delta F_\alpha}{\delta a_\beta} \cdot \delta a_\beta \equiv e_{\alpha\beta} \delta a_\beta \quad , \quad \alpha, \beta = 1 \ldots n. \tag{2}$$

where $e_{\alpha\beta}\left(\{a_\alpha(t)\}\right)$ - elements of the evolutionary matrix $E\{e_{\alpha\beta}\}$, depending on dynamic variables $\{a_\alpha\}$ and time t.

If all time derivatives in (1) are negative, then the perturbations decay and the system is Lyapunov stable. If at least one positive derivative exists, then the phase trajectories scatter, the system is unstable. The ratio of the signs of temporary derivatives also makes it possible to determine the possibility of chaotic behavior and the formation of complex localized structures in the phase space - attractors [2-5] (see fig.1)

Fig-1. Lorentz attractor for "classical" parameter values σ=10,b=8/3, r=28.



Namely: if the signs of the eigenvalues (λ_1 , λ_2 , λ_3) - temporary derivatives (for three-dimensional systems) are

$$a)\quad Sgn(\lambda_1, \lambda_2, \lambda_3) \Rightarrow (-,-,-)$$

$$b)\quad Sgn(\lambda_1, \lambda_2, \lambda_3) \Rightarrow (0,-,-) \quad , \quad (3)$$

$$c)\quad Sgn(\lambda_1, \lambda_2, \lambda_3) \Rightarrow (-,0,+)$$

then the dynamic mode has a character, accordingly

a – sustainable point;

b – limit cycle;

c – attractor (chaotic dynamics).

I.e., chaotic behavior is characterized by the presence of a special saddle-focus point in the spectrum and is expressed in irregular unstable oscillations.

Chaos is divided into dissipative and active. Dissipative chaos is associated with the presence of attracting centers in the phase plane and a decrease in the phase volume of the system (for example, chaos in the Lorentz model), with active chaos the phase trajectories scatter and the phase volume increases (see the Rössler model [4]).

The condition of dissipativity-activity of chaos is the divergence of the velocity vector of the -system $\vec{B}(\partial_t x, \partial_t y, \partial_t z)$ at the same time being one of the conditions of stability

$$divB = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \quad (4)$$

If $divB<0$ phase volume decreases, trajectories approach each other - dissipative chaos. If $divB>0$ then the phase volume increases, the paths scatter - the system is active. Condition (4) reduces to the condition

$$divB \rightarrow \sum \lambda_i ,$$

where λ_i - eigenvalues.

According to the classical spectral method, the signs of temporary derivatives in system (2) correspond to the signs of the roots of the spectral equation - (SE),

which is a solvability condition for system (2) and has the form of a polynomial relative to the spectral parameter λ

$$D = \det\left[\delta_{\alpha,\beta}\lambda - E_{\alpha,\beta}\right] = \lambda^n + \sum_{i=1}^{n} a_i \cdot \lambda^{n-i} = 0 \quad (5)$$

Therefore, classical spectral analysis reduces to an analysis of the signs of the roots of the spectral equation. Equations for the eigenvectors of the evolutionary matrix $\{\chi_i(\lambda_k)\}$ and solutions of system (2), in this case, accordingly, have the form

$$(\delta_{im}\lambda_k - e_{im}) \cdot \chi_{im}(\lambda_k) = 0$$

$$a_i = C_m \cdot \chi_{im}(\lambda_k) \cdot \exp(\lambda_k t) \quad (6)$$

In works [12, 13] a number of methods are proposed for practical analysis of the stability of systems described by equations (1) demonstrated on tasks with different types of dynamics including chaotic. The essence of these methods is to determine by the coefficients of the dynamic equations (2) or by the coefficients of the spectral equation (4) the neutral surface - the boundary dividing the stability and instability regions in the parameter space.

Note the NSE method used below.

a) The NSE method (neutrality, separation, exclusion)

is based on spectral equation (4) and is implemented by the following scheme

($\lambda = Re\,\lambda + i\omega,\ \Delta\lambda = \lambda - \lambda_{cr},\ \Delta a_s = a_s - a_{s,cr}$)

$$\left\{\begin{array}{l} D(\lambda, \mathrm{p}, a_s) = 0 \\ \lambda_{cr} = i\,\omega_{cr} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} D_1(\omega, \mathrm{p}, a_s) = 0 \\ D_2(\omega, \mathrm{p}, a_s) = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} S(\mathrm{p}, a_s) = 0 \\ \omega_{cr} = \omega(\mathrm{p}, a_s) \end{array}\right\} \Rightarrow \mathrm{Re}(\Delta\lambda) = -\left[\frac{\partial_a D}{\partial_z D}\right]_{cr} \Delta a_s \ \frac{>^{НУ} 0}{<^{У} 0}$$

(7.1) , (7.2) , (7.3) . (7.4)

(7.1) – neutrality - the condition that the real part of the roots of SE be equal to zero;

(7,2) – separation of the spectral equation into two under the condition of neutrality;

(7.3) – exclusion of frequency or one of the parameters from these equations $\{p\}$ and getting a neutral surface – $S(p,a_s)$ and critical frequency – $\omega_{cr}(p,a_s)$;

(7.4)- indication of areas of stability - instability relative to a neutral surface.

The NSE scheme is fully implemented for polynomial SE, the general conditions of neutrality (7.3) in this case have the form

$$S(\mathrm{p}, a_s) = R\left[\, D_1(\omega, \mathrm{p}, a_s), D_2(\omega, \mathrm{p}, a_s) \,\right] = 0$$
$$\omega - \omega_{cr} = NOD\left[\, D_1(\omega, \mathrm{p}, a_s), D_2(\omega, \mathrm{p}, a_s) \,\right] = 0$$

where R – resultant, a NOD – largest common polynomial divisor of D_1 and D_2 .

A special case is the "Hamiltonian" systems (systems without dissipation), the spectral equation (5) of which has the form:

$$D(\lambda) = \lambda^n + ib_1\lambda^{n-1} + a_2\lambda^{n-2} + ib_3\lambda^{n-3} + \ldots = 0 \qquad (8)$$

and the roots are purely imaginary, or pairwise symmetrical about the imaginary axis.

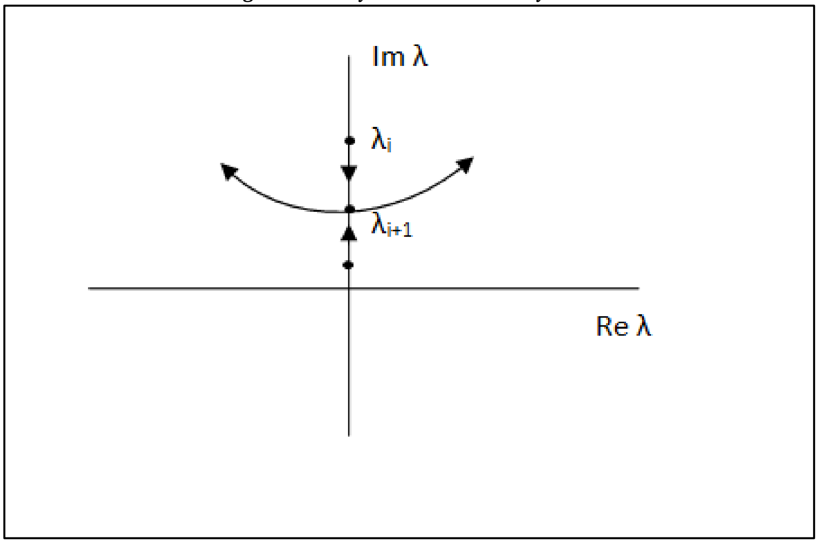
In this case, purely imaginary roots of SE correspond to stable states, and the point of change of stability (i.e., the condition of neutrality) is the fusion of pairs of imaginary roots and the appearance of roots with symmetrical real parts i.e. the appearance of a saddle-focus (see fig. 2). I.e. the instabilities of such systems always have the character of chaotic oscillations.

To determine the neutrality conditions in this case, a modified scheme of the NSE method is used. The modification consists in taking into account the frequency multiplicity at the neutral point i.e. replacing equations (7.2) in the NSE scheme (7) with equations

$$D_1(\omega) = (-i)^n D(i\omega, p) = 0$$
$$D_2(\omega) = \partial D_1/\partial\omega = 0 \qquad (9)$$

Thus, the scheme (7) in which the equations for D_1 and D_2 have the form (9), is a modification of the NSE method for Hamiltonian systems.

Fig-2. Neutrality of "Hamiltonian" systems



The classical approach to spectral analysis is well applicable for linear systems (they can be reduced to one higher-order equation with one spectral parameter value). However, the application of the classical spectral method to nonlinear systems, especially to systems with chaotic dynamics, raises a number of questions:

a) n-dimensional systems with chaotic dynamics are characterized by n number of real parameters - Lyapunov indicators, while the spectral method gives a smaller amount (for example, for three systems with chaos - three Lyapunov indicators and, according to the spectral method, one real root and two complex conjugate i.e. two real parameter);

b) it follows from (6) that all components of the eigenvector are equally dependent on time, while in reality the behavior of the components is different. In traditional analysis, this is partially removed by a linear combination of eigenvectors (6);

c) the classical approach indicates only the boundary of stability-instability (including the transition to chaos), but in reality there is an alternation of regions of regular and chaotic dynamics, i.e. several different dynamic modes.

The above indicates the insufficiency of the spectral method (in the traditional version) for describing systems with chaotic dynamics and the need for its modernization.

Note that the L-criterion method [13] describes the dynamics of n - dimensional systems by n - real parameters.

To describe systems with chaotic dynamics, we formulate a method of various eigenvalues – VE.

3. The VE Method

First, we note the following: it is known that third-order systems reflect the main features of the dynamics of nonlinear systems, so further we will only talk about them.

Suppose that in the equation for perturbations (2) the temporal derivatives are characterized by different spectral parameters λ, then the spectral equation for the three systems takes the form

$$\det\left[\delta_{ik}\lambda_i - e_{ik}\right] = \lambda_1\lambda_2\lambda_3 - \left(\lambda_1\lambda_3 e_{22} + \lambda_1\lambda_2 e_{33} + \lambda_3\lambda_2 e_{11}\right) + \sum_{i=1}^{3}\lambda_i A_i + D = 0$$

$$A_1 = e_{22}e_{33} - e_{23}e_{32}\ ,\ A_2 = e_{11}e_{33} - e_{13}e_{31}\ ,\ A_3 = e_{11}e_{22} - e_{12}e_{21}\ ,$$

$$D = e_{11}\left(e_{23}e_{32} - e_{22}e_{33}\right) + e_{33}e_{12}e_{21} + e_{22}e_{13}e_{31} - e_{12}e_{23}e_{31} - e_{13}e_{32}e_{21} \qquad (9)$$

Note that, the spectral equation in this case does not have the form of a polynomial relative to one spectral parameter, and is an algebraic function of three interdependent spectral parameters $-\lambda_i$.

Assuming for example, $\lambda_1 = Re\,\lambda_1 = \lambda\ ,\ \lambda_{2,3} = \alpha \pm i\omega$ (chaos is instability with a saddle-like focus and irregular oscilations) from SE (6) after extracting the real and imaginary parts of the equation, we obtain:

$$\left(\alpha^2 + \omega^2\right)\cdot\left(\lambda - e_{11}\right) - \lambda\alpha\left(e_{22} + e_{33}\right) + A_1\lambda + \alpha\left(A_2 + A_3\right) + D = 0$$

$$\omega\left[\lambda\left(e_{22} - e_{33}\right) + \left(A_2 - A_3\right)\right] = 0 \qquad (10)$$

From (10) for $\lambda_1,\ \alpha$ we have

$$\lambda_1 = \frac{A_3 - A_2}{e_{22} - e_{33}}\quad ,\quad \alpha_{2,3} = \frac{-M \pm \sqrt{M^2 - 4NG}}{2N}$$

$$N = \lambda_1 - e_{11}\ ,\mathrm{M} = -\lambda_1\left(e_{22} + e_{33}\right) + A_2 + A_3\ ,\mathrm{G} = \omega^2\left(\lambda_1 - e_{11}\right) + \lambda_1 A_1 + D \qquad (11)$$

Relations (11) give us three real parameters, two of which depend on the frequency - ω and limitation for ω - ($M^2 - 4NG \geq 0$). It is also possible to obtain similar (10, 11) equations for the cases $\lambda_2 = Re\,\lambda_2 = \lambda\ ,\ \lambda_3 = Re\,\lambda_3 = \lambda$. This gives an additional two more dynamic modes. The dependence of spectral parameters on frequency and additional dynamic modes form regions of instability and chaos.

Let us demonstrate these methods on the Lorentz model system (classical spectral and VE).

4. Lorentz Model

The Lorentz problem is interesting in that the dynamic equations of a number of real physical systems are reduced to nonlinear equations of the Lorentz model: convection in a liquid layer heated from below, a single-mode laser, a water wheel, etc. In addition, it clearly demonstrates the emergence of chaotic dynamics. The equations of the Lorentz model have the form Kuznetsov [4]; Shilnikov [7].

$$\partial_t x = \sigma\left(y - x\right)$$

$$\partial_t y = rx - y - xz$$

$$\partial_t z = -bz + xy \qquad (12)$$

Where *x,y,z* – dynamic quantities, σ, r, b - parameters, and the controlling parameter that plays the sense of intensity is the parameter $r > 0$.

System (12) has three stationary solutions - stationary states $(x_s, y_s, z_s)_1$, $(x_s, y_s, z_s)_2$, zero and two symmetric (see fig. 1 stationary points)

$$\begin{array}{lll} \partial_t x = 0 & x_s = 0 & x_s = y_s \\ \partial_t y = 0 \;\Rightarrow 1) & y_s = 0 \;\; ; \;\; 2,3) & x_s = \pm\sqrt{bz_s} = \pm\sqrt{b(r-1)} \\ \partial_t z = 0 & z_s = 0 & z_s = r-1 \end{array} \qquad (13)$$

Linearization of system (12) relative to the solution $(\tilde{x}, \tilde{y}, \tilde{z})$, for which any, including stationary, can be chosen, gives a system of equations for perturbations (2), where the evolution matrix is

$$\hat{E} = \begin{pmatrix} -\sigma & \sigma & 0 \\ -\tilde{z}_+ & -1 & -\tilde{x} \\ \tilde{y} & \tilde{x} & -b \end{pmatrix} \qquad (14)$$

The spectral equation and its coefficients in the stationary case are equal ($\tilde{z}_+ \equiv \tilde{z} - r$)

$$\det\left[\delta_{\alpha,\beta}\lambda - E_{\alpha,\beta}\right] = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 ;$$

$$a_1 = (1+b+\sigma) \quad , \quad a_2 = b + (1+b+z_+)\cdot\sigma + x_s^2 \quad , \quad a_3 = \sigma\left[x_s^2 + x_s y_s + b(1+z_+)\right] \qquad (15)$$

The NSE method for SE (15) gives two critical (neutral) modes

$$1) \quad \omega = 0 \quad , \quad a_3 = 0 \qquad (16)$$

$$2) \quad \omega^2 = a_2 \quad , \quad a_1 a_2 = a_3 \qquad (17)$$

Mode (16) is realized for the first stationary state (zero point) and corresponds to its instability at $r \geq 1$, eigenvalues (spectral parameters) in this case are equal

$$\lambda_1 = -b \quad , \quad \lambda_{2,3} = -\frac{\sigma+1}{2} \pm \sqrt{\left(\frac{\sigma+1}{2}\right)^2 + b(r-1)} \qquad (18)$$

Mode (17) is realized for the second and third stationary states and, for classical values of the Lorentz parameters, the critical values of frequency and parameter *r* are equal

$$\omega_{cr} = \sqrt{b(r+\sigma)} = 9.62 \quad , \quad r_{cr} = \frac{\sigma(3+b+\sigma)}{\sigma-1-b} = 24.7 \qquad (19)$$

At $r > r_{cr}$ the second and third stationary states (stationary points in fig. 1) become unstable, while ($r = r_{cr}$ $\lambda_1 = -13{,}7$, $\lambda_{2,3} = +0 \pm 9.62i$) the real part of the second and third eigenvalues becomes positive i.e. weakly growing chaotic oscillations appear. For example, when

$$r = 24.8 \;\Rightarrow\; \lambda_1 = -13.62 \quad , \quad \lambda_{2,3} = 1.902\cdot 10^{-3} \pm 9.636i .$$

The coefficients of the spectral equation by the VE method (9, 10) for the evolution matrix (14) are equal ($x_s^2 = \frac{8}{3}\cdot(r-1)$ see 13)

$$A_1 = \frac{8}{3} + x_s^2 \quad , \quad A_2 = \frac{80}{3} \quad , \quad A_3 = 0 \quad , \quad D = 20\cdot x_s^2 \qquad (20)$$

According to the VE method there are three dynamic modes in this system (10,11):

First dynamic mode ($\lambda_1 = Re\,\lambda_1 = \lambda$, $\lambda_{2,3} = \alpha_{2,3} \pm i\omega_{2,3}$)

1. $$\lambda_1 = -16 \quad , \quad \alpha_{2,3} = \frac{8 \pm \sqrt{64+1.5G}}{-3} \quad , \quad G = 4x_s^2 - 43 - 6\omega^2 \quad , \quad \omega_{cr} \leq \frac{4}{3}\sqrt{r-5} \qquad (21)$$

From the condition of neutrality ($\alpha_2 = 0 \to G = 0$)follows the expression for the critical frequency - ω_{cr}, and the frequency interval in this mode follows from materiality α and equal $\omega \in (0;8)$. Dissipativity condition $\sum \lambda_i < 0$

$$\lambda_1 + \alpha_2 + \alpha_3 = -16 - 2\cdot\frac{8}{3} = -\frac{40}{3}$$

in this mode is performed automatically. From the condition of materiality $\alpha_{2,3}$ follows the upper frequency limit

$$4+1.5G\geq 0 \quad \rightarrow \quad \omega_{\max}=\sqrt{\left(21+4x_s^2\right)/6}=\sqrt{\frac{31}{18}}\sqrt{1+\frac{32}{31}r}\,.$$

It follows from (21) that at $r>5$ oscillations with frequencies arise and amplify in the system $\omega\in\left(0\,;\omega_{\max}\right)$, and a combination of signs of eigenvalues corresponds to chaotic oscillations. The intervals of possible frequencies and the parameter r (which also include the boundary parameters according to classical analysis) indicate a certain set of states with chaotic oscillations.

Second dynamic mode ($\lambda_2=Re\,\lambda_2=\lambda$, $\lambda_{1,3}=\alpha_{1,3}\pm i\,\omega_{1,3}$). From formulas (9, 10) it follows

$$\lambda_2=\frac{3\cdot x_s^2+8}{22}\quad,\quad \alpha_{1,3}=\frac{-M\pm\sqrt{M^2-4NG}}{2N}\quad,\quad N=\frac{3(10+x_s^2)}{22}\quad,$$
$$M=\frac{30\left(x_s^2+8/3\right)}{11}\quad,\quad G=\frac{1}{22}\left[520x_s^2+640+3\omega^2\left(x_s^2+10\right)\right]\quad. \tag{22}$$

It follows from (22) that in the system for $r\in\left(1\,;53\right)$ there are unstable oscillations depending on the frequency, which also lies in a limited range, this corresponds to instability at r>1 in the classical analysis. Combination of signs of eigenvalues (+,-,-)_indicates the absence of chaos in this mode.

Third dynamic mode ($\lambda_3=Re\,\lambda_3=\lambda$, $\lambda_{1,2}=\alpha_{1,2}\pm i\,\omega_{1,2}$).From formulas (9,10 with 20 taken into account), we have

$$\lambda_3=\frac{x_s^2-24}{9}\quad,\quad \alpha_{1,2}=\frac{-M\pm\sqrt{M^2-4NG}}{2N}\quad,\quad N=\frac{1}{9}x_s^2\quad,$$
$$G=\frac{1}{9}(20+\omega^2)\,x_s^2\quad,\quad M=\frac{20}{9}x_s^2 \tag{23}$$

From (23) it can be seen that this regime is realized in the range $r\in\left(1\,;77\right)$ Instability occurs when $x_s^2>24 \quad \rightarrow r>9$ in the form of amplified oscillations in the frequency range $\omega\in\left(0\,;\sqrt{80}\right)$, moreover, low-frequency oscillations amplify faster. As in the second mode, the combination of the signs of the eigenvalues (+,-,-) indicates the absence of chaotic oscillations.

5. Hamiltonian Systems

As an example of a "Hamiltonian" system — a system without dissipation, we consider the flow of a weakly ionized gas plasma in the approximation of high velocities and gradients.

Flows of weakly ionized plasma are formed upon the expiration of combustion products in the chambers of jet engines and were considered using various models in a number of scientific works [8, 9, 11, 14].

In works [10, 14, 15], a three-fluid hydrodynamics model is used to describe the flow of a weakly ionized gas plasma consisting of interacting electronic, ionic, and neutral components. On the basis of which a system of equations of motion is constructed for component velocities, mass and charge density.

Also, in works [10, 14] results of a numerical flow study are presented, in Perevoznikov, *et al.* [15] flow stability is considered analytically. General activisation conditions under the influence of inhomogeneities, ionization, induced and external fields of various instabilities are obtained. In this paper, based on the results of [10, 14, 15], the flow dynamics in the presence of high-frequency, high-gradient $\left(\omega\,,k\gg 1\right)$ perturbations is considered, and the flow acquires the properties of "Hamiltonian" systems — systems without dissipation, and the spectral equation takes the form (8) (see. [15]

$$D(z,k)=\lambda^4+ik\left|A_1\right|\lambda^3+\left(ik\right)^2\left|A_2\right|\lambda^2+\left(ik\right)^3\left|A_3\right|\lambda+\left(ik\right)^4\left|A_4\right|=0 \tag{24}$$

$$|A_1| = 1.5(\tilde{U}_e + \tilde{U}_i) + \tilde{U}_s \quad ;$$

$$|A_2| = \left[\tilde{U}_e \tilde{U}_i + 0.5(\tilde{U}_e + \tilde{U}_i)\left(\tilde{U}_e + \tilde{U}_i + 3\tilde{U}_s\right) - 0.5\frac{\eta}{\alpha}\frac{m_a}{m_e}\tilde{T} \right] \quad ;$$

$$|A_3| = 0.5\tilde{U}_i\tilde{U}_e\left(\tilde{U}_e + \tilde{U}_i + 2\tilde{U}_s\right) + 0.5(\tilde{U}_e + \tilde{U}_i)^2\tilde{U}_s - 0.5\tilde{T}\left[\tilde{U}_e + \tilde{U}_s + \frac{\eta}{\alpha}\frac{m_a}{m_e}(\tilde{U}_e + \tilde{U}_i)\right] \quad ;$$

$$|A_4| = \tilde{U}_s\left[0.5\tilde{U}_i\tilde{U}_e(\tilde{U}_e + \tilde{U}_i) + 0.5 \quad \tilde{T}\left(\tilde{U}_e + \frac{\eta}{\alpha}\frac{m_a}{m_e}\tilde{U}_i\right)\right] \quad . \tag{25}$$

Where ($\tilde{\nabla} \Rightarrow ik$), k – Fourier transform parameter - dimensionless gradients, $\tilde{U}_{e,i,s}$ - dimensionless velocities of the electronic, ionic, and neutral components, $m_{e,a}$ – particle mass of electronic and neutral components, T – dimensionless temperature, α , η degree of ionization and relative electron concentration, accordingly [15].

Introducing a new variable $y = \lambda / ikU_s$, equation (24) is converted to the form

$$D(y) = y^4 + \tilde{A}_1 y^3 + \tilde{A}_2 y^2 + \tilde{A}_3 y + \tilde{A}_4 = 0 \tag{26}$$

where the coefficients $\tilde{A}_i = |A| / \tilde{U}_s$ accordingly equal

$$\tilde{A}_1 = 2.5 + 1.5\bar{U}_e \quad ; \quad \tilde{A}_2 = 2 + 3\bar{U}_e + 0.5\bar{U}_e^2 - 0.5\xi$$

$$\tilde{A}_3 = 1.5\bar{U}_e + \bar{U}_e^2 - 0.5\bar{U}_e\xi \quad ; \quad \tilde{A}_4 = 0.5\left(\bar{U}_e^2 + \xi\right) \quad , \tag{27}$$

And the notations of relative electronic velocity are introduced - $\bar{U}_e$ and relative temperature ξ

$$\bar{U}_e \equiv \tilde{U}_e / \tilde{U}_s \quad ; \quad \xi \equiv \frac{\eta}{\alpha}\frac{m_a}{m_e}\frac{\tilde{T}}{\tilde{U}_s^2} ,$$

and also taken into account that m_e / m_a <<1, and that in the absence of external fields and the proximity of the masses $m_i / m_a \approx 1$, the velocity of the particles of the ionic component differs little from the velocity of the particles of the neutral component and, accordingly, the mass one, i.e. $U_i / U_s \sim 1$.

Values of $\tilde{\lambda} = \lambda / k\tilde{U}_s = iy$, expressed through the roots of equation (26) depending on $\bar{U}_e$ and relative temperature ξ, are given in tables 1, 2.

Table-1. The dependence of the roots of the spectral equation on relative electronic speed $\bar{U}_e$

$\bar{U}_e / \xi$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
0.1/1	0.860-i1.49	-0.860-i1.49	0.429+i0.165	-0.429+i0.165
0.5/1	0.641-i1.61	-0.525-i1.6	0.538-i0.014	-0.538-i0.014
1/1	0.612-i1.79	-0.612-i1.79	0.612-i0.21	-0.612-i0.21
2/1	0.390-i2.21	-0.390-i2.21	0.636-i0.538	-0.636-i0.538
2.4/1	0.076-i2.41	-0.076-i2.41	0.613-i0.643	-0.613-i0.643
2.45/1	0-i2.55	0-i2.34	0.610-i0.655	-0.610-i0.655
5/1	0-i2.90	0-i5.20	0.351-i0.952	-0.351-i0.952

Table-2. The dependence of the roots of the spectral equation from relative temperature ξ

$\bar{U}_e / \xi$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
0.1/10	0.860-i2.358	-0.860-i2.358	i0.518	i1.549
0.1/8	0.871-i1.223	-0.871-i1.223	i0.566	i1.244
0.1/7	0.871-i2.161	-0.871-i2.161	i0.627	i 1.045
0.1/6.355	0.866-i2.113	-0.866-i2.113	i0.765	i 0.811
0.1/6.345	0.866-i2.112	-0.866-i2.112	i0.787+0.011	i0.787-0.011
0.1/5	0.762-i2.499	-0.762-i2.499	i0.874+0.152	i0.874-0.152

From tables 1, 2 it follows,

a) that in a stream of weakly ionized three-component plasma there are high-gradient, high-frequency, chaotic oscillations, depending both on the relative electron velocity and temperature.

At moderate temperatures and velocities, there are two types of chaotic oscillations that differ in frequency. With increase of temperature and electronic velocity, chaos disappears, first for some oscillations, then for others, passing into undamped oscillations of close frequencies in accordance with fig. 1.

b) at neutral points (stability changes) $(\overline{U}_e/\xi)_1 \in$ (2.4/1 – 2.54/1) и $(\overline{U}_e/\xi)_2 \in$ (0.1/6.3.55 – 0.1/6.345) the oscillation frequencies converge and are accordingly reduced to the points $\omega_{cr,1} =$ -2.41 $\omega_{cr,2} =$ 0.787.

6. Conclusion

- Proposed VE method gives a more complex picture of the dynamics in the Lorentz model than the classical spectral.
- Chaos exists in the first dynamic mode at lower thresholds values of r, than in classical linear theory.
- The presence of intervals of possible frequencies and parameters and the multiplicity of dynamic modes indicates the existence of regions of instability and chaos, that is actually has a place in reality.
- On the whole, the proposed method for studying the dynamics of nonlinear systems can be considered as additional to the methods based on calculations of Lyapunov indicators and classical spectral one.
- The example of dynamics of short-wave, high-frequency perturbations in a stream of weakly ionized gas plasma is used to consider the conditions for the instability of “Hamiltonian” systems — systems without dissipation.
- It is shown that the instability condition in this case is the merging of pairs of imaginary roots and the formation of saddle-focus points in the phase space. And instabilities in such systems are realized in the form of chaotic oscillations.
- The calculation of the plasma disturbance spectrum depending on the relative electronic velocity and temperature was given.
- And the results obtained in this consideration can be useful in studying the properties of weakly ionized plasma flows in high nonequilibrium states.

References

[1] Andronov, V. A., Witt, V. A., and Haikin, S. B., 1981. "Oscillation theory, M. Nauka." p. 587.
[2] Anischenko, V. S., 1990. "Complex oscillations in simple systems, M. Nauka." p. 312.
[3] Anischenko, V. S., 1997. *Attractors of dynamic systems* vol. 5. University News, applied nonlinear dynamics, pp. 109-127.
[4] Kuznetsov, S. P., 2006. "Dynamic chaos. M. Fizmatgiz." p. 355.
[5] Mun, F., 1990. "Chaotic oscillations, Mir." p. 312.
[6] Neymark, U. I. and Landa, P. S., 1987. "Stochastic and chaotic oscillations, M.Nauka." p. 424.
[7] Shilnikov, L. P., 1980. *Bifurcation theory and lorentz model, comp*. Bifurcation of Birth Cycle, M.Mir, p. 386.
[8] Comp, "Combustion Instability in LRE," ed. ed. by Harrier D.T.., Reedon F.G.// M. Mir, 1975, p. 869.
[9] Natanzon, M. S., 1986. "Combustion instability, M. Mashinostroenie." p. 292.
[10] Pinchuk, V. A., 1997. "Motor electrification as a development reflecting phenomenon of charge instability in the environment of combustion products at the expiration." *ZhTF,* vol. 62, pp. 21-26.
[11] Vatazhin, A. B., Grabovsky, V. I., Lichter, W. A., and Shulman, V. I., 1983. "Electro-gas-dynamic flows. M. Nauka." p. 345.
[12] Perevoznikov, E. N. and Mikhailova, O. A., 2015. "Neutrality criteria for stability analysis of dynamical systems through lorentz and rossler model problems." *Journal of Applied Mathematics and Physics,* vol. 03, pp. 569-576.
[13] Perevoznikov, E. N., 2006. *Stability analysis methods for nonequilibrium systems*. University News, Physics, pp. 34-39.
[14] Pinchuk, V. A., 1994. "Low-temperature plasma in external acoustic conditions impacts." *IFZh,* vol. 67, pp. 112-118.
[15] Perevoznikov, Pinchuk, V. A., and Shveiko, U. V., 2002. "Disturbance dynamics of weakly ionized plasma in the conditions of chambers of thermal energy converters." *IFZh,* vol. 75, pp. 14-18.