Academic Journal of Applied Mathematical Sciences
ISSN(e): 2415-2188, ISSN(p): 2415-5225
Vol. 7, Issue. 3, pp: 143-148, 2021
URL: https://arpgweb.com/journal/journal/17
DOI: https://doi.org/10.32861/ajams.73.143.148

Academic Research Publishing Group

Original Research **Open Access**

Different Types of Curvature and Their Vanishing Conditions

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Article History
Received: 22 February, 2021
Revised: 8 April, 2021
Accepted: 27 April, 2021
Published: 2 May, 2021

Abstract

In the present paper, I studied different types of Curvature like Riemannian Curvature, Concircular Curvature, Weyl Curvature, and Projective Curvature in Quarter Symmetric non-Metric Connection in P-Sasakian manifold. A comparative study of a manifold with a Riemannian connection is done with a P-Sasakian Manifold. Conditions for vanishing for different types of curvature are also a part of the study. Some necessary properties of the Hessian operator are discussed with respect to all curvatures as well.

Keywords: Riemannian curvature; Concircular curvature; Weyl curvature; Projective curvature and Hessian operator.

1. Introduction

In 1963, Kobayashi and Nomizu [1], gave the basics of differential geometry. Later in 1970 sufficient conditions of Riemannian manifolds were discussed by Sekigawa and Tanno [2]. In 1975, Golab [3], defined and studied quarter-symmetric connection in a differentiable manifold with affine connection. In 1977, Adati and Matsumoto [4], defined Para-Sasakian and Special Para-Sasakian manifolds which are special classes of an almost paracontact manifold introduced by Sato [5], Sato [6]. Para-Sasakian manifolds have been studied by De and Sheikh [7], Burman [8], and many more.

This paper gives a brief detail of different types of curvature and its nature in Riemannian manifold and a manifold with quarter symmetric non-metric connection. Though lot of study has been done on different types of connections, but this paper gives brief idea of different types of curvatures and their vanishing conditions. We have done this study on Para-Sasakian manifold.

A linear connection as defined in Mondal and De [9] and Mondal and De [10], $\tilde{\nabla}$ on an n-dimensional Riemannian manifold M is called a quarter-symmetric connection if its torsion tensor T of the connection $\tilde{\nabla}$

$$T(X, Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X, Y] \quad (1)$$

satisfies

$$T(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y \quad (2)$$

where η is a 1-form and ϕ is a (1,1) tensor field.

In particular, if $\phi(X) = X$, then the quarter-symmetric connection reduces to a semi-symmetric connection. Thus, the notion of quarter-symmetric connection generalizes the notion of the semi-symmetric connection as given by Agashe and Chafle [11], If a quarter-symmetric connection $\tilde{\nabla}$ satisfies the condition.

$$(\tilde{\nabla}_X g)(Y, Z) = 0 \quad (3)$$

for all $X, Y, Z \in T(M)$, where T (M) is the Lie algebra of vector fields of the manifold M, then $\tilde{\nabla}$ is said to be a quarter-symmetric metric connection; otherwise, it is said to be a quarter-symmetric non-metric connection.

Sato [5], Sato [6], introduced the notion of an (almost) paracontact structure, either P-Sasakian or SP-Sasakian and gave a lot of interesting results about such manifolds. In current paper we generalize some results on curvature tensors. A Riemannian manifold is called locally symmetric if its Riemannian curvature tensor R satisfies $\nabla R = 0$, where ∇ denotes the operator of covariant differentiation.

The paper is organized as follows: In section 2, we give brief account of P-Sasakian manifold. In section 3, we give relation between Riemannian manifold and Quarter sym- metric non-metric connection. In section 4 and 5, we study curvature tensor R, Ricci tensor S, Concircular curvature tensor C, Weyl / conformal curvature tensor W and Projective curvature tensor P, with respect to quarter symmetric non-metric connection. In section 6, we study necessary and sufficient condition for vanishing of these tensors in Riemannian manifold. In section 7, we study necessary and sufficient condition for vanishing of these tensors in manifold with quarter-symmetric non-metric connection. In section 8, we study the properties of Hessian operator with respect to vanishing of these tensors.

2. P-Sasakian Manifold

An n dimensional differentiable manifold M is said to admit an almost para contact Rie- mannian structure (ϕ, ξ, η, g) where ϕ is a (1, 1) -tensor field, ξ is a vector field, η is a $1-form$ and g is a Riemannian metric on M such that

$$(a) g(\xi,\xi)=1,$$
$$(b) \phi^2 X = X - g(\xi, X)\xi, \quad (4)$$
$$(c) g(\phi X, \phi Y) = g(X, Y) - g(\xi, X)g(\xi, Y),$$

According to (a) and (c), put $X, Y = \xi$

$$g(\phi\xi, \phi\xi) = g(\xi, \xi) - g(\xi, \xi)g(\xi, \xi) = 1 - 1 = 0$$

Similarly, from (b)

$$\phi^3 X = \phi X - g(\xi, \phi X)\xi$$

which implies

$$\phi^3 X = \phi X$$
$$g(\phi X, \xi) = g(\phi^3 X, \xi) = g(\phi^2 \phi X, \xi)$$
$$= g(\phi X - g(\phi X, \xi)\xi, \xi) = g(\phi X, \xi) - g(\phi X, \xi)g(\xi, \xi) = 0$$

Thus, we have from (4)

$$\phi\xi = 0,\ \eta\phi = 0,\ \eta(\xi) = 1,\ g(\xi, X) = \eta(X), \quad (5)$$

for all the vector fields $X, Y \in T(M)$. The equation $\eta(\xi) = 1$ is equivalent to $|\eta| = 1$, and then ξ is just the metric dual of η, where g is the Riemannian metric on M. If (ϕ, ξ, η, g) satisfy the following equations as Oguzhan [12].

$$d\eta = 0,\ \nabla_X \xi = \phi X, \quad (6)$$
$$(\nabla_X \phi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \quad (7)$$

Then M is a Para Sasakian manifold or briefly a P-Sasakian manifold. A P-Sasakian manifold M is called a special Para-Sasakian manifold or briefly a SP-Sasakian manifold if M admits a $1-f$ orm η satisfying

$$(\nabla_X \eta)(Y) = -g(X, Y) + \eta(X)\eta(Y), \quad (8)$$

Let S and r denote the Ricci tensor and the scalar curvature tensor of M respectively. The operator Q and the (0,2) tensor S are defined by

$$S(X, Y) = g(QX, Y) \quad (9)$$

Let (M, g) be an n-dimensional Riemannian manifold. Then the Concircular curvature tensor C and the Weyl conformal curvature tensor W is given by Yadav and Dhruvanarain [13].

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}(g(Y,Z)X - g(Z,X)Y) \quad (10)$$

$$W(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\{S(Y,Z)X - S(Z,X)Y + g(Y,Z)QX - g(X,Z)QY\} + \frac{r}{(n-1)(n-2)} g(Y,Z)X - g(X,Z)Y \quad (11)$$

for all $X, Y, Z \in T(M)$, respectively, where r is the scalar curvature of M and Q is the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor S.

A Riemannian manifold of quasi constant curvature was given by Chen and Yano as a conformally flat manifold with the curvature tensor R of type (0, 4) which satisfies the condition

$$R(X, Y, Z, W) = a\{g(Y, Z)g(X, W)\} + b\{g(Y, Z)T(X)T(W) - g(X, Z)T(Y)T(W) + g(X, W)T(Y)T(Z) - g(Y, W)T(X)T(Z)\} \quad (12)$$

where $R(X, Y, Z, W) = g(R(X, Y)Z, W)$, a, b are scalars, T is a nonzero 1-form defined by $T(X) = g(X, \rho)$, and ρ is a unit vector field.

It can be easily seen that if the curvature tensor is of the form (12), then the manifold is conformally flat. If $b = 0$, then it reduces to a manifold of constant curvature.

An n-dimensional P-Sasakian manifold is said to be η-Einstein if the Ricci tensor S satisfies

$$S = ag + b\eta \otimes \eta \quad (13)$$

where a and b are smooth function on the manifold. If $b = 0$, then the manifold reduces to an Einstein manifold.

3. Relation between Riemannian Manifold and Quarter Symmetric Non - Metric Connection

Let $\tilde{\nabla}$ be a linear connection and ∇ be a Riemannian connection of a P-Sasakian manifold M [13] such that

$$\tilde{\nabla}_X Y = \nabla_X Y + U(X, Y) \quad (14)$$

where U is a tensor of type (1, 2). For $\tilde{\nabla}$ to be a quarter-symmetric connection in M, we have

$$U(X,Y) = \frac{1}{2}[T(X,Y) + T'(X,Y) + T'(Y,X)] \quad (15)$$

where

$$g(T'(X, Y), Z) = g(T(Z, X), Y) \quad (16)$$
$$T'(X, Y) = g(\phi Y, X)\xi - \eta(X)\phi Y \quad (17)$$

From (2) and (4) we get and using (2), (4) and (8) we obtain

$$U(X, Y) = -\eta(X)\phi Y \quad (18)$$

Hence a quarter-symmetric connection $\tilde{\nabla}$ in a P-Sasakian manifold is given by

$$\nabla_X Y = \nabla_X Y - \eta(X)\phi Y \quad (19)$$

Using (14), (18) and (19) the torsion tensor of the connection $\tilde{\nabla}$ is given by

$$T(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y] = \eta(Y)\varphi X - \eta(X)\varphi Y \quad (20)$$

Above equation shows that the connection $\tilde{\nabla}$ is quarter-symmetric.

Also, we have

$$(\tilde{\nabla}_X g)(Y,Z) = Xg(Y,Z) - g(\tilde{\nabla}_X Y, Z) - g(Y, \tilde{\nabla}_X Z)$$
$$= \eta(X)[g(\phi Y, Z) + g(\phi Z, Y)]$$
$$= 2\eta(X)g(\phi Y, Z) \quad (21)$$

In virtue of (20) and (21) we conclude that $\tilde{\nabla}$ is a quarter-symmetric non-metric connection.

Therefore (20) is the relation between the Riemannian connection and the quarter- symmetric non-metric connection on a P-Sasakian manifold.

4. Introduction to Curvature Tensors

We define the curvature tensor of a P-Sasakian manifold with respect to the quarter-symmetric non-metric connection $\tilde{\nabla}$ by

$$\tilde{R}(X,Y)Z = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z \quad (22)$$

Using (19) we obtain

$$\tilde{R}(X,Y)Z = R(X,Y)Z - (\nabla_X \eta)(Y)\phi Z + (\nabla_Y \eta)(X)\phi Z$$
$$-\eta(Y)(\nabla_X \phi)Z + \eta(X)(\nabla_Y \phi)Z \quad (23)$$

which in view of (7) and (8) yields

$$\tilde{R}(X,Y)Z = R(X,Y)Z + \eta(Y)g(X,Z) - \eta(X)g(Y,Z)\xi$$
$$+\{\eta(Y)X - \eta(X)Y\}\eta(Z) \quad (24)$$

A relation between the curvature tensor of M with respect to the quarter-symmetric non-metric connection $\tilde{\nabla}$ and the Riemannian connection ∇ is given by the relation (23).

So, from (23) and (6) we have

$$\tilde{R}(X, \xi)Y = R(X, \xi)Y + g(X, Y) - 2\eta(X)\eta(Y)\xi + \eta(Y)X \quad (25)$$
$$\tilde{R}(X, Y)\xi = 2\{\eta(Y)X - \eta(X)Y\} \quad (26)$$

Taking inner product of (23) with W we have,

$$\tilde{R}(X, Y, Z, W) = R(X, Y, Z, W) + \{\eta(Y)g(X, Z) - \eta(X)g(Y, Z)\}\eta(W)$$
$$+\{\eta(Y)g(X, W) - \eta(X)g(Y, W)\}\eta(Z) \quad (27)$$

where $\tilde{R}(X, Y, Z, W) = g(\tilde{R}(X, Y, Z), W)$

From (26) we can state the following.

Proposition 4.1. If the manifold is of constant curvature with respect to the Levi- Civita connection, then the manifold is of quasi constant curvature with respect to the quarter-symmetric non-metric connection.

Also, from (27) clearly

$$\tilde{R}(X,Y,Z,W) = -(\tilde{R}(X,Y,Z),W) \quad (28)$$

But

$$\tilde{R}(X,Y,Z,W) \neq -(\tilde{R}(X,Y,Z),W) \quad (29)$$

From (23) it is obvious that

$$\tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y = 0 \quad (30)$$

Hence, we can state that the curvature tensor with respect to the quarter-symmetric non-metric connection satisfies first Bianchi identity.

Contracting (27) over *X* and *W*, we obtain

$$\tilde{S}(Y,Z) = S(Y,Z) - g(Y,Z) + \eta(Y)\eta(Z) \quad (31)$$

where $\tilde{S}$ and S is the Ricci tensors of the connection $\tilde{\nabla}$ and ∇, respectively. So, in a P -Sasakian manifold the Ricci tensor with respect to the quarter-symmetric non-metric connection is symmetric. Also, if M is Einstein or η-Einstein with respect to the Rieman- nian connection, then M is η-Einstein with respect to the quarter-symmetric non-metric connection.

Again contracting (31) we have $\tilde{r} = r$, where $\tilde{r}$ and r are the scalar curvature of the connection $\tilde{\nabla}$ and $\tilde{\nabla}$, respectively. So, we have the following

Proposition 4.2. For a P-Sasakian manifold M with the quarter-symmetric non- metric connection $\tilde{\nabla}$

(a) The curvature tensor is given by (22),

(b) The Ricci tensor is given by (31)

(c) The first Bianchi identity is given by (30),

(d) $\tilde{r} = r$

(e) The Ricci tensor $\tilde{S}$ is symmetric,

(f) If M is Einstein or η-Einstein with respect to the Riemannian connection, then M is η-Einstein with respect to the quarter-symmetric non-metric connection.

5. Different Curvature Tensors

We observe immediately from the definition of the Concircular curvature tensor that

Riemannian manifolds with vanishing Concircular curvature tensor are of constant curvature. Thus, one can think of the Concircular curvature tensor as a measure of the failure of a Riemannian manifold to be of constant curvature. Also, necessary and sufficient condition that a Riemannian manifold be reducible to a Euclidian space by a suitable Concircular transformation is that its Concircular curvature tensor vanishes. Conformal curvature tensor plays an important role in differential geometry.

We define the **Concircular curvature tensor** $\tilde{C}$ on a P-Sasakian manifold with respect to the quarter-symmetric non-metric connection $\tilde{\nabla}$ by

$$\tilde{C}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{\tilde{r}}{n(n-1)}(\tilde{g}(Y,Z)X - \tilde{g}(X,Z)Y),$$

$$\tilde{C}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{r}{n(n-1)}(g(Y,Z)X - g(X,Z)Y), \qquad (32)$$

Using (9), (31) reduces to

$$\tilde{C}(X,Y)Z = C(X,Y)Z - (g(X,Z)\eta(Y) - g(Y,Z)\eta(X)) + (\eta(Y)X - \eta(X)Y)\eta(Z) \qquad (33)$$

Now if we consider $\tilde{C} = C$, then from (33) we have

$$g(X,Y)=n\,\eta(X)\eta(Y) \qquad (34)$$

We define the **Weyl / Conformal curvature tensor** $\tilde{W}$ on a P-Sasakian manifold with respect to the quarter-symmetric non-metric connection $\tilde{\nabla}$ by

$$\widetilde{W}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{1}{n-2}\{\tilde{S}(Y,Z)X - \tilde{S}(X,Z)X\} + g(Y,Z)\tilde{Q}X - g(X,Z)\tilde{Q}Y\} + \frac{\tilde{r}}{(n-1)(n-2)}\{\tilde{g}(Y,Z)X - \tilde{g}(X,Z)Y\} \qquad (35)$$

$$\widetilde{W}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{1}{n-2}\{\tilde{S}(Y,Z)X - \tilde{S}(X,Z)X\} + g(Y,Z)\tilde{Q}X - g(X,Z)\tilde{Q}Y\} + \frac{\tilde{r}}{(n-1)(n-2)}\{\tilde{g}(Y,Z)X - \tilde{g}(X,Z)Y\} \qquad (36)$$

for all $X, Y, Z \in T(M)$, respectively, where $\tilde{r}$ is the scalar curvature, and $\tilde{Q}$ is the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor $\tilde{S}$ with respect to quarter-symmetric non-metric connection.

The Projective tenso is another important tensor. If there exists a one to one correspondence between each coordinate neighborhood of M and a domain in Euclidian space such that any geodesic of the Riemannian manifold corresponds to a straight line in the Euclidian space, then M is said to be locally projectively flat. For $n \geq 3$, M is locally projectively flat iff the well known Projectively curvature tensor P vanishes. Here P is defined by

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}\{S(Y,Z)X - S(X,Z)Y\} \qquad (37)$$

or $X, Y, Z \in T(M)$, where R is the curvature tensor and S is the Ricci tensor. M is projectively flat iff the manifold is of constant curvature. Thus, the Projective curvature tensor is a measure of the failure of a Riemannian manifold to be of constant curvature. Since the Ricci Tensor $\tilde{S}$ of the manifold with respect to quarter symmetric non-metric connection $\tilde{\nabla}$ is symmetric from (31) and the Projective curvature tensor $\tilde{P}$ becomes

$$\tilde{P}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{1}{n-1}\{\tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y\} \qquad (38)$$

$$\tilde{P}(X,Y)Z = R(X,Y)Z + \eta(Y)g(X,Z) - \eta(X)g(Y,Z)\xi + \{\eta(Y)X - \eta(X)Y\}\eta(Z) - \frac{1}{n-1}\{[S(Y,Z) - g(\phi Y,Z)]X - [S(X,Z) - g(\phi X,Z)]Y\} \qquad (39)$$

$$\tilde{P}(X,Y)Z = R(X,Y)Z + \eta(Y)g(X,Z) - \eta(X)g(Y,Z)\xi + \{\eta(Y)X - \eta(X)Y\}\eta(Z) - \frac{1}{n-1}\{[-g(\phi Y,Z)]X + [g(\phi X,Z)]Y\} \qquad (40)$$

6. Necessary and Sufficient Conditions for Vanishing of These Tensors with Riemannian Connection

6.1. Vanishing of Curvature Tensor *R*

If $R = 0$ then the Riemannian manifold is a flat manifold

6.2. Vanishing of Concircular curvature tensor *C*

From (10)

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\{g(Y,Z)X - g(Z,X)Y\} \qquad (41)$$

Let us assume that

$C(X,Y)Z = 0$. Then we have,

$$R(X,Y)Z - \frac{r}{n(n-1)}\{g(Y,Z)X - g(Z,X)Y\} = 0$$

As, $R(X,Y)Z = g(X,Z)Y - g(Y,Z)X$, above equation becomes,

$$R(X,Y)Z = \frac{r}{n(n-1)}\{-R(X,Y)Z\}$$

which implies

$$\frac{r}{n(n-1)} = -1 \quad (42)$$

The necessary and sufficient condition for vanishing of Concircular curvature tensor C is (42)

6.3. Vanishing of Weyl Curvature Tensor *W*

From (11) let us assume that $W(X, Y)Z = 0$

Then we have

$$R(X,Y)Z = \frac{1}{n-2}\{S(Y,Z)X - S(Z,X)Y + g(Y,Z)QX - g(X,Z)QY\}$$

$$-\frac{r}{(n-1)(n-2)} g(Y,Z)X - g(X,Z)Y \quad (43)$$

On simplification we have,

$$\frac{r}{(n-1)(n-2)} = 1 \quad (44)$$

6.4. Vanishing of Projective Curvature Tensor P

From (37), let us assume $P(X,Y)Z = 0$

$$R(X,Y)Z = \frac{1}{n-1}\{S(Y,Z)X - S(X,Z)Y\} \quad (45)$$

So, the necessary and sufficient condition for vanishing of Projective curvature tensor P is (45)

7. Necessary and Sufficient Conditions for Vanishing of These Tensors with Quarter Symmetric Non-Metric Connection $\tilde{\nabla}$

7.1. Vanishing of Curvature Tensor $\tilde{R}$

From (24), let us assume that $\tilde{R}(X, Y)Z = 0$. Then we have

$$R(X,Y)Z + \eta(Y)g(X,Z) - \eta(X)g(Y,Z)\xi + \{\eta(Y)X - \eta(X)Y\}\eta(Z) = 0 \quad (46)$$

So, the necessary and sufficient condition for vanishing of Projective curvature tensor P is (46)

7.2. Vanishing of Projective Curvature Tensor $\tilde{P}$

From (38) let us assume that, $\tilde{P}(X,Y)Z = 0$.

Then, we have

$$\tilde{R}(X,Y)Z = \frac{1}{n-1}\{\tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y\} \quad (47)$$

$$S(Y, Z) = g(Y, Z) - \eta(Y)\eta(Z) \quad (48)$$

So, the necessary and sufficient condition for vanishing of Projective curvature tensor S is (48).

7.3. Vanishing of Concircular Curvature Tensor $\tilde{C}$

From (33) let us assume that, $\tilde{C}(X, Y)Z = 0$

Then, we have

$$\tilde{R}(X,Y)Z - \frac{\tilde{r}}{n(n-1)}(\tilde{g}(Y,Z)X - g(X,Z)Y) = 0 \quad (49)$$

So, the necessary and sufficient condition for vanishing of curvature tensor $\tilde{C}$ is (49)

7.4. Vanishing of Weyl Curvature Tensor $\tilde{W}$

From (35) let us assume that $\tilde{W}(X, Y)Z = 0$

Then, we have

$$\tilde{R}(X,Y)Z = \frac{1}{n-2}\{\tilde{S}(Y,Z)X - \tilde{S}(X,Z)X\} + g(Y,Z)\tilde{Q}X - g(X,Z)\tilde{Q}Y\} - \frac{\tilde{r}}{(n-1)(n-2)}\{\tilde{g}(Y,Z)X - \tilde{g}(X,Z)Y\}$$

Further we have,

$$\tilde{R}(X,Y)Z = \frac{1}{n-2}\{S(Y,Z)X + g(\phi Y,Z)X - S(X,Z)Y - g(\phi X,Z)Y + g(Y,Z)\{QX + \phi X\} - g(X,Z)\{QY + \phi Y\}\} -$$

$$\frac{r}{(n-1)(n-2)}\{g(Y,Z)X - g(X,Z)Y\} \quad (50)$$

So, the necessary and sufficient condition for vanishing of curvature tensor $\tilde{W}$ is (51).

8. Hessian Operator on Tensors

Let us assume that the Hessian operator $D2(X)$ is defined by

$$D2(X)(Y, Z) = Y.(Z.X) - (Y.Z).X \quad (52)$$

where $X.Y = D_X Y$

which implies

$$D^2(X)(Y,Z) = \nabla_Y(\nabla_Z X) - \nabla_{(\nabla_Y Z)} X$$

Let us assume

$$D2(X)(Y, Z) = 0 \quad (53)$$

which means

$\nabla_Y(\nabla_Z X) = \nabla_{(\nabla_Y Z)} X$ (54)

Similarly,

$\nabla_Z(\nabla_Y X) = \nabla_{(\nabla_Z Y)} X$ (55)

Then subtracting (54) from (53) gives

$\nabla_Y(\nabla_Z X) - \nabla_Z(\nabla_Y X) = \nabla_{(\nabla_Y Z)} X - \nabla_{(\nabla_Z Y)} X$ (56)

This implies $\nabla_Y \nabla_Z - \nabla_Z \nabla_Y = \nabla_{(\nabla_Y Z)} - \nabla_{(\nabla_Z Y)}$

$[\nabla_Y, \nabla_Z] = \nabla_{[Y,Z]}$ (57)

Acknowledgment

The author is thankful to Dr. Michel Nguiff Boyom for his suggestions to enrich the paper.

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