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Original Research

Introduction of New Spiral Formulas from ROTASE Model and Application to Natural Spiral Objects

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Abstract

A new set of spiral formulas is introduced as a new member of the spiral formulas' family, people with interest in mathematics and natural spirals may use the new spiral formulas to simulate natural spiral objects or generate their own spirals. The new formulas are derived from a proposed new hypothesis called Rotating Two Arm Sprinkler Emission model (ROTASE) for the formation of spiral galaxies. In this paper, the derivation of the new spiral formulas is illustrated with boats moving across a circular river from central round island. The formulas have only one parameter and one variable, the parameter can change with time in any format, the morphology of the spiral is decided by the behavior of the parameter. 4 real spiral galaxies are precisely simulated shown as examples. It is demonstrated in this paper that the new spiral formulas can be also applied to simulate Earth natural objects such as plants, animals and hurricanes which have spiral patterns. The result shows that the new formulas seem more universal spiral formulas which can produce various different spirals patterns, can be applied to architecture, artworks and industry design. The new spiral formulas which can produce various different spirals patterns, can be applied to architecture, artworks and industry design. The new spiral formulas

Keywords: New spiral formulas; ROTASE model; Spiral objects; Spiral simulation.

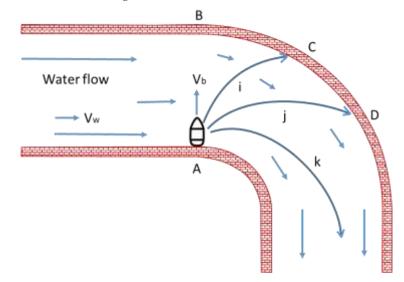
1. Introduction

The nature is full of spirals in different scale from large as spiral galaxies to small as DNA double helix. Good examples include Milky Way galaxy, hurricanes, nautilus shells, land snail shells and vine tendrils, etc. Spirals are one of most prevalent morphologies adapted in the nature, they are also widely used in architecture, industry and artwork creations. The characteristics of spirals have been extensively studied in the entire history of science, various mathematical spiral formulas have been invented, such as Archimedean spiral, Fermat's spiral, logarithmic spiral, golden spiral as special case of logarithmic spiral, Fibonacci spiral, Euler's spiral, etc. The Wikipedia website lists about 28 spiral formulas [1] at current time. Recently, the author proposed a hypothesis called Rotating Two Arm Sprinkler Emission model (for short, ROTASE model) to explain the spiral arm formation of galaxies, a set of 4 equations are derived from the model, which can precisely simulate spiral patterns of most (if not all) of disc-like spiral galaxies with various types of patterns, galaxies with very special spiral patterns such as spirals broken connection with the ends of galactic bars, spiral arm crossings can be naturally explained [2, 3]. This paper will introduce the new spiral formulas to the mathematical community and illustrate the application of those formulas to natural spiral objects. The new spiral formulas may be useful for architecture, artwork creations and industry design.

2. A Boat Moves across a Circular River with Flat Water Flow Velocity

It will be much easier to understand the new spiral formulas with the very simple knowledge about how a boat moves across a circular river with constant water flow in entire river.

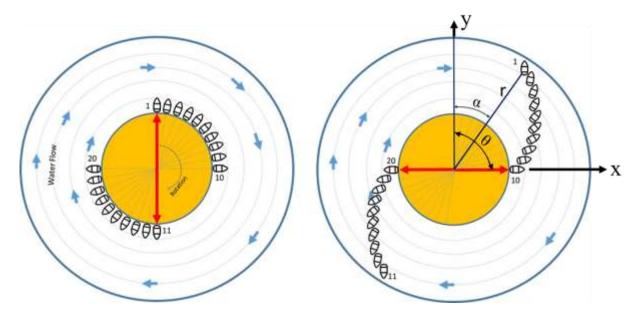
Figure-1. A boat moves across curved river



First, let's think how a boat moves across a curved river shown in Figure 1. Assume that the boat is initially located at the position A, the captain wants to drive the boat across the river to other side B (north) with the velocity V_b pointing to north during entire trip. However, after leaving the position A, the river water flow current will drag the boat with it, so the boat will not reach B, instead, it will reach C or D following moving lines i, or j depending on how fast the boat's speed V_b is; if the V_b is too small, it will follow the moving line k and never reach other side.

Let's have more fun with a scenario illustrated in Figure 2. Figure 2 left shows a circular river with constant water flow velocity V_r , a round island is in the center, 10 boats are evenly parked along the central island shore from 12 clock position to 3 clock position; 10 boats are evenly parked from 6 clock position to 9 clock position, a double-side red clock needle is initially set at vertical position and touches boat 1 and 11. The needle will rotate clockwise with a constant rotation speed, each boat will start to move at the radial direction across river when the red needle touches the boat.

Figure-2. left, Boats parked at along the central island. Right: Boats sequentially move across the river triggered by the clock needle and dragged by the water flow



Each boat will move along the curve (i, or j or k) line shown in the Figure 1, when the clock needle reaches boat 10, all other boats already reach their positions shown in Figure 2 right, a spiral line is formed by the boats 1 to boat 10; boat 11 to boat 20 form another spiral line symmetrically with respect to boat 1 to boat 10. The mathematical formulas to calculate the boat spiral line can be derived which are listed in the next section.

3. The Mathematical Formulas for the Spirals

If desired, please refer the reference papers for the detail of the ROTASE model which has same mathematics as the boat across the circular river, and the derivation of the spiral formulas will not be repeated here to respect the copy right [2, 3].

The motion of the boat can be described by the following differential equation:

$$\begin{cases}
dx = R * \cos(\alpha)d\theta \\
dy = R * (\rho - \sin(\alpha)) d\theta
\end{cases}$$
(1)
The initial values of x and y are set as:

$$\begin{cases}
x_0 = 0 \\
y_0 = R
\end{cases}$$
(2)

Refer to Fig. 2 right, where the α is the angle between the distance r of the current boat location to the center and the initial axis from which the boat starts to move; R is the initial distance (the radius of the round island) of the boat to the center before moving; θ is the rotation angle of the clock needle after the boat moves to current location, it is equivalent to time scale, the motions of all boats are synchronized with the rotation angle θ of the clock needle. The parameter ρ is defined as the ratio of the boat initial velocity V_b over the water flow velocity V_r:

$$\rho = \frac{\dot{v}_b}{v_r} \tag{3}$$

The equation (1) has the following solutions: R (4)

$$r = \frac{1}{1 - \rho * \sin(\alpha)}$$
For $\rho > 1$:
$$\left(\frac{1}{\rho^2 - 1}\right) \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \left(\frac{R}{\sqrt{\rho^2 - 1}}\right) ln |r\sqrt{\rho^2 - 1} + \frac{R}{\sqrt{\rho^2 - 1}} + \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} | \right\} - \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\sqrt{\rho^2 - 1}} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 - 1} \left\{ \sqrt{(\rho^2 - 1)r^2 + 2rR - R^2} - \frac{R}{\rho^2 - 1} \right\} + \frac{1}{\rho^2 -$$

$$\left(\frac{R}{\rho^{2}-1}\right) \left\{ \sqrt{(p^{2}-1)}r^{2} + 2R - R^{2} - \left(\frac{1}{\sqrt{\rho^{2}-1}}\right) ln \left[r\sqrt{\rho^{2}-1} + \sqrt{(p^{2}-1)}r^{2} + 2R - R^{2}\right] \right\} - \left(\frac{R}{\rho^{2}-1}\right) \left(\rho - \frac{1}{\sqrt{\rho^{2}-1}} \ln\left|\frac{\rho^{2}R}{\sqrt{\rho^{2}-1}} + \rho R\right|\right) = R\theta$$

$$(5)$$

For
$$\rho = 1$$
:

$$\frac{\sqrt{2}}{3\sqrt{R}}(r+R)\sqrt{r-\frac{R}{2}-\frac{2}{3}R} = R\theta$$
(6)

For $\rho < 1$

$$\frac{R}{(1-\rho^2)^3} \arcsin\left(\frac{(1-\rho^2)r-R}{\rho R}\right) - \frac{1}{(1-\rho^2)}\sqrt{2rR-R^2-(1-\rho^2)r^2} - \frac{R}{(1-\rho^2)^2} \arcsin(-\rho) + \frac{\rho R}{1-\rho^2} = R\theta$$
(7)

The formulas (4) to (7) assume that the boat leaves the shore with initial velocity V_b and the V_b remains constant during its trip. The unit of arcsin is in radian not degree. Each boat has its own starting position, r, α , θ and ρ .

All boats are independent from each other, i.e., the motion of each boat has no any effect on the motion of other boats. The initial velocity $V_{\rm b}$ of the boats can be same or different. It is the only interest of this paper that the initial velocities of boats change following a pattern which can be described by a mathematical formula. For example, all boats may have the same initial velocity, or the initial velocity linearly increases (or decreases) from boats 1 to boat 10, or the follows other mathematic equations (exponential, Gaussian, etc.). Apparently, different mathematical equations of velocities will produce different spiral patterns in Figure 2. The V_b will be the function of time, so does the ρ .

$$\rho = f(\theta) \tag{8}$$

The θ represents time in equation (8). When the velocity V_b of the boat is smaller than the river water flow velocity V_r , i. e., $\rho < 1$, the maximum distance of the boat to the center is limited by the equation:

$$r(limit) = \frac{\kappa}{1-\rho} \tag{9}$$

The boat will never reach other side if the outer radius of the circular river is greater than r(limit).

The new spiral formulas have only one parameter ρ and one variable r. For general applications, the ρ can be viewed as a growth factor of the spirals representing the initial growth velocity of the spiral component, which can change with time in any format, the morphology of the spiral is decided by the behavior of p change with time.

4. Calculation of the Spirals

The steps to calculate the spiral pattern is outlined here:

- Select the right equation to calculate the rotation angle θ of the clock needle according to the value of ρ . For 1. example, if $\rho > 1$, select equation (5) to calculate the θ with variable r. r = R represents the current time. If ρ changes with time (rotation angle θ represents time), then use θ as variable to calculate ρ with equation (8), then, insert the θ and the calculated ρ into the right equation to find the matched r.
- Use equation (4) to calculate the X and Y coordinates of the spiral arm: 2.

$$x = r * \sin(\alpha) = \frac{r - \kappa}{\rho}$$
(10)

$$y = r * \cos(\alpha) = \frac{\sqrt{(r\rho)^2 - (r-R)^2}}{\rho}$$
(11)

Rotate x and y coordinates backward for the rotation angle θ : 3.

$$\begin{aligned} x' &= x * \cos(-\theta) + y * \sin(-\theta) \end{aligned} \tag{12} \\ y' &= -x * \sin(-\theta) + y * \cos(-\theta) \end{aligned} \tag{13}$$

$$y' = -x * \sin(-\theta) + y * \cos(-\theta)$$
⁽¹³⁾

4. Plot the spiral arm pattern with x' and y'.

The backward rotation by equations (12) and (13) is critical as explained in detail in the reference [4] and will not be repeated here. Please note, for $\rho < 1$, when the r reaches its limit, continues to use equation (10) and (11) to calculate x and y with r(limit), and rotates x and y with equations (12) and (13), because θ keeps change, it will produce a beautiful spiral-ring pattern which has widely application in the nature, will be seen in the following sections

Additional Euler rotation is needed for simulation of spiral galaxies to match the orientation of the line of sight. The Euler rotation matrix $R(\phi\sigma\psi)$ used in this paper is:

 $R(\varphi\sigma\psi) = \begin{bmatrix} \cos\phi \cos\psi - \cos\sigma \sin\psi \sin\phi & -\sin\psi \cos\phi - \cos\sigma \sin\phi \cos\psi & \sin\sigma \sin\phi \\ \sin\phi \cos\psi + \cos\sigma \cos\phi \sin\psi & -\sin\phi \sin\psi + \cos\sigma \cos\phi \cos\psi & -\sin\sigma \cos\phi \\ \sin\sigma \sin\psi & \sin\sigma \cos\psi & \cos\sigma \end{bmatrix}$ (14)

The differential equation (1) and its solutions (4) to (7) are derived under assumption that the boat starts to move position at Y axis as boat 1, however, the boat 2 to the boat 10 start to move at their own Y axis, so the Y axis is rotating. The rotation by equations (12) and (13) is to compensate such Y axis rotation with the last boat 10 as the reference time zero point. Readers may write a mini simple computer program to calculate the new spirals with the formulas and steps outlined above.

5. Simulation of Spiral Patterns of Galaxies with Various Morphologies

Total 15 spiral galaxies with substantially different types of morphology have been simulated in the reference papers [2, 3], this paper will show only 4 spiral pattern simulations of galaxies as examples to minimize the overlap with reference papers. Readers may refer the reference papers for the detailed simulations and descriptions of other spiral galaxies [2, 3]. The model for the spiral is developed with assumption that the current double side clock needle position is at Y axis, and rotates clockwise. However, for spiral galaxies, the galactic bars may be located at X axis and rotate counterclockwise. Therefore, after the spiral calculation, it may be necessary to exchange the x with y or x with -x, etc., to match the real images of the galaxies. Spiral galaxies normally have two central symmetric spiral arms, one spiral simulation line can be calculated by the steps above, the other simulation line can be generated through symmetry.

Figure 3 left shows the simulation of the galaxy UGC 12158 by formulas (4) and (6), this galaxy is a regular spiral galaxy with constant $\rho = 1$. The yellow spiral arm starts at bar end A, extends to B and C, the arm is about to fade away beyond the C. The red line arm has the same characteristics. Figure 3 right shows the simulation of galaxy ESO325-28 by formulas (4) and (7) with constant $\rho = 0.65$, it shows a nice spiral-ring pattern with ring radius limited by formula (9), which cannot be simulated by all other available models for the spiral arm formation. The yellow line arm starts at bar end A, extends to B and C, at C it merges to red line arm and becomes a perfect circle, section CD is a perfect ring circle; the red line has the same characteristics. Constant ρ means that the central supermassive black hole emits the X-matter with a stable velocity.

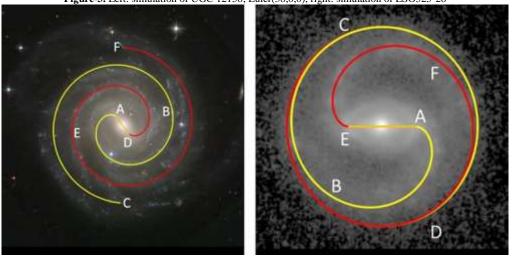
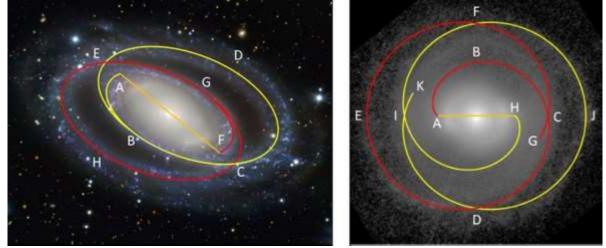




Figure 4 left is the simulation of the NGC 7098 by formulas (4) and (7) with Gaussian growth factor equation: $\rho = 0.55 * \exp(-0.0001 * (\theta - 360)^2)$ (15)

Figure-4. Left: simulation of NGC 7098, Euler (25,60,25), right: simulation of NGC 1079



This galaxy has two beautiful ring pattern and is described in Wikipedia website as a smaller inner ring wrapped around the galactic bar and a big well-defined outer ring located outside of the inner ring. Such description is possibly the most accepted description in the astronomy community. However, the simulation by ROTASE model shows completely different picture, the yellow line spiral arm starts from bar end A, extends to the small half ring B, then to point C, at C it crosses the red line spiral arm, but the luminosity of the yellow line arm is stronger than the red line arm, so the yellow arm looks on "top" of the red line arm because the yellow arm is younger than red line arm; then, the yellow line arm couter half line D, then to point E, at E it crosses red line spiral arm again, however, at E, the yellow line arm looks "below" the red line arm because the yellow line arm is older than red line arm. The whole spiral pattern is made of two identical rings (the yellow line ring and the red line ring), each ring is made of a half small inner ring and a half large outer ring, the two rings cross each other twice in chain-link style.

The chain-link style spiral arm crossing is illustrated in Figure 5. In Figure 5, a gold ring is linked with a silver ring, they cross each other twice. At the cross-point N, the gold ring is on top of the silver ring; at the cross-point M, the silver ring is on top of the gold ring. The two-ring structure of the NGC 7098 matches the Figure 5 perfectly. One can see that the ROTASE model provides more accurate description.

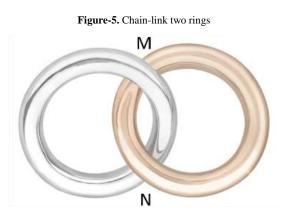


Figure 4 right shows the simulation of the NGC 1079 by formulas (4) and (7) with parameter ρ change with time (rotation angle) following Gaussian growth factor equation:

 $\rho = 0.655 * \exp(-0.00001 * (\theta - 450)^2)$

(16)

The simulation clearly reveals that this galaxy has two identical rings, each ring cross other ring twice and cross itself once. For the red line arm, it starts at bar end A, extends to B and C, then crosses yellow line arm at point D, extends to E, crosses yellow line arm again at F, extends to C again and crosses itself at C. The red line arm has the same characteristics. This is a really amazing artwork-like galaxy made by nature, and perfect simulation by ROTASE model is just a miracle.

6. Simulation of Spiral Patterns of Earth Natural Objects

Although the ROTASE model was developed to describe the formation of spiral arms of galaxies, the spiral formulas can be applied to other natural objects. The Earth ecosystem is full of spiral objects, which can be found everywhere around us. In this section, it will be demonstrated that the new spiral formulas can be used to nicely simulate many natural objects. The parameter ρ can be viewed as a growth parameter, every point of the spiral line can still be viewed as "generated" at the starting point and then grows outwardly with its own growth velocity, however, the real situation may be totally different. In some cases, the whole spiral patterns of some creatures are formed in the beginning. For example, the spiral pattern of the land snail shell is the same from a baby to an adult,

only size difference (ignore possible small change during its growth); in some other cases, the spiral patterns are gradually formed during growth, the spiral Aloe Polyphylla and hurricanes are good examples. Some animals and insects have organs which can be curled or uncurled depending on situations.

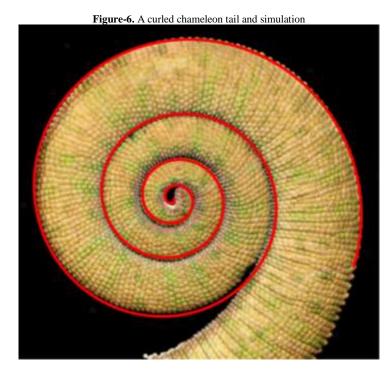
6.1. Curled Chameleon Tail

The chameleon's tail is an important part of its body, and it can be used in various purposes. Chameleons curl their tails to hold objects, balance themselves, make themselves look smaller, and to express their mood such as aggressive action. The tightly coiled tail pattern of Figure 6 shows no any empty gaps between the curled tail loops; therefore, the spiral curling is the most efficient space saving (looks smallest possible) method in this case, this should be the result of the natural evolution.

Figure 6 shows a tightly curled chameleon tail, which can be precisely simulated by equations (4) and (5) with the following exponential growth factor equation:

 $\rho = 1^* \exp(0.0008^*\theta)$

(17)



6.2. Nautilus Shell

The nautilus is a pelagic marine mollusc of the cephalopod family Nautilidae, it is a distant cousin to squids, octopi. The name "nautilus" literally means "sailor" from ancient Greek. The species survived have relatively unchanged for hundreds of millions of years as a good example of living fossils. Its outer shell has one of the finest natural examples of a logarithmic spiral. Figure 7 shows the simulation of the Nautilus shell by equations (4) to (7) with the growth factor following the exponential equation:

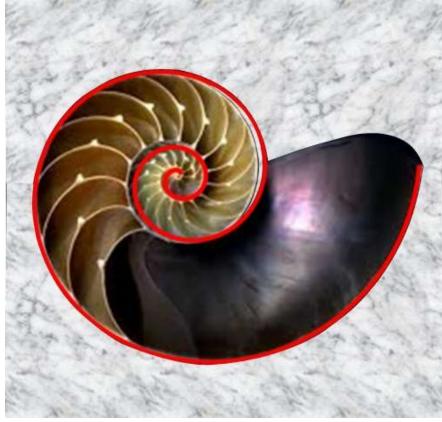
 $\rho = 0.42 * \exp(0.0015 * \theta)$

(18)

The ρ changes from less than 1 to greater than 1, so, all 4 spiral formulas have to be used in the calculation. The simulation at the outer shell is very good. The adaption of spiral shell makes the whole body being the most efficient streamline geometry with the least fluid resistance during movement in the water, it is the result of evolution.

The nautilus resides in the shell's largest chamber (black area in Figure 7), while the other chambers function like the ballast tanks of a submarine, the amount of air in those chambers can be adjusted for moving up or down. This is the secret to how the nautilus swims.

Figure-7. Simulation of Nautilus shell



6.3. Sunflower Seed Head

The sunflower seed head is a very interesting object in which the alignment of the seeds in the head forms two different spiral lines: clockwise and counterclockwise spirals. Spiral lovers believe that the number of spirals formed by sunflower seeds follows the Fibonacci sequence, in which each number is the sum of the previous two (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...), the Fibonacci sequence numbers are found in everything from pineapples to pine cones. For the sunflower seed head, a published research showed that the Fibonacci sequence numbers (13, 21, 34, 55 and 89) indeed dominate the spiral counting from 657 sunflower heads in statistics, only one in five of the flowers had non-Fibonacci spiraling patterns [5]. The spirals of the sunflower seed head can be nicely simulated by the new spiral formulas (4) and (5) with the following growth factor equations, shown in Figure 8:

Red line: $\rho = 4 * \exp(-0.00015 * \theta)$ Blue line: $\rho = 4 * (1 + 0.0005 * \theta)$

(19) (20)

Michael Naylor illustrated by mathematics how the seed spirals and flower spirals are developed in different cases such as the Golden flowers, π -flowers, Fibonacci flowers, and $\sqrt{2}$ flowers [6]. However, two critical factors are not considered which are the volume and geometry of each individual element in those cases. Take the Sunflower seed head as an example from the Figure 8, the seed near the center is very young with almost empty soft shell and has much smaller volume than the seed near the edge which is well developed, the seed density (the number of seeds in unit area) near the center is much higher than the seed density near the edge. The outer shell of the seeds has unique geometry which makes sure that in any stage of the growth of the sunflower seed head, the spiral arrangement of the seeds has the maximum space efficiency, i.e., the sunflower head can hold maximum number of seeds with minimum space, one can see in Figure 8 that the sunflower seed head is perfectly packed without any wasted space from center to edge. This is the result of natural evolution.

Figure-8. Spirals in the sunflower seed head and simulations by new spiral equations



6.4. Spiral Aloe Polyphylla

Aloe Polyphylla has a beautiful spiral pattern, is a famous model of spiral plants. The pattern can be nicely simulated shown in Figure 8 by the formulas (4) and (5) with a special growth factor equation with double exponential decay components:

 $\rho = 5^{*}(1+1.2^{*}\exp(-0.001^{*}\theta) - 0.2^{*}\exp(-0.01^{*}\theta))$

(21)

The author has a living Aloe Polyphylla plant, with careful observation, it is found that the leaves of the Aloe Polyphylla do not rotate during growth, they grow outwardly in radial direction straight out, it is the generation of the new leaves at the center in the rotation fashion with well-defined sequential rotation angle, it seems that the Aloe Polyphylla "has" a very tiny "rotation bar" at the center, which "rotates" with constant rotating speed, the leaves are generated at the "end of bar" at regular rotation angle intervals, very similar to the spiral arm formation in the recently proposed ROTASE model. Many cacti have spiral patterns formed by their spine clusters, such spine clusters are sequentially generated at the center with well-defined rotating angle intervals; after formation of the spine clusters, they will grow straight outwardly in the radial direction. This type of spiral arrangement is also the most efficient way to hold the maximum number of objects with minimum space. The number of spirals in the plants is decided by the size and the shape of each object. For Sunflower seed head, the size of the seeds gradually increases from center to outer edge, it is fully packed without any gaps between seeds, this can be achieved only by spiral arrangement. The Aloe Polyphylla has larger leaves, it can only hold 5 spirals for the most efficient growth. So, the adaption of spiral arrangement of the plants is the result of evolution for maximum efficiency.

Figure-9. Spiral Aloe Polyphylla and simulation



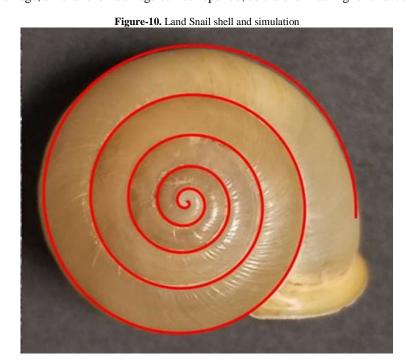
6.5. Land Snail Shell

Land snails have hard shells which provide a good housing and protection shield against the predators. A snail's shell has a spiral concave suture with logarithmic spiral pattern. Most snail shells are right-handed or <u>dextral</u> in coiling. Shell-coiling is important because a snail's sexual organs are usually twisted and it is difficult for snails of opposite handedness to reproduce. In addition, the way a snail catches prey depends on the handedness of the micro-organism or plant. The Figure 10 shows the image of the land snail shell and the simulation by the spiral formulas (4) and (5) with following linear growth factor equation:

 $\rho = 2*(1+0.0011*\theta)$

(22)

The simulation with the new spiral formulas is very good. The growth factor linearly increases with the rotation angle. The spiral concave suture structure on the shell enforces the mechanical strength of the shell, the shell will crack or be broken much easier without the spiral concave suture structure with external impact, the snail will have more chance to die. With spiral concave suture structure, the same impact force will be less harmful, may cause smaller or no shell damage, smaller shell damage can be repaired, so the snail has higher chance to survive.



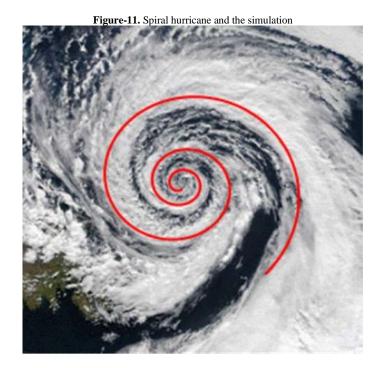
6.6. Spiral Hurricane

A hurricane is rapidly rotating moisture and air system with a low-pressure eye center, carries strong winds and heavy rain or thunderstorm by heavy thick clouds, the heavy thick clouds normally have spiral patterns. The rotation of the Hurricanes is the result of Earth's rotation following the conservation of angular momentum. Figure 11 shows a well-developed hurricane with nice spiral pattern. Its spiral pattern can be fairly simulated by the new spiral formulas (4) and (7) with the following exponential growth factor equation:

 $\rho = 0.5 * \exp(0.001 * \theta)$

(23)

(24)



6.7. Spiral Medicago Orbicularis Seed Pod

This world is full fantasies and surprises, the strange morphology of the Medicago Orbicularis seed pod is one of them. Figure 12 left is the original image of the Medicago Orbicularis seed pod. At the center, it looks like a bird head with a curved beak. Its face profile shown in the image has a spiral-ring pattern similar to galaxy ESO 325-28, the spiral is extended with helix style like a thread, an unique and beautiful structure in the nature. Its face spiral pattern can be nicely simulated by the new spiral formulas (4) and (5) with the following exponential decay growth factor equation:

$$\rho = 4 \exp(-0.001 * \theta)$$

Figure-12. Left: Original image of Medicago Orbicularis seed pod Right; simulation



It seems that there is no good explanation why it takes this special shape, however, it must be the result of natural evolution through the history which makes it survived well.

7. Discussion

The above examples show that the new spiral formulas can be applied to various different natural objects with nice results as long as a suitable growth factor equation is found which is, in sometime, brain challenge, the growth factor provides much broad flexibility and make the new spiral formulas more universal than other available spiral formulas. Some spirals cannot be simulated with one parameter equation, multiple section simulations may be used like the galaxy M51 which has to be simulated with two sections by different parameter equations [2]. Those new spiral formulas can be a new member of the spiral equation family. The formulas have only one parameter and one variable, the spiral pattern is decided by the behavior of the parameter, could be useful to the architecture, industry design and artwork creation. The natural species adapt the spiral geometry with many reasons by the principle of the evolution selection. The spirals can make the biological system most efficient with minimum cost, can make the architecture of the hard structure mechanically stronger. Butterflies drink through a tube-like tongue called a proboscis, they uncoil the proboscis to sip liquid food, and then coils up again into a spiral when not feeding, the spiral arrangement is the best way to store the long and thin proboscis tube with space saving without tangling. It will be really fun to simulate the natural spirals or generate your own spirals with those formulas. In summary, the spiral is one of the most important morphological patterns adapted by nature with beauty, efficiency and diversity.

8. Conclusion

The new spiral formulas obtained from ROTASE model are more universal and can be applied to simulate natural objects with various spiral morphologies; The new spiral formulas can be a new addition to the current family of spiral mathematical formulas.

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