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1-D Optimum Path Problem and Its Application

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Abstract

The 1-D optimum path problem with two end-points fixed or one end-point fixed, the other end-point variable reduces to vector integral equations of Fredhom / Volterra type and is hard to solve. Translating it to scalar components equations would be an easier way of solving it. Here, the solution of the optimum path problem is recommended by connecting it with the Principle of minimum Energy Release (PMER). A lot of optimum path problems with path function $E=cu^2$, where E is the released energy, u is the velocity, c is constant, can be solved by PMER, e.g., the Great Earthquake, the denotation of a nuclear weapon, the strategy of sports games. The one end-point fixed, the other end-point variable is studied for wing moving. High lights: The pulse-mode of nuclear denotation releasing energy is the same as Earthquake, Yun [1], shows that the derivative of wind velocity with respect to time in proportion to the derivative of temperature with respect to the track. Which conforms with the weather forecast in winter that strong wind companies with low temperature for cold wave coming, and also suits for the motion of mushroom cloud [2].

Keywords: 1-D optimum path problem; Vector integral equation; Principle of minimum energy release (PMER).

1. Introduction

There are many 1-D (one dimension) optimum path problems existing in universal and diary life. For example, optimum path in logistics navigation, optimum path in military design-attacking target, optimum path in wind moving track, and typhoon track, etc.

The 1-D optimum path problem with constraint(s), usually, it can be changed to un-constraint problem by method of Lagrange multipliers [3]. Therefore we just study the un-constraint problem.

The optimum problem of integrand with given scalar function have been summery in mathematical hand books, e.g., [4].

There are many principles on energy relating to mechanical problems or relating to scientific problems. These principles have no connection with the optimum path problem.

This paper aims to set up relationship between the 1-D optimum path problem and the “Principle of Minimum Energy Release” (PMER), and studies its applications.

§2, the 1-D optimum path problem of two end-points fixed is reduced to a vector integral equation of Fredholm type. It is hard to solve, therefore, translating to 3-D scalar components is a better choice. Then, it is directly connected to PMER.

A lot of the 1-D optimum path problem with energy release as $E_k = cu^2$ (where E_k is the kinetic energy, c is a constant, u is the velocity) can be solved shown in §3.

§3Results

§**3.1** the application of Principle of Minimum Energy Release (PMER) to great Earthquake [5]. The Earthquake and the speed of Earth’s self-rotation are two problems. Scientists used statistic method to study the relationship between these two problems. While the author used conservation law of energy (CLE) to study these relationship [5], and proved that the energy release of Earthquake is proportioned to the square of velocity of the Earth’s rotation. Furthermore, the pulse-mode of great Earthquake had also been proved via PMER. Testing examples of Japan 2011-3-11earthquake, 1960 Great Chilean earthquake, were given.

§**3.2** the application of PMER to denotation of unclear weapon [2].

§**3.3** the application of PMER to strategy of sport games [5].

§**3.4** the optimum path problem of one end-point fix and the other end-point variable for cases of Cartesian coordinates is studied with the development of wind moving [1].

§4. Discussion

§5. Conclusion

2. Material and Method

2.1. The Optimum Path Problem of 1-D Two End-Points Fixed

The optimum path problem of 1-D (one variable) two end-points A and B fixed is reduced to an optimum of an vector integral equation of Fredholm type, i.e.,

$$\min_p \int_A^B p d\mathbf{s} = Y, \tag{2-1}$$

Where $p = f(\mathbf{s})$ is the path function, continuous or segmental continuous, A and B are two end-points of $p = f(\mathbf{s})$ is an unknown function, and $\mathbf{s}$ is a variable position vector.

Instead of (2-1), the above optimum path problem can also lead to a maximum of Y, i.e.,

$$\max_p \int_A^B p d\mathbf{s} = Y, \tag{2-2}$$

The stationary point of (2-1) or (2-2) is the same.

The physical meaning of the integral shown in (2-1), (2-2) is that if p represents a force (vector), then the integral represents the total work done by force on displacements from A to B.

Since the vector integral equation is hard to solved, while the solution of scalar integral equation is easier. Therefore, the 1-D vector integral equation is translated to its 3-D scalar components integral equations of Cartesian coordinates.

The 2-D optimum path problem is that where $p = f(s, t)$ is a 2-D unknown function, if t represents time, then the problem deduced to PDE.

The 2-D optimum path problem is not discuss here.

2.2. The 1-D Vector Integral is Translated to its Scalar Component Integral of Cartesian Coordinates

The relationship between scalar components in Cartesian coordinates and its position vector $\mathbf{s}$ is:

$$\mathbf{s} = (x, y, z) = s\mathbf{I} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \tag{2-3}$$

$$\mathbf{f} = (f_x, f_y, f_z) = f\mathbf{I} = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}, \tag{2-4}$$

where $\mathbf{I}$ is unit vector, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are components unit vectors, the **bold** face denotes **vector,** the non-bold face denotes scalars.

Rewrite (2-1) in vector form

$$\mathbf{Y} = \min_f \int_A^B \mathbf{f}(\mathbf{s}) d\mathbf{s} = \min_{(f_x\mathbf{i}+ f_y\mathbf{j}+f_z\mathbf{k})} \int_A^B (f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}) d(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}), \tag{2-5}$$

Since vector optimum problem is hard to solve, we use scalar components form to express the optimum problem.

The necessary condition for Y to be minimum, is:

$$\frac{\partial \mathbf{Y}}{\partial \mathbf{f}} = \frac{\partial (Y_x\mathbf{i} + Y_y\mathbf{j} + Y_z\mathbf{k})}{\partial \mathbf{f}} = \frac{\partial Y_x}{\partial \mathbf{f}}\mathbf{i} + \frac{\partial Y_y}{\partial \mathbf{f}}\mathbf{j} + \frac{\partial y_z}{\partial \mathbf{f}}\mathbf{k} =$$

$$\frac{\mathbf{i}}{\frac{\partial(f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k})}{\partial Y_x}} + \frac{\mathbf{j}}{\frac{\partial(f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k})}{\partial Y_y}} + \frac{\mathbf{k}}{\frac{\partial(f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k})}{\partial Y_z}} =$$

$$\frac{\partial Y_x}{\partial f_x} + \frac{\partial Y_y}{\partial f_y} + \frac{\partial Y_z}{\partial f_z} = 0. \tag{2-6}$$

Where

$$\mathbf{Y} = Y_x\mathbf{i} + Y_y\mathbf{j} + Y_z\mathbf{k}. \tag{2-7}$$

$$Y_x = Y.\cos\alpha_Y,\ Y_y = Y.\cos\beta_Y,\ Y_z = Y.\cos\gamma_Y, \tag{2-8}$$

$$Y_x^2 + Y_y^2 + Y_z^2 = Y^2\left[(\cos\alpha_y)^2 + (\cos\beta_Y)^2 + (\cos\gamma_Y)^2\right] = Y^2, \tag{2-9}$$

Where Y_x, Y_y, Y_z are the components (projection) of Y, $(\alpha_Y, \beta_Y, \gamma_Y)$ are the angles between vector $\mathbf{Y}$ and the x-, y-, z-axis, respectively.

$$f_x = f.\cos\alpha_f,\ f_y = f.\cos\beta_f,\ f_z = f.\cos\gamma_f, \tag{2-10}$$

$$f_x^2 + f_y^2 + f_z^2 = f^2[(\cos\alpha_f)^2 + (\cos\beta_f)^2 + (\cos\gamma_f)^2] = f^2, \tag{2-11}$$

Where f_x, f_y, f_z are the components (projection) of f, $(\alpha_f, \beta_f, \gamma_f)$ are the angles between vector $\mathbf{f}$ and the x-, y-, z-axis, respectively.$\mathbf{i}.\mathbf{i} = \mathbf{j}.\mathbf{j} = \mathbf{k}.\mathbf{k} = 1$, $\mathbf{i}.\mathbf{j} = \mathbf{i}.\mathbf{k} = \mathbf{j}.\mathbf{k} = 0$.

2.3. The 1-D Two End-Points Fixed Problem Directly Translating To Optimum Energy Release Problem With $E_r = cu^2$

Now, the 1-D two end-points fixed problem is directly transformed to optimum energy release problem and solved by PMER.

$$\text{Let } p = f(s) = E_r(u) = cu^2, \tag{2-12}$$

Where E_r is the energy release, u is the velocity, c is a constant. (2-12) represents those problems of energy release proportions to the square of the motion velocity. A lot of such cases, are studied in the following.

3. Results

3.1 Application of Principle of Minimum Energy Release (PMER) to Great Earthquake

The Earthquake and the speed of Earth's self-rotation are two problems. Scientists used statistic method to study the relationship between these two problems. While the author used conservation law of energy to study these relationship [5], and proved that the energy release by Earthquake is proportioned to the square of velocity of the Earth's rotation. Furthermore, the pulse-mode of great Earthquake has also been proved by the author via Principle of Minimum Energy Release (PMER).

Here, we repeated the derivation briefly [5].

The CLE states that the change of energy of an isolated system (the Earth) is zero. i.e.,

$$\Delta E = \Delta E_k + \Delta E_p + \Delta E_q = 0, \quad (3.1\text{-}1)$$

Where $\Delta E = E(t_1) - E(t_0)$, $\Delta E_k = E_k(t_1) - E_k(t_0)$, $\Delta E_p = E_p(t_1) - E_p(t_0)$, $\Delta E_q = E_q(t_1) - E_q(t_0)$ are the change of energy, kinetic energy, potential energy and heat and other energies of the Earth respectively, t_0 is the time just before Earthquake, t_1 is the time at Earthquake over.

$$\Delta E_k = \frac{1}{5}[\omega_c^2(t_1) - \omega_c^2(t_0)]R_e^2 m_e, \quad (3.1\text{-}2)$$

Where ω_c is the rotation angora velocity of the crust, R_e and m_e are the radius and mass of the Earth.

Since the potential energy E_p (including gravity-buoyancy potential energy E_g, deformation energy E_{def}, the heat and electromagnetic energy E_q) inside the crust are nearly impossible to calculated accurately. The author used K to represent these energy, i.e.,

$$K = \Delta E_k = \Delta E_g + \Delta E_{def} + \Delta E_q, \quad (3.1\text{-}3)$$

Note that the (3-15) of (Yun, 2019) ΔE_k is miss typed as ΔE_p, (3.13) is correct.

Substituting (3.1-3) into (3.1-2), we have

$$\omega_c^2(t_1) - \omega_c^2(t_0) = -\frac{5K}{R_e^2 m_e}, \quad (3.1\text{-}4)$$

If $K = 0$, then, $\omega_c(t_0) = \omega_c(t_1)$, it means no Earthquake happened.

If $K = -$, defines the system losing energy, then $\omega_c^2(t_1) > \omega_c^2(t_0)$, it means that Earthquake fastens the Earth self-rotation.

If $K = +$, defines the system absorbing energy.

(3.1-4) shows that the square of changing of Earth rotation velocity is proportion to the release of energy of the Earthquake.

How to measure or to calculate K? In Yun [5], K is calculated by formula of Hanks & Kanamon, a half empirical formula.

Now, we use PMER to prove the mode of energy release of Earthquake is a pulse-mode.

Construct a path function $p(t) = \omega_c^2(t)$, such that

$$Y = \int_{t_0}^{t_1} p(t)dt = \int_{t_0}^{t_1} \omega_c^2(t)dt, \quad (3.1\text{-}5)$$

There are many paths $p(t) = \omega_c^2$satisfying (3.1-5), For example, a straight line or a curve line passing through t_0 and t_1, etc. The actual one must be that with minimum energy release.

Proof: this is a optimum problem, i.e.,

$$\min_u Y = \min_u \int_{t_0}^{t_1} \omega_c^2(t)\, dt, \quad (3.1\text{-}6)$$

The necessary condition for this problem to be minimum is:

$$dY/d\omega_c = 0, \quad (3.1\text{-}7)$$

Then, we have

$$2\int_{t_0}^{t_1} \omega_c(t)\frac{d\omega_c(t)}{dt}dt = 0, \quad (3.18)$$

Since t_0 and t_1 can be arbitrary chosen, by Newton-Leibniz formula, the integrand of (3-8) must be zero while $\omega_c(t) \neq 0$, then, we have

$$\omega_c(t) = const = \omega_c(t_0), \quad (t_0 \le t < t_1) \quad (3.1\text{-}9)$$

$$\omega_c(t) \to \omega_c(t_1), \quad (t \to t_1) \quad (3.1\text{-}10)$$

Eqs. (3.1-9) and (3.1-10) show that $\omega_c(t)$ is a broken line for $(t_0 \le t < t_1)$, $\omega_c(t) = \omega_c(t_0)$; for $(t \to t_1)$, $\omega_c(t) \to \omega_c(t_1)$, That means that the energy release of a great Earthquake suddenly at a very short time interval $(t \to t_1)$ like a few seconds or minutes. i.e., the Earthquake happens in a pulse-mode. □

3.2. The Application of PMER to Denotation of Unclear Weapon [2].

Using model like hot air bloom with zero-weighted membrane wrapped hot air, surrounding by cold air, the paper [2] set up a partial differential equations (PDE) of motion of mushroom cloud by modifying Navier-Stockes (N-S) equations. It was a vector PDE. It stated that the derivative of velocity respect to time proportions to the gradient of temperature respect to trace. Its solution was obtained by method of separating of variables for scalar function. These results had been compared with well agreement with literature [1].

3.2.1. Combination of Boyle's Law, Chares' Law and Einstein Mass-Energy Equation

Since both dimensions of combination of Boyle's law and Chares' law and Einstein mass-energy equation were the same, (i.e., N.m or J), therefore, it can be combined these laws into one equation, i.e., :

$$pV = RT = E = \lim_{u \to c} mu^2, \quad (3.2\text{-}1)$$

where p, V, R, T, E, m, and u^2 represent the pressure, volume, constant, temperature, the total energy, and the square of velocity of the mushroom cloud, respectively.

3.2.2. Gravity-Buoyancy Field

In gravity-buoyancy field, gravity acts vertically on cold air; while buoyancy acts vertically and horizontally on hot air.

3.2.2.1. Potential Energy E_p

The capability of work done by position changing of object is called the potential energy

$$E_p = \iiint_V [-\rho_c g z + \rho_h (du_z/dt)z + 0 - \rho_h (du_r/dt)r]dv, \quad (3.2\text{-}2)$$

Where $dm_c = \rho_c dv$, $dm_h = \rho_h dv$, and ρ_h, ρ_c are the masses, and density of hot air and cold air of dv, respectively; g is the gravity acceleration (opposite to the displacement z); V is the volume of bloom; u_z and u_r are the components of the velocity u in z-direction and r-direction of the bloom, respectively; $\mathbf{u} = u_r\mathbf{e_r} + u_z\mathbf{K}$, or $u^2 = u_r^2 + u_z^2$. $E_p$is the potential energy of the bloom. Where the first term shows the work done by force on opposite direction of displacement and is defined as negative work. The second term shows the work done by force on same direction of displacement z. The third term 0 shows that gravity does not acting on the horizontal level of hot air. The fourth term shows the work done by horizontal buoyancy, which against the direction of horizontal displacement due to denotation.

3.2.2.2. The kinetic energy E_k

$$E_k = (1/2)(m_c + m_h)u^2, \quad (3.2\text{-}3)$$

3.2.2.3. The heat and other energies E_T

$$E_T = E - E_p - E_k = E_k - E_p, \quad (3.2\text{-}4)$$

3.3. The Principle of Minimum Energy Release (PMER) for Optimum Path Problem

The (PMER) states that a path connected points A and B, and an object goes from A to B with releasing energy proportioning to the square of velocity on the path; there are many possible paths to reach B from A, the actual one path that carried out (or the best) is that which releases minimum total energy.

Now we use PMER to prove **the mode of energy release by nuclear fusion is a pulse-mode.**

Rewrite (3.1-5) as

$$Y = \Delta E = m(u^2(t_1) - u^2(t_0)), \quad (3.2\text{-}5)$$

Where Y represents energy release, $u \to c$ is the velocity, t_1 and t_0 are the time of end and beginning of the explosion respectively.

Construction a paths function $p(t) = mu^2(t)$, such that

$$Y = \int_{t_0}^{t_1} p(t)\,dt = \int_{t_0}^{t_1} mu^2(t)\,dt, \quad (3.2\text{-}6)$$

There are many paths $p(t)$ satisfying (3.2-6). For example, a straight line or a curve line from t_0 to t_1. The actual one must be that with minimum energy release.

Proof :

This is an optimization problem, i.e.,

$$\min_p Y = \min_u Y = \min_u \int_{t_0}^{t_1} mu^2(t)\,dt, \quad (3.2\text{-}7)$$

The necessary condition for the problem to be minimum is:

$$dY/du = 0, \quad (3.2\text{-}8)$$

Then, we have

$$2m\int_{t_0}^{t_1} u(t)\,dt = 0, \quad (3.2\text{-}9)$$

Since $t_0 \neq t_1$, and t_1 can be arbitrary chosen, by Newton-Leibniz formula, the integrand must be zero, then, we have:

$$u(t) = 0, (t_0 \le t < t_1) \text{ and } u(t_1) = c, \quad (3.2\text{-}10)$$

Eq. (2.5-6)[3] shows that the path is a broken line, $u(t) = 0$, $(t_0 \le t < t_1)$ and $u(t_1) = c$. This is a pulse-mode, the velocity suddenly increases to infinity at $t = t_1$. Like a great earthquakes releasing energy in a very short time (Yun, 2019), the detonation releasing energy is also in a pulse-mode. □

High lights:

1. The pulse-mode of nuclear denotation releasing energy is the same as Earthquake,
2. Yun [1] shows that the derivative of wind velocity respect to time is proportion to the derivative of temperature respect to the track, also suits for motion of mushroom cloud [2], although the model, the based scientific laws, the derivation, the solution [6] of both problem are different.
3. The wind speed equation is un-visible, while the motion of mushroom cloud is visible, both equations and solutions can be used to check or compare each other and thus promoting the believable of calculation.

3.4.The Application of PMER to Strategy of Sport Games [5]

There are many sport games with energy release proportions to the square of velocity, for examples, the long-distance-running, long-distance-swimming, etc. The strategy of how to distribute the energy release in the process to obtain best results? This problem can be reduced to optimum path problem by setting the path function $p(t) = cu^2(t)$, then, we have

$$\min_u Y = \min_u \int_{t_0}^{t_1} cu^2(t)dt, \quad (3.3\text{-}1)$$

Similar to above (3.2-6) – (3.2-10), we have the solution

$$u(t) = u(t_0) = \text{const}, \quad (t_0 \le t < t_1) \quad (3.3\text{-}2)$$

$$u(t) \to u(t_1), \ (t \to t_1) \quad (3.3\text{-}3)$$

Eqs.(3.3-2), (3.3-3) show that the optimum path is a broken line, at first time t_0 sports use constant velocity $u(t_0)$, until the last time t_1, sports use biggest velocity $u(t_1)$.

People usually seen that sports keep constant velocity in the early stage, while release all energy in the final stage. This strategy confirms that the Principle of Minimum Energy Release (PMER) is nice guiding for sports.

3.5. The 1-D Optimum Path Problem of one End-Point Fixed and the other End-Point Variable for Cases of Cartesian Coordinates

The 1-D optimum path problem of one end-point A fixed and the other end-point B variable is reduced to a vector integral equation of Volterra's type. Since the vector integral equation is hard to solve, therefore, it is translated to components scalar integral equations as shown in §**2.3**

In this section, the development of wind moving is studied. Let us recalled [1].

3.5.1. Wind Speed Equation of a Point (Mass) in Air [1]

The wind speed equation of a point (mass) in air is shown as:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{g} = k\,\frac{\partial \mathbf{T}}{\partial s}, \quad (3.4\text{-}1)$$

$$\frac{\partial \mathbf{u}}{\partial \mathbf{t}} = k\,\frac{\partial \mathbf{T}}{\partial \mathbf{s}}, \quad (3.4\text{-}1)_a$$

Where **u** is the vector of wind speed, **T** is the temperature vector, **s** is the position vector, **g** is the acceleration of gravity, $k = R/m$ is a constant, m is the point mass. (3.4-1) and $(3.4\text{-}1)_a$ are the vector form of wind speed equation and its approximate equation respectively.

Eq. (3.4-1) reveals that the derivative of wind speed respect to time plus a constant is proportional to the derivative of temperature respect to the trace. Eq. $(3.4\text{-}1)_a$ reveals that the derivative of wind speed respect to time is proportional to the derivative of temperature respect to the trace.

The wind speed equation (3.4-1) expressed in scalar component form is:

$$m\frac{\partial u_x}{\partial t} = R\frac{\partial T_x}{\partial x}, \quad (3.4\text{-}2)$$

$$m\frac{\partial u_y}{\partial t} = R\frac{\partial T_y}{\partial y}, \quad (3.4\text{-}3)$$

$$m\frac{\partial u_z}{\partial t} + mg = R\frac{\partial T_z}{\partial z}, \quad (3.4\text{-}4)$$

$$u_x = u\cos\alpha_u,\ u_y = u\cos\beta_u,\ u_z = u\cos\gamma_u, \quad (3.4\text{-}5)$$

$$\alpha_u^2 + \beta_u^2 + \gamma_u^2 = 1, \quad (3.4\text{-}6)$$

$$u^2 = u_x^2 + u_y^2 + u_z^2, \quad (3.4\text{-}7)$$

$$T_x = T\cos\alpha_T,\ T_y = T\cos\beta_T,\ T_z = T\cos\gamma_T, \quad (3.4\text{-}8)$$

$$\alpha_T^2 + \beta_T^2 + \gamma_T^2 = 1, \quad (3.4\text{-}9)$$

$$T^2 = T_x^2 + T_y^2 + T_z^2, \quad (3.4\text{-}10)$$

where u_x, u_y, u_z are the components of **u** in x-, y-, z-direction, T_x, T_y, T_z are the components of **T** in x-, y-, z-direction, respectively, m represents the mass of the point (mass) or a group of masses with its center moving the same trace.

Now, we connect the optimum path problem with wind speed equation.

$$\text{Let } \frac{\partial T_x}{\partial x} = f_x,\ \frac{\partial T_y}{\partial y} = f_y,\ \frac{\partial T_z}{\partial z} + g = f_z, \quad (3.4\text{-}11)$$

The optimum path problem in components form, by (2-6) is:

$$Y = \min_{(f_x\mathbf{i}+f_y,\mathbf{j}+f_z\mathbf{k})} \int_A^B \left(f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}\right) d(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}), \quad (3.4\text{-}12)$$

The necessary condition for Y to be minimum or maximum, by (2-7), is:

$$\frac{\partial Y_x}{\partial f_x} + \frac{\partial Y_y}{\partial f_y} + \frac{\partial Y_z}{\partial f_z} = 0, \quad (3.4\text{-}13)$$

Where f_x, f_y, f_z are path functions, passing through two end-points $(A_x = A_x(f_x),\ A_y = A_y(f_y), A_z = A_z(f_z))$, and $(B_x = B_x(f_x), B_y = B_y(f_y), B_z = B_z(f_z))$.

Let us looking at the first term of (6-13), where $Y_x = \int_{A_x}^{B_x} f_x dx$ is a compound function, using Newton-Leibniz formula and chain rule, we have:

$$\frac{\partial Y_x}{\partial f_x} = \frac{\partial}{\partial f_x}\int_{A_x}^{B_x} f_x\,dx = [B_x(f_x) - A_x(f_x)].[B_x]' = [B_x(f_x) - A_x(f_x)], \quad (3.4\text{-}14)$$

Similarly, we have:

$$\frac{\partial Y_y}{\partial f_y} = [B_y(f_y) - A_y(f_y)], \qquad (3.4\text{-}15)$$

$$\frac{\partial Y_z}{\partial f_z} = [B_z(f_z) - A_z(f_z)], \qquad (3.4\text{-}16)$$

Substituting (6-11) into (6-13) – (6-16), we have

$$B_x\left(\frac{\partial T_x}{\partial x}\right) - A_x\left(\frac{\partial T_x}{\partial x}\right) + B_y\left(\frac{\partial T_y}{\partial y}\right) - A_y\left(\frac{\partial T_y}{\partial y}\right) + B_z\left(\frac{\partial T_z}{\partial z}\right) - A_z\left(\frac{\partial T_z}{\partial z}\right) = 0, \qquad (3.4\text{-}17)\square$$

(3.4-17) shows that the gradient of temperature in $A(A_x, A_y, A_z)$ and $B(B_x, B_y, B_z)$ are the same, such that no wind moving between A and B.

The developing of wind, becoming stronger or weaken, depends on subsequent power of wind supplying.

3.6. More Applications of CLE, Can be Found in Yun and Li [7], and Yun [8]

In Yun and Li [7], they calculated the In-Plane-Hinge-Joint Rigid Sloping Piles. A question arose: the problem has not been researched before, no previous study can be used to compare, and it is not possible to do an experiment, how to check the accuracy of the calculation? The “Principle of Minimum Work” based on CLE had been used to check the accuracy of the calculation.

In Yun [8], the seeking range of missing plane MH370 had been determined via CLE, where a non-powered flying path with most far from an origin in time interval [0, t] is proved. The problems corresponding to two ranges of velocity reduced to Riccati equation and Volterra equation and solved, respectively. Finally, the relative motion in longitude direction due to self-rotation of earth was added to the calculation range. A simple calculation was given for reference.

4. Discussion

Q = Question**, A** = Answer

Q1: How to check the correctness of the <wind speed equation>?

A1: It is examined by well agreement with the weather forecast in China every winter. In winter the weather forecast always alerts that **cold wave is coming, beware the strong wind company with cold temperature down.** It confirms the description of the wind speed equation that **the derivative of wind speed respect to time is proportional to the derivative of temperature respect to the trace.**

Furthermore, this equation is again confirmed by highly agreed with the equation of motion of mushroom cloud [2], although its derivation and solution [6] are different and based on different model (modifying N-S equation).

Q2: How to apply the pulse-mode of Earthquake ?

A2: The process of a great Earthquake is at first the earthquake releases energy to store in fasting Earth rotation, then in a short time the Earth rotation returns to normal and the stored energy releases to Earth crust destroying everything. **People should run away out of the dangerous region before the kinetic energy returning.**

5. Conclusion

The 1D optimum path problem for two end-points fixed, or one end-point fixed the other end-point variable is a vector integral equation of Fredholm/Volterra type, and is hard to solve. Translating them to its scalar components equations would be easier to solve. This paper solves the problem by connecting it to PMER for some case with $p(t) = E = cu^2$, is particularly effected.

The applications of 1-D optimum path problem in wide fields, e.g., in logistics, wind developing, military march, etc.

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