

About a “Certain-uncertain” Inconsistency within the Generally Accepted Experimental Procedures of Behavioral Economics

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Abstract

Old problems of the mathematical description of the economic behavior of a man are briefly reviewed. They are the comparison of choices of a man between uncertain and sure games and the radically different behavior of a man in different domains. The proposed solution of the problems consists of purely mathematical methods and models. They are briefly reviewed in the Appendix. In the present article, the main attention is paid to the analysis of the experimental support of this possible solution. The generally accepted random incentive experimental procedures are discussed. A “certain-uncertain” inconsistency between the certain type of choices and the uncertain type of incentives is revealed and analyzed.

Keywords: Utility; Prospect theory; Experiment; Incentive; Random-lottery incentive system; Prelec.

1. Introduction

There are several problems related to the mathematical description of the behavior of a man. Examples of these are the underweighting of high and the overweighting of low probabilities, the Allais paradox (Allais, 1953), risk aversion, loss aversion, the equity premium puzzle, the fourfold pattern of risk preferences, etc.

1.1. Comparison of Choices of a Man between Uncertain and Sure Games

One of the problems of this mathematical description is a comparison of the choices of a man between uncertain and sure games.

The essence of this problem consists of biases of choices of people (subjects) for the uncertain and sure games in contradiction with the predictions of the theory of probability. This problem is crucial and well-known. It is most important in behavioral economics in prospect and utility theories and also in psychology, decision theory, and the social sciences. It is pointed out in a wealth of works.

For example, we see in Kahneman and Tversky (1979)(pp. 265):

“PROBLEM 1: Choose between

A: 2,500 with probability .33, // 2,400 with probability .66, // 0 with probability .01;

B: 2,400 with certainty.

N = 72 [18] [82].”

My note. We see the clear inconsistency: 18% for A, that is less than 82% for B (for 72 trials) in opposition with the expectations $2,500 \times .33 + 2,400 \times .66 = 2,409$ for A, that is more than 2,400 for B.

For example, we see in Starmer and Sugden (1991)(pp. 974):

“... a choice between two lotteries R' (for “riskier”) and S' (for “safer”). R' gave a 0.2 chance of winning £10.00 and a 0.75 chance of winning £7.00 (with the residual 0.05 chance of winning nothing); S' gave £7.00 for sure.”

My note. $£10.00 \times 0.2 + £7.00 \times 0.75 = £7.25$ for R' and £7.00 for S' . Here the expectations are £7.25 for R' that is more than £7.00 for S' , but the results were 13 choices for R' that is less than 27 choices for S' .

1.2. Radically Different Behavior of a Man in Different Domains

An additional problem (that is probably a lot harder than the previous one) is, moreover, the radically different behavior of a man in different domains.

Thaler (2016)(pp. 1581–1582) wrote (the **boldface type** is my own):

“Kahneman and Tversky’s research documented numerous choices that **violate any sensible definition of rational**. ...subjects were **risk averse in the domain of gains** but **risk seeking in the domain of losses**.”

My note: at high probabilities.

For example, the data in [Kahneman and Tversky \(1979\)](#) (pp. 268) in Table 1 can be represented as:

Problem 3: (4,000 at 0.80) > (3,000 at 1.00) leads to choices [20%] < [80%].

Problem 3': (-4,000 at 0.80) < (-3,000 at 1.00) leads to choices [92%] > [8%].

My note. These data lead to the undoubted deduction of the clear inconsistency between the behavior of subjects in the domains of gains and losses.

The above problems were stated first in [Allais \(1953\)](#) and have not been solved at present (see, e.g., [Baccelli, 2024](#); [Kahneman and Thaler, 2006](#); [Thaler, 2016](#)).

1.3. The Present Article's Purpose and Central Argument

A man is the central subject of the economic theory. The above problems related to the mathematical description of the behavior of a man are therefore the significant issue of the theory. The present article's purpose is to make an essential step to solve these problems.

The central argument of the article is an intrinsic and inalienable inconsistency (that is revealed here) in the generally accepted experimental procedures of behavioral economics.

1.4. About the Methodology of the Presented Research

The methodology of the presented research includes the following.

The essence of research consists of an analysis of the experimental procedures of behavioral economics. The analysis includes revealing of the incentives used in the procedures and comparing them with the outcomes of the experiments.

The problems that are considered are the problems of the mathematical description of the behavior of a man. These problems were revealed in 1952 by [Allais \(1953\)](#). They were intensively studied till the present day see, e.g., [Thaler \(2016\)](#), [Baccelli \(2024\)](#), but are still unsolved. The data used in the research are the experimental data of the established scientists.

The purpose of the research is to make an essential step to solve the above problems. The objectives of the research are to answer purely logical and mathematical questions (see below), analyze experiments, and reveal inconsistencies.

The type of the presented research is theoretical and analytical. The general method of the research is the theoretical, logical analysis.

The data used in research are the experimental data of the established scientists. Logical tools and techniques are used to analyze the data.

The logical method is used to conclude the research.

2. Purely Logical Question

2.1. The Qualitative Difference between Certain and Uncertain Outcomes

As was mentioned above, one of the problems of behavioral economics is to compare certain (sure, guaranteed) and uncertain (probable) outcomes. The essence of these problems consists of biases between choices of people for certain and uncertain outcomes in contrast to the predictions of the theory of probability.

Some (little or big) qualitative difference between certain and uncertain outcomes is intuitively understood. Certain and uncertain outcomes can be moreover understood as opposite ones in some cases.

In the content of this article, it should be stressed that this qualitative difference takes place in any case regardless of the particular numerical results or values of such outcomes. It should be stressed especially that the particular numerical results (or values) of such outcomes can be very close to each other, but the qualitative difference remains true.

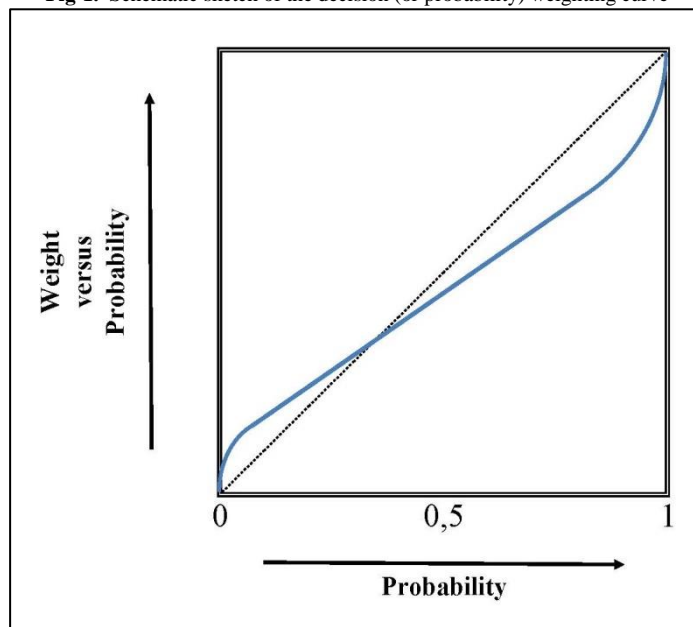
What creates the qualitative difference between certain and uncertain outcomes? The basis for a certain outcome is some guarantee that this outcome will come. This guarantee should be supported by some additive efforts.

So the origin of this qualitative difference is based on some additive efforts that are intended to guarantee certain outcomes.

2.2. Qualitative Difference Versus the Quantitative Equalities

Let us consider schematically (see fig. 1) the probability weighting (or Prelec's) curve (see, e.g., [Kahneman and Tversky \(1979\)](#), [Prelec \(1998\)](#)).

Fig-1. Schematic sketch of the decision (or probability) weighting curve



This curve displays the weight of the reaction of subjects (people) versus probability in experiments with probable (non-guaranteed) outcomes at probabilities from 0 to 1.

It is an inverse S-shaped function $\pi(p)$, that is first concave $\pi(p) > p$ at low probabilities and then convex $\pi(p) < p$ at high probabilities. Two features can be mentioned that should attract our attention.

1. The first feature. The curve deviates from the straight line $\pi(p) = p$ of the predictions of the probability theory near the boundaries of the probability scale.

2. The second feature. At the end-points of the boundaries, the curve tends to the straight line $\pi(p) = p$ of the predictions of the probability theory and touches it exactly at the boundaries. So $\pi(0) = 0$ and $\pi(1) = 1$.

Thus we see that the curve tends to the quantitative equalities at the end-points while the qualitative difference remains true. This fact was mentioned also in [Kahneman and Tversky \(1979\)](#) (pp. 282-283).

2.3. Purely Logical Question

2.3.1. Should They Cause?

The above features (especially the second one) can lead one to ask: Should qualitatively different situations cause the same quantitative result? One can easily give the evident answer “No” to this evident question.

2.3.2. Can They Cause?

If one asks:

Can qualitatively different situations lead to the same quantitative results? Then one can answer: Yes. They can. But they can only be under some special conditions. The discussed quantitative results nevertheless aren't special ones, they are widespread and well-established.

If there are the facts that qualitatively different situations lead to the same quantitative results, then, at least, three evident consequences can be derived from these facts.

1. The situations are not qualitatively different.

There is little likelihood of this consequence.

2. The quantitative results are not equal to each other (are not the same).

There is little likelihood of this consequence because plenty of skilled and independent research groups have repeated these results.

3. The situations have, in a sense, very little influence on the results.

Therefore one can come to a question:

2.3.3. Why Do They Cause?

As the result, we come to a purely logical question:

Why do the different qualitative situations cause the same quantitative result at the end points of the probability weighting curve?

3. Purely Mathematical Restrictions

3.1. One Possible Way to Solve the Problems

One (and the first possible) way to solve the above problems is widely discussed, e.g., by [Schoemaker and Hershey \(1992\)](#), [Hey and Orme \(1994\)](#), [Chay et al. \(2005\)](#), and [Butler and Loomes \(2007\)](#). The essence of this way

consists of proper attention to noise, uncertainty, imprecision, and other reasons that might cause dispersion, scattering, and spread of the data.

3.2. Another Possible Way. The Aczél-Luce question whether $W(1)=1$

Another possible way to solve the problems of prospect and utility theories is to consider the vicinities of the boundaries of the probability scale, e.g. at $p \sim 1$ (see [Aczél and Luce \(2007\)](#)).

[Aczél and Luce \(2007\)](#) emphasized a fundamental question: whether $W(1)=1$ or, equivalently, $\pi(1)=1$ (whether the Prelec's weighting function (see [Prelec \(1998\)](#)) $W(1)$ is equal to 1 at $p=1$). From now on, I refer to this question as the Aczél-Luce question (or, in short, as the Luce question).

3.3. Additional Definitions

There are a number of the evidences of the qualitative difference between subjects' treatment of the probabilities of probable and certain outcomes (see, e.g., [Kahneman and Tversky \(1979\)](#), [McCord and de Neufville \(1986\)](#), [Halevy \(2008\)](#)). Therefore, in a general case, one should distinguish between the values of the probability weighting function of the certain outcome and of the limit of the probability weighting function of the uncertain outcome when the probability of this uncertain outcome tends to 1.

Let us additionally define or specify a value $W_{Certain}$ of the probability weighting function $W(p)$ for the certain outcome. $W_{Certain}$ may be assumed to be equal to 1 or else values of $W(p)$ may be normalized by $W_{Certain}$.

Let us additionally specify a value $W(1)$ as the limit of the probability weighting function $W(p)$ for the probable outcome when p tends to 1

$$W(1) \equiv \lim_{p \rightarrow 1} W(p).$$

If $W(1)=W_{Certain}$, then $W(p)$ is continuous (at $p=1$). In the general case, this has not been proven. So, if W_{Unreal} is defined similarly for the impossible case,

$$W(p) = \begin{cases} W_{Unreal} & p = 0 \\ W(p) & p \in]0, 1[\\ W_{Certain} & p = 1 \end{cases}$$

(see also [Aczél and Luce, 2007](#)) and $W(p)$ can be continuous or discontinuous.

3.4. Modification of the Question. A synthesis of the two ways

Let us reformulate and modify the Aczél-Luce question whether $W(1)=1$ into the question whether $W(1)=W_{Certain}$ or whether $W(p)$ is continuous at $p=1$.

To answer the question and to prove or disprove the continuity of $W(p)$ at $p=1$ one should determine and measure the difference

$$W_{Certain} - W(1) = ?$$

Note, that this does not put a question whether $W_{Certain}$ is equal to 1.

The answer $W(1) \neq W_{Certain}$ to the modified Aczél-Luce question means that the function can have a discontinuity at $p=1$. This is not only a quantitative but a qualitative, moreover, a topological feature also. So, the answer to the question can qualitatively change the situation in the prospect and utility theories, at least in their mathematical aspects.

In any case, one may suppose that a synthesis of these two possible ways can be of some interest. This idea of the synthesis turned out to be useful indeed. It has successfully explained, at least partially or qualitatively, the underweighting of high and the overweighting of low probabilities, risk aversion, and some other problems (see, e.g., [Harin, 2012a;2012b](#)). There are also works providing experimental support for this synthesis (see, e.g., [Steingrimsson and Luce \(2007\)](#), [Harin \(2014\)](#)).

A purely mathematical research (see, e.g., [Harin, 2012a;2022](#)) combines, synthesizes the two above-mentioned ways. That is, it considers the dispersion of data (or the influence of the dispersion of the data) near the boundaries of the probability scale.

3.5. Existence Theorem and its Consequences

A purely mathematical theorem was proved, e.g., in [Harin \(2022\)](#). It states that near every boundary of the probability scale, there is some non-zero restriction on the expectation of a real-valued random variable under the condition that its variance is non-zero. These restrictions may be considered as some forbidden zones for the expectation (see the Appendix of the present article).

The theorem answers the above logical question (strictly speaking, it cancels this question). Indeed, the theorem states that the probability weighting curve cannot tend to the straight line $\pi(p)=p$ exactly at the endpoints of the probability scale. In particular, the expectations cannot take values within this forbidden zone and are therefore biased from the boundaries of the probability scale to its middle. So the considered particular qualitatively different situations should not and cannot lead to the same quantitative results.

The theorem allows moreover to explain the above problems, at least partially. Namely, an applied mathematical method of biases of expectations, qualitative mathematical models, and a special qualitative mathematical model is proposed by [Harin \(2022\)](#). The special qualitative mathematical model explains in particular

the above problems. The method, general models, and special model are briefly reviewed in the Appendix of the present article.

4. Pure Mathematical Question

4.1. Preliminaries

As has been mentioned in the preceding subsection, the theorem answers the above logical question. Indeed, the theorem states that, due to the forbidden zones, the probability weighting curve cannot tend to the straight line $\pi(p)=p$ exactly at the endpoints of the probability scale.

Nevertheless, the experiments seem to contradict the above logic and are seemed to don't manifest the predictions of the theorem.

For example, in the experiments at $p \sim 1$, in particular, the tendency of the experimental (Prelec's) probability weighting curve to $\pi(p)=p$ (see, e.g., Bruhin *et al.* (2010), Abdellaoui *et al.* (2011)), seems to do not support the theorem.

One can see that a question arises:

4.2. Pure Mathematical Question

So a pure mathematical question arises:

Why the mathematics has not been manifested and confirmed by the experiments?

Note that, strictly speaking, one should name the question "purely mathematical" only in that parts of it those are not concerned with experiments and not in parts that cannot be named as "purely mathematical".

So the theorem needs experimental confirmation.

5. Analysis of a Typical Detail of the Experiments

5.1. Random, Uncertain Incentives

Let us analyze a feature of the abovementioned experiments. Let us consider some typical descriptions of the prospect and utility theories' experiments. One can see in the literature (the boldface type is my own):

Loewenstein and Thaler (1989), page 188: "The students ... were told that the experimenter would select and implement one of their choices at random."

Tversky and Thaler (1990), page 206: "The subjects are told that one of these pairs will be selected at random at the end of the session and that they will play one of these bets."

Kahneman *et al.* (1991), page 195: "One of the four market trials would subsequently be selected at random and only the trades made on this trial would be executed". Page 197: "One of the accepted offers (including the original endowment) was selected at random at the end of the experiment to determine the subject's payment."

Vossler *et al.* (2012), page 158: "Participants are instructed that one of the 12 choice sets will be randomly chosen at the end of the experiment, each with equal probability."

Baltussen *et al.* (2012), page 424: "In the WRIS treatment, subjects play the game ten times, one of which for real payment. In the BRIS treatment, subjects play the game only once with a one-in-ten chance of real payment."; page 425: "In both RIS treatments, a ten-sided die was thrown individually by each subject to determine her payment."

Such a procedure can be seen not only in the field of the prospect and utility theories but also in other fields of economics, see, e.g., Larkin and Leider (2012), page 193: "Subjects made fifteen choices between a lottery and a fixed payment. ... Subjects were paid for one randomly selected decision".

5.2. Certain Outcomes

So, subjects are stimulated by random incentives. This is a well-known feature of the experiments including in the field of prospect and utility theories. But let us consider this feature more closely. One can see fine detail in the literature (the boldface type and underlining are my own):

Andreoni and Sprenger (2012), page 3365: "One choice for each subject was selected for payment by drawing a numbered card **at random**. Subjects were told to treat each decision as if it were to determine their payments." and page 3366: "Section I provided a testable hypothesis for behavior across certain and uncertain intertemporal settings."

Gaudecker von *et al.* (2011), page 669: "Additionally, for one in every ten participants in these two treatments, one lottery was **randomly** selected and played out, and the payoff of that lottery was paid out." and page 667: "the probabilities of the high payoff in each option vary from 25 percent to 100 percent".

Holt and Laury (2002), page 1646: "... subjects began by indicating a preference, Option A or Option B, for each of the ten paired lottery choices in Table 1, with the understanding that one of these choices would be selected **at the random** post and played to determine the earnings for the option selected." and "Even the most risk-averse person should switch over by decision 10 in the bottom row, since Option B yields a sure payoff of \$3.85 in that case" and page 1645: TABLE 1 "Option A: 10/10 of \$2.00, 0/10 of \$1.60. Option B: 10/10 of \$3.85, 0/10 of \$0.10"

Harrison *et al.* (2005), page 898: "We undertook a new series of experiments that build closely on the basic design features of HL," ("HL" means here Holt and Laury (2002)) "but allow an identification of the extent to which

the apparent scale effects on risk aversion are order effects.” and from TABLE 1: **Probability=1** Payoff=\$2 **Probability=0** Payoff=\$1.60 **Probability=1** Payoff=\$0.10.

Bruhin *et al.* (2010), page 1380: “At the end of the experiment, one-row number of one decision sheet was **randomly** selected for each subject, and the subject’s choice in that row determined her payment.” and “The subjects had to indicate in each row of the decision sheet whether they preferred the lottery or the **certain** payoff.” and page 1378: “We elicited **certainty equivalents for** a large number of two-outcome lotteries.”

Abdellaoui *et al.* (2011), page 704: “Each series involved a choice between a prospect and an ascending range of **sure** payments, ... At the beginning of the experiment, each subject was told that one of his choices would be **randomly** drawn and then played for real.”

Starmer and Sugden (1991), page 974: “Subjects in groups B and C knew that they were taking part in a **random**-lottery experiment in which questions 21 and 22 had equal chances of being for real.” and “One problem, which we shall call P, required a choice between two lotteries R' (for "riskier") and S' (for "**safer**"). R' gave a 0.2 chance of winning £10.00 and a 0.75 chance of winning £7.00 (with the residual 0.05 chance of winning nothing); S' gave £7.00 **for sure**.”

So, the random incentive procedures are used not only in the probable but in certain situations too. Let us consider this detail more closely.

6. “Certain-uncertain” Inconsistency of the Random-lottery Incentive Systems

6.1. Inconsistency between the Uncertain Incentives and Certain Outcomes

First, let us note that the stimulation by the random payment of one from two or more alternatives may be named as a random, uncertain stimulation. One may name it also as the stimulation by an uncertain incentive.

Further, let us consider the stimulation of this uncertain incentive for uncertain and certain choices.

Suppose, that subjects choose an uncertain choice, that is the choice, which probability is strictly less than 1 (and strictly more than 0). In this case, the choice and the incentive are of the same type.

Suppose, that the subjects choose a certain choice, that is the choice, which probability is strictly equal to 1 (or strictly equal to 0). In this case, the choice and the incentive are of qualitatively different types. The choice is certain but the incentive is uncertain.

Hence, there is an evident inconsistency between the certain type of choice and the uncertain type of incentive.

One may name this problem as a “certain-uncertain” inconsistency.

So one can define the “certain-uncertain” inconsistency as the inconsistency between a certain type of choice and the uncertain type of the incentive within the random incentive experimental procedure systems.

This “certain-uncertain” inconsistency is evident but is not still mentioned in the literature (see, e.g., Andreoni and Sprenger (2012), Vossler *et al.* (2012), Baltussen *et al.* (2012), Harin (2005-2024)). The inconsistency was revealed (discovered) in the report by Harin (2014).

6.2. Role of the Incentives

Incentives are widely discussed in economics (see, e.g., Starmer and Sugden (1991), Fehr and Falk (2002), Holt and Laury (2002), Baltussen *et al.* (2012), Larkin and Leider (2012)). Do incentives influence the choice of the subjects in the prospect and utility theories and models?

The correct answer to this question needs special research. However, one may be sure, that if incentives would not have any significant influence on the choice of the subjects, then there would be no reason to use such incentives.

So, one may not exclude that an incentive can influence a choice of a subject, at least partially.

6.3. Random-Lottery Incentive Systems as the Generally Accepted Experimental Procedure Systems

The discussed random incentive procedures are usually referred to as the random-lottery incentive systems (or the random lottery incentive systems or random incentive systems (RIS), etc.).

Baltussen *et al.* (2012), page 219: “Savage (1954, p. 29) credits W. Allen Wallis for first proposing the RIS.”

Page 420: “RISs are known under several names, including random lottery incentive system (Starmer and Sugden, 1991), random lottery selection method (Holt, 1986), random problem selection procedure (Beattie and Loomes, 1997), and random round payoff mechanism (Lee, 2008). The different names apply to particular types of experiments (risky choice or social dilemma), rewards (lotteries or outcomes), or tasks (composite or single-choice)”

The random-lottery incentive systems are well-known mechanisms in experiments in the prospect and utility theories and models.

Starmer and Sugden (1991), page 971: “This is the random lottery procedure. This incentive system has several attractive features. It allows the experimenter to collect a considerable amount of data from each subject, thus economizing on the costs of recruiting subjects and allowing tests that compare a subject's responses to two or more tasks. At the same time, it avoids the problem of reference points and wealth effects that would be created if subjects were paid according to their performances on each of several tasks. (A subject's response to one task might be affected by the amount he or she had won on a previous task.)”

Andreoni and Sprenger (2012), page 3365: “random-lottery mechanism, which is widely used in experimental economics” Starmer (2000), page 371: “Although most experiments involve real — usually monetary — incentives, the most common reward mechanism is the random lottery incentive system. In experiments with this design, subjects are rewarded according to their response to one task which is randomly selected at the end of the experiment.”

Wakker (2007): “The random-lottery incentive system has become the almost exclusively used incentive system for individual choice, and numerous studies have used and tested it. It is used by people well recognized in experimental economics ... Without it, real incentives for individual choice are no longer well possible.”

Baltussen *et al.* (2012), page 419: “If a subject performs multiple tasks in an experiment where each task is for real, then income and portfolio effects will arise (Cho and Luce, 1995; Cox and Epstein, 1989). The RIS is the only incentive system known today that can avoid such effects. In addition, for a given research budget and with the face values of the monetary amounts kept the same, RISs allow for a larger number of observations.”

So, the random-lottery incentive systems are the prevailing and generally accepted experimental procedures in the prospect and utility theories and models. The tests of the Aczél-Luce questions are often connected with these systems.

7. To Consider Certain Outcomes as Uncertain

So, one may conclude:

1) The random-lottery incentive systems are widespread in the prospect and utility theories and models. They are moreover the generally accepted experimental procedure systems.

2) The essence, in particular the incentives of these random-lottery incentive systems, corresponds to the random, uncertain character of their name.

3) These incentives are used for all the outcomes including certain ones. This causes the evident inconsistency between the uncertain incentives and certain outcomes. One may name this inconsistency as the “certain-uncertain” inconsistency.

4) The specific “certain-uncertain” inconsistency of the random-lottery incentive systems is a natural feature of these systems. But it has still not been considered as it deserves.

In many works (see, e.g., Gaudecker von *et al.* (2011), (Vossler *et al.*, 2012), (Harin, 2005-2024)) one can find the elaborated researches of correctness of the random-lottery incentive systems. However, the author of the present article has found no mention of what one can name the “certain-uncertain” inconsistency.

5) So, the (direct and non-direct) tests of the modified Luce question that have been performed to date (those, maybe, have not been analyzed thoroughly enough) can lead to the misleading opinion that, at $p \sim 1$, the probable outcomes are quantitatively the same as the certain ones. That is, it can lead to the misleading opinion that the probability weighting function $W(p)$ tends to W_{certain} when p tends to 1.

However, there are evidences of the qualitative difference between the probable and certain outcomes (see, e.g., Kahneman and Tversky (1979), McCord and de Neufville (1986), Halevy (2008)). The existence theorem supports this qualitative difference and the answer $W(1) \neq W_{\text{certain}}$ to this question.

6) Because of this evident “certain-uncertain” inconsistency, the deductions from the random lottery incentive experiments, which include certain outcomes, cannot be unquestionably correct.

In particular, at present, the existing random-lottery incentive systems seem unable to determine the quantitative difference between certain and probable outcomes at $p \sim 1$.

So, these deductions need additional proof or an amendment.

7) In any case, additional independent analyses and/or research of the “certain-uncertain” inconsistency are needed.

8) The following feature should be stressed: the uncertain incentives can call certain outcomes into question.

That is, the uncertain incentives of the generally accepted experimental procedure systems can mislead the subjects. It can mislead the subjects to consider (to some extent, indirectly or directly, unconsciously or consciously) the certain outcomes, that are stimulated by these uncertain incentives, as some uncertain outcomes.

8. Confirmations of the Theorem

Let us consider some of the many confirmations of the occurrence of the discussed forbidden zones. Let us consider also the usefulness of these zones and SQMM (see the Appendix) in behavioral economics. Let us also find adequate answers to the logical and mathematical questions.

8.1. Practical Everyday Life Evidence of the Occurrence of the Considered Forbidden Zones

8.1.1. Boat and Waves

Consider a calm or mirror-like sea and a small rigid boat or any other small rigid floating body at rest in the sea. Suppose that this boat or body rests right against (or is constantly touching) a rigid moorage wall. As long as the sea is calm, the expectation of its side can touch the wall.

Suppose there is a heavy sea. Consider a small rigid boat or any other small rigid floating body that oscillates on the waves in the heavy sea. Suppose that this boat or body oscillates on the waves near this rigid moorage wall.

When the boat is oscillated by sea waves, then its side oscillates also (both up–down and left–right) and it can touch the wall only in the (nearest) extremity of the oscillations. Hence the expectation of the side cannot touch the wall. Hence the expectation of the side is biased away from the wall.

So, one can say that, in the presence of waves, a forbidden zone exists between the expectation of the side and the wall.

This forbidden zone biases the expectation away from the wall. The width of the forbidden zone is roughly one-half of the amplitude of the oscillations.

8.1.2. Washing Machine, Drill

Consider a washing machine or drill (or any other rigid body) that can vibrate when it works. Suppose its rigid (and not sharp, thick) side (or some rigid limiter of the movement of its side) is located near a rigid surface or wall.

If the machine or drill is at rest, then the expectation of this side can be located right against (constantly touching) the wall.

If the machine or drill vibrates, then the expectation of this side is biased and kept away from the wall due to the vibrations.

So, in the presence of the vibrations, a forbidden zone exists between the expectation of the side of the rigid body and the rigid wall. This is true for any rigid body near any rigid surface or wall.

8.2. Numerical Examples of the Forbidden Zones in Behavioral Economics

8.2.1. Practical numerical example. Gains

Suppose that the parameters of the special practical qualitative mathematical model (see the Appendix) for the gains are: the presupposed bias for the choices for the uncertain game is equal to \$2, and for the sure game it is equal to \$1.

The typical examples (see, e.g., [Correa et al. \(2013\)](#), [Zhang and Siemsen \(2019\)](#)) can be simplified to the special qualitative situations similar to that of [Harin \(2022\)](#) and [Kumar and Goyal \(2015\)](#). The enumeration corresponds to the Appendix.

Imagine that you face the following pair of concurrent games (a sure game and an uncertain game). Choose between:

A) A sure gain of \$99.

B) A 99% chance to gain \$100 and a 1% chance to gain or lose nothing.

In the ideal case, without taking into account the dispersion of the data, the expectations μ_{sure} and μ_{uncert} are equal to each other: $\mu_{sure} = \$99 \times 100\% = \99 and $\mu_{uncert} = \$100 \times 99\% = \99 . So, in the ideal case, we have $\$99 = \99 , that is, the uncertain and sure games are equally preferable.

In the real case, one should take into account some dispersion of the data, and hence the minimal non-zero variance (3) caused by this dispersion, and the forbidden zones (4) caused by this variance, at least for the uncertain games.

One can consider the real case of a non-zero variance of the data, the corresponding forbidden zones, and presupposed biases.

The biases are $\Delta ch - \mu_{uncert} = \2 and $\Delta ch - \mu_{sure} = \1 . Sowe have $\mu_{sure} - \Delta ch - \mu_{sure} = \$99 \times 100\% - \$1 = \98 and $\mu_{uncert} - \Delta ch - \mu_{uncert} = \$100 \times 99\% - \$2 = \97 . The expectation μ_{uncert} is biased more than μ_{sure} and

We see the evident difference between the resulting expectations (with their biases caused by the forbidden zones of the theorem) and their correspondence with the salient and unequivocal choices of the subjects.

8.2.2. Practical Numerical Example. Second Domain. Losses

The case of gains has been explained many times and in a lot of ways. But a uniform explanation for both gains and losses, without any additional suppositions (as, e.g., in [Correa et al. \(2013\)](#)), had not been nevertheless recognized by the author of the present article (see slightly similar work by [Arnold et al. \(1983\)](#)).

SQMM (Special Qualitative Mathematical Model, see the Appendix) turns out to be useful for such a uniform explanation.

The case of losses was considered under the same suppositions as for the case of gains.

Imagine that you face the following pair of concurrent games (a sure game and an uncertain game). Choose between:

A) A sure loss of -\$99.

B) A 99% chance to lose -\$100 and a 1% chance to lose or gain nothing. In the ideal case, for the sure game $\mu_{sure} = -\$99 \times 100\% = -\99 and for the uncertain one $\mu_{uncert} = -\$100 \times 99\% = -\99 . So they are exactly equal to each other: $-\$99 = -\99 :

Therefore both choices (games) should be equally preferable.

Consider the real case. The forbidden zone biases the expectation from the boundary of the interval to its middle (see also, e.g., [Kahneman and Thaler \(2006\)](#), [Madansky \(1959\)](#), [Zhang and Siemsen \(2019\)](#)). Therefore, at high probabilities, the biases are subtracted from the absolute values for both cases, gains and losses. That is, due to the opposite signs of the values for gains and losses, the bias is subtracted for the gains and added for the losses.

Note. This is not a supposition but a rigorous conclusion. Hence the conditions of the SQMM are naturally uniform for more than one domain.

The forbidden zones were considered under the same suppositions as for the gains, that is for the same, uniform parameters.

The biases are $\Delta_{ch-\mu.uncert} = \2 and $\Delta_{ch-\mu.sure} = \$1$. So we have $\mu_{sure} + \Delta_{ch-\mu.sure} = -\$99 \times 100\% + \$1 = -\98 and $\mu_{uncert} + \Delta_{ch-\mu.uncert} = -\$100 \times 99\% + \$2 = -\97 . The expectation μ_{uncert} is biased more than μ_{sure} and $-\$98 < -\97 .

The expectation for the uncertain game is biased more than that for the sure one, as was also the case for the gains, but here the bias increases the preferability of the uncertain loss and it is (due to the obvious difference between the resulting expectations) more preferable than the sure one.

So here is the clear difference between the resulting expectations and their correspondence with the salient preferences and choices. So the SQMM explains the domain of losses as well. Moreover, this explanation is uniform for both domains of gains and losses.

9. Answers to the Questions, etc.

Let us find adequate answers to the logical and mathematical questions.

9.1. Logical Question

The purely logical question was: Why do the different qualitative situations cause the same quantitative result at the end points of the probability weighting curve?

The above analysis and confirmations lead to the statement that “the same quantitative result” is at least under question. The theorem states that it is moreover impossible.

The discussed different qualitative situations don't (and cannot) cause the same quantitative result. Feasibly this is due to some mistake in the interpretation of the results of the experiments.

So the purely logical question is cancelled.

9.2. Mathematical Question

The pure mathematical question was:

9.2.1. Why the Mathematics

Has not been manifested and confirmed by the experiments?

The above analysis and confirmations lead to the statement that mathematics is confirmed by everyday experience and (at the correct interpretation of the experiments) by the experiments in behavioral economics.

So the pure mathematical question is answered.

9.3. Policy Suggestions

One can make the following policy suggestions in behavioral economics.

The analysis and conclusions of the present research should be thoroughly tested by more than one independent researcher or research team. The same testing should also be done for the mathematical methods and models proposed in [Harin \(2022\)](#).

If at least a part of these analyses and conclusions are true, then an open discussion should be initiated about the important items that are discussed here.

In any case, the revealed “certain-uncertain” inconsistency within the generally accepted experimental procedure systems of behavioral economics should be publicly discussed. This is necessary due to the evidence of this inconsistency and the total lack of any public discussion and even references on this important item.

If the results of the present article and [Harin \(2022\)](#) are true, then these results can be used in the existing theories and models to correct the interpretations of their data and results. This can be made by, e.g., simple corrections of the probability weighting curve.

10. Conclusions

The old problems of behavioral economics and a new approach to their possible solution are briefly reviewed. The problems are concerned with the mathematical description of the behavior of a man. The first problem of this mathematical description is the comparison of the choices of a man between uncertain and sure games. The second is the radically different behavior of a man in different domains.

The basis of the possible solution of these problems is the purely mathematical theorem of the existence of some forbidden zones for the expectations of any real-valued random variable. These forbidden zones are located near the boundaries of the probability scale. The consequences of the theorem are the hypothesis of presupposed biases in the behavior and decisions of people, the applied mathematical method of biases of expectations (MMBE), and general and special (SQMM) qualitative mathematical models.

In the present article, the main attention is paid to the analysis of the experimental support of this possible solution.

The generally accepted and prevailing random incentive experimental procedures are analyzed and the “certain-uncertain” inconsistency between the certain type of the outcomes and the uncertain type of the incentives is revealed.

This inconsistency can lead (in the minds of people) to the (hidden) misleading understanding of certain outcomes as some uncertain ones. This misleading understanding can explain the incorrect interpretation of the

experiments at very high and very low probabilities where the forbidden zones bias the behavior and choices of people.

The confirmations of the occurrence of the discussed forbidden zones and the usefulness of them and SQMM are considered as well.

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A. Appendix. A new Applied Mathematical Method and Models for Behavioral economics

This Appendix summarizes some of the results of Harin (2022). The notation and numbering of expressions are maintained.

A.1. Existence theorem in a little more detail

A set $\{X_i\}$, $i = 1, \dots, n$, of real-valued random variables X_i whose values lie within an interval $[a, b]$ was considered in, e.g., Harin (2020), Harin (2022). Under the normalizing condition

$$\sum_{k=1}^K p_X(x_k) + \int_{-\infty}^{+\infty} f_X(x) dx = \sum_{x_k \in [a, b]} p_X(x_k) + \int_a^b f_X(x) dx = 1, \quad (1)$$

where K is the number of the discrete value(s) of X , $p_X(x_k)$ is the probability mass function and $f_X(x)$ is the probability density function, the expectation and variance of X , and their relations were analyzed.

A proof was made that, for the variance σ^2 of any real-valued random variable X_i with range $[a, b]$ and expectation μ ,

$$\sigma^2 \leq (\mu - a)(b - \mu). \quad (2)$$

Suppose a set $\{X_i\}$, $i = 1, \dots, n$, of random variables X_i whose values lie within an interval $[a, b]$. If $0 < (b-a) < \infty$ and there exists a forbidden zone (or a lower bound) of a non-zero width σ_{\min}^2 for the variances σ_i^2 of X_i , such that for all i ,

$$\sigma_i^2 \geq \sigma_{\min}^2 > 0. \quad (3)$$

The theorem states: certain forbidden zones (or boundary bounds, or restrictions) of a non-zero width r_μ exist for the expectations μ_i of each X_i such that

$$a < (a + r_\mu) \leq \mu_i \leq (b - r_\mu) < b. \tag{4}$$

The proof uses inequalities (2) and (3) to obtain inequalities including

$$\mu_i \geq a + \frac{\sigma_i^2}{b-a} \geq a + \frac{\sigma_{\min}^2}{b-a},$$

to obtain restrictions r_μ or boundary bounds or forbidden zones of the width

$$r_\mu \equiv \frac{\sigma_{\min}^2}{b-a} \leq \frac{\sigma_i^2}{b-a}, \tag{5}$$

and inequalities

$$a < \left(a + \frac{\sigma_{\min}^2}{b-a} \right) \leq \mu_i \leq \left(b - \frac{\sigma_{\min}^2}{b-a} \right) < b \tag{6}$$

those are equivalent to (4).

A.2. Hypothesis of Presupposed Biases

The above practical examples of section 6 illustrate possible forbidden zones of the theorem. Similar examples are widely disseminated in real life. Due to this dissemination, subjects (people) can keep in mind the feasibility of such forbidden zones and the biases of the expectations caused by the zones. This can influence the behavior and choices of the subjects.

In consequence of this consideration, I propose a statement that can be named as a behavioral hypothesis of presupposed biases:

“People, as economic subjects, behave and decide (at least to a considerable degree) as if there were some presupposed (hypothetical) biases of the expectations for games.”

Note. This hypothesis can be supported by the reason that such biases may be proposed and tested even from a purely formal point of view.

This hypothesis can be found in hidden forms in the literature or derived from it (see, e.g., [Kumar and Goyal \(2015\)](#), [Correa et al. \(2013\)](#), [Schoemaker and Hershey \(1992\)](#), etc.) in this particular field, and an explicit form in neighboring fields (see, e.g., [Aldashev et al. \(2011\)](#), [Meier and Sprenger \(2010\)](#)). Nevertheless, one should state it here in an explicit form and emphasize it.

A.3. Applied Mathematical Method of Presupposed Biases of Expectations

A.3.1. Propositions

Two main propositions can be suggested for a mathematical method of solution to the above problems.

Proposition 1. *Subjects behave and decide (at least to a considerable degree) as if there were some presupposed biases of the expectations for games.*

Due to this proposition, the method (approach) can be called an Applied Mathematical Method of Presupposed Biases of Expectations, or AMMPBE, or shortly MMBE. The MMBE is to explain not only the objective situations but also and mainly the subjective behavior and choices of subjects.

Proposition 2. *Explanation by the forbidden zones of the theorem.*

That is these biases (real biases or subjective reactions and choices of the subjects) can be explained (at least to a considerable degree) with the help of the forbidden zones of the theorem.

A.3.2. Notation

The real expectations for the games were denoted by

$$\mu_{\text{sure}} \quad \text{and} \quad \mu_{\text{uncert}} \equiv \mu_{\text{uncertain}}.$$

The presupposed biases (of the expectations) that are required to obtain the data corresponding to the choices of the subjects were denoted by

$$\Delta_{\text{ch-}\mu.\text{uncert}} \equiv \Delta_{\text{choice-}\mu.\text{uncertain}} \quad \text{and} \quad \Delta_{\text{ch-}\mu.\text{sure}} \equiv \Delta_{\text{choice-}\mu.\text{sure}}.$$

That is the resulting expectations (i.e., expectations including these biases) for the observed choices of the subjects can be written as $\mu_{\text{uncert}} + \Delta_{\text{ch-}\mu.\text{uncert}}$ for the uncertain games and as $\mu_{\text{sure}} + \Delta_{\text{ch-}\mu.\text{sure}}$ for the sure ones.

A.3.3. General Relations

1. *Condition for the MMBE.* Due to the first main proposition, the method of biases of expectations can be useful only if these biases for the choices for the uncertain (see the subparagraph “3. Biases for sure games” below) games are non-zero

$$|\Delta_{\text{ch-}\mu.\text{uncert}}| > 0 \quad \text{or} \quad \text{sign}(\Delta_{\text{ch-}\mu.\text{uncert}}) \neq 0. \tag{7}$$

2. *Forbidden zones as, at least, one of the origins of biases.* The presupposed bias $\Delta_{ch-\mu.uncert}$ may be introduced and considered purely formally. The question is not only whether $\Delta_{ch-\mu.uncert}$ can explain the problems. Due to the above second proposition, $\Delta_{ch-\mu.uncert}$ itself should be explained by the forbidden zones of the theorem, at least partially.

First of all, the theorem should be applicable. Therefore some minimal variance is required to exist and inequality is required to be true.

Further, the bias caused by the forbidden zone of the theorem was denoted by $\Delta_{theorem}$. The sign of the presupposed bias should coincide with that of the bias caused by the theorem $\text{sign}(\Delta_{ch-\mu.uncert}) = \text{sign}(\Delta_{theorem})$.

Then the conditions for the explanation can be written as $\Delta_{ch-\mu.uncertain} \approx \Delta_{theorem}$ in the case when the forbidden zones are the main source of the biases. If these zones are only one of the essential sources of the biases, then these conditions can be represented as $\Delta_{ch-\mu.uncert} = O(\Delta_{theorem})$.

So the relations of the explanation by the theorem are

$$\sigma^2 \geq \sigma^2_{\min} > 0 \quad \text{and also} \quad \Delta_{ch-\mu.uncert} \approx \Delta_{theorem} \quad \text{or at least} \quad \Delta_{ch-\mu.uncert} = O(\Delta_{theorem}). \quad (8)$$

3. *Biases for sure games.* The above considerations about noise suppression and sure games emphasize the condition that the sure games are guaranteed by some guaranteeing efforts. Due to these efforts, the biases for the sure games can be suppressed and reduced. They can be moreover equal to zero.

Therefore the presupposed biases of the data for the sure games were assumed to be equal to zero or, more generally, are strictly less than the presupposed biases for the corresponding uncertain games.

So, the relation for the sure and uncertain games is

$$|\Delta_{ch-\mu.uncert}| > |\Delta_{ch-\mu.sure}|. \quad (9)$$

A.4. General and Special Qualitative Mathematical Models

A.4.1. General Qualitative Mathematical Models

A *qualitative problem* was defined for the models. It is defined as a problem such that the sign of the difference between the resulting expectations for the choices of the subjects for the uncertain and sure games is distinct from the sign of the difference between the real expectations for these games.

This type of problem was chosen as the example that is usual in experiments (see, e.g., [Correa et al. \(2013\)](#), [Zhang and Siemsen \(2019\)](#), [Schoemaker and Hershey \(1992\)](#)). It can manifest a qualitative representation of the problems and can be a background for further research. Such problems will be the item of the first stage of the MMBE.

The inalienable feature of the analyzed qualitative problems is the necessary change of the sign. There can be only three combinations of the signs: the expectation for the uncertain game (or outcome) can be: 1) greater than, 2) less than, or 3) equal to that for the sure game. So, the signs of their differences can be correspondingly positive, negative, or zero.

In other words, when the difference between the real expectations is, e.g., positive (that is, $\text{sgn}(\mu_{uncert} - \mu_{sure}) > 0$), then, to obtain the observed data, the difference for the choices (resulting expectations) should be non-positive. That is, $\text{sgn}(\mu_{uncert} + \Delta_{ch-\mu.uncert} - \mu_{sure} - \Delta_{ch-\mu.sure}) \leq 0$. When it is negative, then the difference between the choices should be non-negative. When the difference between the real expectations is equal to zero, then the difference between the choices should be undoubtedly positive or negative.

This feature can be represented by

$$\text{sgn}(\mu_{uncert} - \mu_{sure}) \neq \text{sgn}(\mu_{uncert} + \Delta_{ch-\mu.uncert} - \mu_{sure} - \Delta_{ch-\mu.sure}). \quad (10)$$

To overcome the real difference between the expectations for the uncertain and sure games, the absolute value of the presupposed bias for the uncertain game should be not less than this real difference. That is

$$|\Delta_{ch-\mu.uncert}| \geq |\mu_{uncert} - \mu_{sure}|. \quad (11)$$

So, relations (10) and (11) constitute an addition to the method. The sum of the method and addition can be named as a family of preliminary general qualitative mathematical models.

A.4.2. Special Qualitative Mathematical Model

A preliminary estimate by [Harin \(2018\)](#) restricts applications of the general model by the secure upper bound (5) for the bias. One of the main questions for future research is to analyze the possible widths of the forbidden zones for various types of distributions.

Let us consider the qualitative problems under the special condition

$$\mu_{uncert} = \mu_{sure} \quad (12)$$

This special condition and relation assert that the expectations for the uncertain games are exactly equal to the expectations of the corresponding sure games. This is a well-known and important case of real experimental situations. Here (12) (keeping in mind (7)) substitutes (10) and (11).

Such a special situation enables avoiding the constraints of preliminary estimate [Harin \(2018\)](#) of the secure upper bound (5) for the bias and making this special model less formal. The biases can be selected to be much less than the secure upper bound (5), and the suppositions will be simpler.

This Special Practical Qualitative Mathematical Model (SPQMM or shortly SQMM) can be considered as a first step of the first stage of the approach (method) MMBE.