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Non-Maturity Deposit Modeling in the Framework of Asset Liability Management

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Abstract: Liquidity risk is one of the major risks inherent in the banking business. It occurs when the bank does not have sufficient liquid assets to meet its commitments at the time of their occurrence. The most critical challenges confronting financial institutions when managing liquidity risk is so-called non-maturity accounts. These accounts are characterized by the fact that they have no specific contractual maturity, and their risk management is complicated by the embedded options that depositors may exercise. As part of an asset-liability management and for the purpose of healthy and prudential management of a liquidity risk, each bank must properly assess the deposits of its customers. Liquidity risk is not the risk that there are massive withdrawals, but the risk they are unanticipated. In this paper, we apply two methods to model non-maturity deposits of a Moroccan commercial bank. We treat separately individual deposits and enterprise deposits aiming an accurate analysis. We then select between the models by means of a selection criteria. Furthermore, we back-test and forecast future deposits using the selected model. Finally, we model the decay rates of non-maturity deposits by elaborating a flowing function of these latter.

Keywords: Non-maturity deposit; liquidity risk; Asset liability management; ARMA; Jarrow-Van Deventer model.
JEL Classification: C50 ; C51 C52 ; C58 ; G20 ; G21

1. Introduction

Competitiveness and international openness of the financial sector play a key role in the growth and development of the Moroccan economy. In recent years, this sector has undergone several reforms to make the Moroccan financial system more efficient and to create a competitive and healthy climate between banks. Financial institutions have therefore realized that they were subject to a number of risks which must be managed well.

Several reforms have emerged to make the banking sector more dynamic and more efficient. The first regulations concerning the banking risks were issued by the Basel Committee. The Basel III accords Basel Committee on Banking Supervision (2013) taking effect in 2015 incite banks to develop a performing system in terms of risk management.

Liquidity risk is one of the major risks inherent in the banking business. It occurs when the bank does not have sufficient liquid assets to meet its commitments at the time of their occurrence. An uncontrolled and excessive risk-taking or even a bad anticipation of the environment changes can threaten not only the financial stability of the bank in question, but also the stability of the banking sector as a whole.

In the literature we find several definitions of liquidity risk. The first definition is given by the Basel Committee: « Liquidity [is] the ability to fund increases in assets and meet obligations as they come due », Basel Committee on Banking Supervision (2008). According to this definition, the position of the bank is sufficiently liquid if it is able to finance an increase in assets and to meet its commitments as they become due. From there, one can formulate the first definition of liquidity risk which result through the inability to meet its commitments on the same date they become due. This risk arises when the bank suffered from unexpected needs and cannot face them by its liquid assets. The Basel Committee, as part of its Basel II agreements, has issued recommendations on liquidity risk management and on the principles that banks should follow for a relevant prudential supervision

Harrington Richard (1987) gave a definition of liquidity risk for a bank which is situated on three levels:

- Funding risk: risk that results in the need for new resources when the resources the bank own in the past is no longer available (for example at time of the massive withdrawal of deposits).

- Time Risk: risk that appears when the bank cannot get expected cash flow (for example inability to repay a loan by a customer).

- Call risk: risk relating to the obtaining of new resources (for example after significant loans on credit lines).

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The various points raised by Harrington, allows clearly to reveal the importance role played by the behavior of customers in the liquidity management of the bank. Indeed, the bank must manage its liquidity by the study of customer behavior:

- By assessing the resources it is sure to have in future dates through investments and deposits made by customers, assessing for example the worst flow it may consider on each station for the amounts available to it today (in terms of withdrawal of amounts placed on deposits of non-repayment of the loans ...)

- By developing its activity with necessary amounts to open new credits of customers when these request them. This requires to apprehend the new productions expected by the bank in the future.

Note that the sources of liquidity risk result mainly:

- From massive withdrawal and/or unanticipated deposits or from the withdrawal of client savings;
- From the general liquidity crisis of the market;
- From the market crisis of confidence with regard to the concerned bank.

Regarding unanticipated massive withdrawal of deposits or clients' savings withdrawal, often liquidity risk occurs when the institution does not have sufficient liquidity to cover unexpected needs. Generally banks are able to meet the withdrawals through their cash, however daily withdrawals are generally well anticipated and adequate reserve funds is retained. Liquidity risk is not the risk that there are massive withdrawals, but the risk they are unanticipated. For this, and due to the fact that the main resource of the bank in terms of liquidity consists of customer deposits, it is very important for the bank that the depositors do not make unanticipated massive withdrawals on their accounts so that the bank can protect themselves from a liquidity crisis that could lead to bankruptcy, or to a deterioration in the financial situation after a penalizing refinancing.

Demand deposits are an important part of commercial banks resources. The modeling of the stock of deposits represents a major challenge for these banks through the management of liquidity risk. These accounts are characterized by the possibility of immediate exigibility by the customers. They are in fact available to them at any time and without any charge. These accounts are considered non-maturities and present to the bank the risk to be payable by the holder at any time. This possibility, however, is to exclude since it is unlikely in normal situations that all customers withdraw their deposits at the same time. So the bank possess always an amount of deposits that is never affected by withdrawals over an agreed or calculated period. As part of an asset-liability management and for the purpose of healthy and prudential management of a liquidity risk, each bank must properly assess the deposits of its customers and the expected behavior of the latter in the normal case and in the case of a crisis.

In the case of the normal activity of the bank, it is sure that a significant part of these deposits will not be required, which allows the bank to convert this resource into remunerating but less liquid assets (loans, investments ...).

In terms of crisis situations that can lead to withdrawals from the accounts, they are the subject of simulated scenarios in "stress test". These methods assess the bank's position in case of serious liquidity problems. Banks must establish refinancing market strategies to address these liquidity crises.

In this paper, we suggest two methods to model the non-maturity deposits of a Moroccan commercial bank. We consider deposits as an example of non-contractual products of the bank and we propose two models to assess both the individual accounts owned by households and enterprise accounts owned by companies. We then select between the models by means of a selection criteria. After that, we apply the model selected to make a Back-Testing and a forecast of future deposits. Finally, we model the flowing function of individual deposits and enterprises deposits. The elaborations of flowing conventions of non-maturity deposit is fundamental in the asset liability management. It consists in describing how the stocks of the balance sheet elapse and consequently permits the measuring of the liquidity gap.

2. Review of Related Literature

In this paper we contribute to the literature of modeling the non-maturity accounts. These accounts have a non-contractual maturity, and their risk management is complicated by the embedded options that depositors may exercise. We apply two types of models to assess non-maturity deposits of a Moroccan commercial bank. The first model is ARMA time series model James and Hamilton (1994) and the second is Stochastic model of Jarrow and Van (1998).

Most papers use the risk-neutral modeling approach introduced by Jarrow and Van (1998) or a discounted cash flow framework under the probability distribution in a real-world. Jarrow and Van (1998) apply a one-factor Heath *et al.* (1992) model (Heath-Jarrow-Morton model) using the short rate dynamics and a product rate process depending on the shifts and changes of market interest rates. Brien (2000) uses a Cox *et al.* (1985) term structure model and an asymmetric partial adjustment model for the product rate. Kalkbrener and Willing (2004) apply a two-factor Heath-Jarrow-Morton model and a piecewise linear function which links product rates to market rates. Dewachter *et al.* (2006) apply a three-factor term structure model and an additional fourth factor for the product rate spread dynamics. We notice that the Hutchison and Pennacchi (1996) model assuming that short rates follow a Vasicek (1977) process can also be incorporated in the risk-neutral valuation approach. At the empirical level, Janosi *et al.* (1999) use the Jarrow and Van (1998) model to analyze a US commercial bank's deposit data and aggregate negotiable orders of withdrawal account data from the Federal Reserve. Hutchison and Pennacchi (1996) apply their models for almost 200 US banks, Brien (2000) for nearly100 US banks and Dewachter *et al.* (2006) for 8 major Belgian banks. The US bank data consists of Money market deposit accounts and negotiable orders of withdrawal accounts and the Belgian bank data consists of savings deposits.Within the discounted cash flow framework, Selvaggio (1996) uses product rates as a function of the one-month money market forward rate minus a constant cost rate, Office of Thrift Supervision (2001) considers product rates as changes in the lagged product rate and forward 3-month money market rate changes and de Jong and Wielhouwer (2000) suppose that product rates follow a stochastic error-correction model. The models of Selvaggio (1996) and de Jong and Wielhouwer (2000) were calibrated on a subset of a US commercial bank's retail deposits and on one Dutch bank data, respectively. The Office of Thrift Supervision (2001) model is applied for the supervision of US savings associations.

3. Data and Methodology

The observations of our analysis are two time series: the individual deposits series of a Moroccan commercial bank denoted by PATICULIERS and the enterprise deposits series of the same commercial bank denoted by ENTREPRISES. The data are monthly and spread over eight years, from January 2007 to December 2014. The 96 monthly observations were collected from the commercial bank web site. Using the Augmented Dickey Fuller Test, we conclude that the series PARTICULIERS and ENTREPRISES are not-stationary and are integrated of order 1. We use thus for our models the differentiated time series of PARTICULIERS and ENTREPRISES denoted respectively by DPARTICULIERS and DENTREPRISES.

In our paper we propose two models to assess both the individual accounts owned by households and enterprise accounts owned by companies. The first model is the ARMA time series model James and Hamilton (1994) defined by:

An Autoregressive Moving Average process of orders p and q is a stochastic process $(X_t)_{t\in\mathbb{Z}}$ which have the form:

$$\Phi(L).X_t = c + \Psi(L).X_t \quad (1)$$

Where $\Phi(L) = \sum_{i=0}^{p} \Phi_i L^i.X_t$ is a lag autoregressive polynomial of order p and $\Psi(L) = \sum_{j=0}^{q} \Psi_j L^j.X_t$ is a lag moving polynomial of order q, with L is a lag operator defined by $L^i.X_t = X_{t-i}$ and c is a constant.

The second model is the stochastic model of Jarrow and Van (1998) defined by:

$$X_t = c_1 \times X_t + c_2 \times t + c_3 R_t + c_4 (R_t - R_{t-1}) + \varepsilon_t \quad (2)$$

Where ε_t is a Gaussian white noise, R_t the weighted average rate of the secondary market and c_i are constants.

We select between the two models by means of a selection criteria. The selected model is used to Back-Test and a forecast the future deposits. We finally conclude our analysis by modeling the flowing function Paul *et al.* (2003) of individual deposits and enterprises deposits. The elaborations of flowing conventions of non-maturity deposit is fundamental in the asset liability management and permits the measuring of liquidity gap.

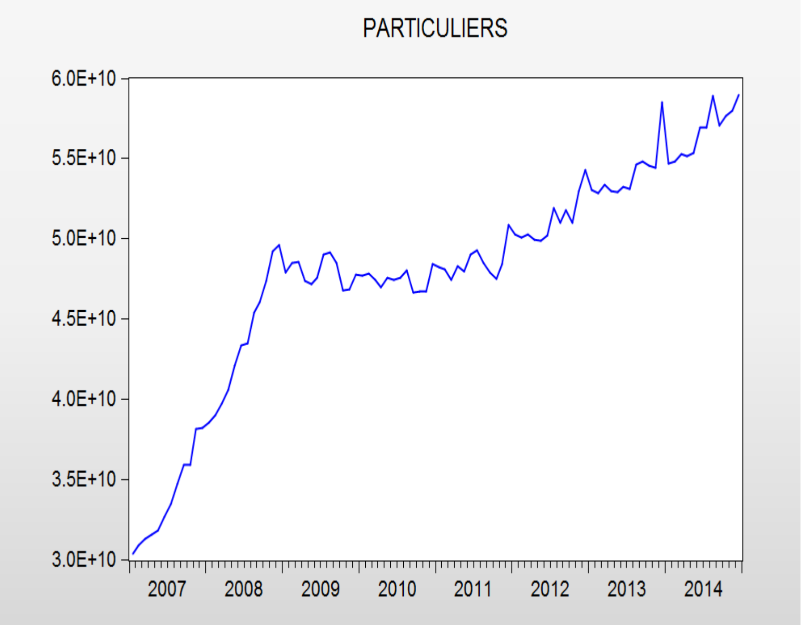
4. Modeling the Demand Deposits

4.1. Studies of Deposits Time Series

4.1.1. Studies of Individual Deposits Time Series

The first time series that we analyze is the individual deposits series. The data are spread over eight years, from January 2007 to December 2014. Thus we have 96 monthly observations. We denote this series by PATICULIERS.

Figure-1. Individual deposits data (PARTICULIERS series) 2007-2014



This graph suggest that the series is non-stationary. We need to check this using the Augmented Dickey Fuller test (ADF) James and Hamilton (1994).

- **Augmented Dickey Fuller Test for the series PARTICULIERS**

Model 3: With constant and trend

Table-1. ADF-Test with constant and trend for PARTICULIERS series

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats

Augmented Dickey-Fuller Unit Root Test on PARTICULIERS

Null Hypothesis: PARTICULIERS has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.654349	0.2579
Test critical values:	1% level	-4.057528	
	5% level	-3.457808	
	10% level	-3.154859	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PARTICULIERS)
Method: Least Squares
Date: 06/01/15 Time: 10:45
Sample (adjusted): 2007M02 2014M12
Included observations: 95 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PARTICULIERS(-1)	-0.095148	0.035846	-2.654349	0.0094
C	4.04E+09	1.35E+09	3.001303	0.0035
@TREND("2007M01")	16855656	8836936.	1.907409	0.0596

R-squared	0.083422	Mean dependent var	3.01E+08
Adjusted R-squared	0.063496	S.D. dependent var	1.04E+09
S.E. of regression	1.01E+09	Akaike info criterion	44.33401
Sum squared resid	9.37E+19	Schwarz criterion	44.41465
Log likelihood	-2102.865	Hannan-Quinn criter.	44.36659
F-statistic	4.186667	Durbin-Watson stat	2.344461
Prob(F-statistic)	0.018188		

The observed t-Statistic is equal to -2.654349. It exceeds the critical value -3.457808 to 5%, so we accept the null hypothesis of non-stationarity of the series at 5% significance.
We now test the hypothesis of nullity of the trend, taking into account that the critical value is 2.79. The t-Statistic of the trend equals 1.907409, which is smaller than the critical value 2.79. So we accept the hypothesis of nullity of the coefficient of the trend. We move now to the model 2 of the ADF test.

Model 2: With constant and without trend

Table-2. ADF-Test with constant and without trend for PARTICULIERS series

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats

Augmented Dickey-Fuller Unit Root Test on PARTICULIERS

Null Hypothesis: PARTICULIERS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.145811	0.2276
Test critical values:	1% level	-3.500669	
	5% level	-2.892200	
	10% level	-2.583192	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PARTICULIERS)
Method: Least Squares
Date: 06/01/15 Time: 10:48
Sample (adjusted): 2007M02 2014M12
Included observations: 95 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PARTICULIERS(-1)	-0.033332	0.015533	-2.145811	0.0345
C	1.89E+09	7.49E+08	2.526408	0.0132

R-squared	0.047175	Mean dependent var	3.01E+08
Adjusted R-squared	0.036930	S.D. dependent var	1.04E+09
S.E. of regression	1.02E+09	Akaike info criterion	44.35174
Sum squared resid	9.74E+19	Schwarz criterion	44.40550
Log likelihood	-2104.707	Hannan-Quinn criter.	44.37346
F-statistic	4.604503	Durbin-Watson stat	2.400061
Prob(F-statistic)	0.034492		

The observed t-Statistic is equal to -2.145811. It exceeds the critical value -2.892200 to 5%, so we accept the null hypothesis of non-stationarity of the series at the 5% significance.

We now test the hypothesis of nullity of the constant. The number of observations is 96, so the critical value is 2.54. It is superior to the t-Statistic of the Constant which is equal to 2.526408. We accept thus the hypothesis of nullity of the constant. So we move to the model 1 of the ADF test.

Model 1: Without constant and without trend

Table-3. ADF-Test without constant and without trend for PARTICULIERS series

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats

Augmented Dickey-Fuller Unit Root Test on PARTICULIERS

Null Hypothesis: PARTICULIERS has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.468462	0.9966
Test critical values:	1% level	-2.589531	
	5% level	-1.944248	
	10% level	-1.614510	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PARTICULIERS)
Method: Least Squares
Date: 06/01/15 Time: 10:52
Sample (adjusted): 2007M02 2014M12
Included observations: 95 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PARTICULIERS(-1)	0.005525	0.002238	2.468462	0.0154

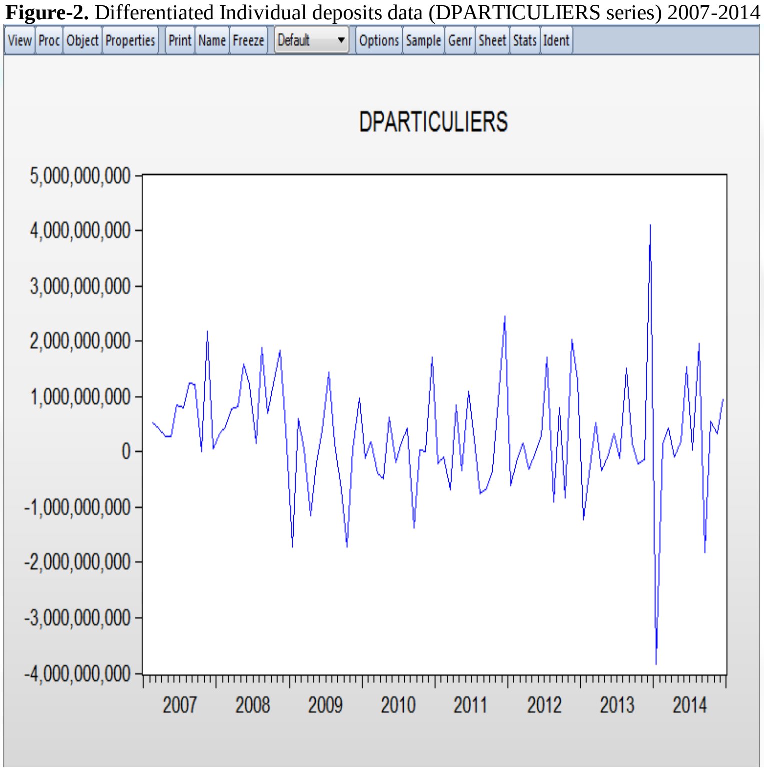
R-squared	-0.018219	Mean dependent var	3.01E+08
Adjusted R-squared	-0.018219	S.D. dependent var	1.04E+09
S.E. of regression	1.05E+09	Akaike info criterion	44.39706
Sum squared resid	1.04E+20	Schwarz criterion	44.42395
Log likelihood	-2107.860	Hannan-Quinn criter.	44.40793
Durbin-Watson stat	2.334748		

The observed t-Statistic is equal to 2.468462. It exceeds the critical value -1.944248 to 5%, so we accept the null hypothesis of non-stationarity of the series at the 5% significance.

We conclude that the series PARTICULIERS is not stationary to the 5% significance.

We will now stationarize the series PARTICULIERS and check the stationarity of the differentiated series by the ADF test. The latter series will be denoted DPARTICULIERS.

Figure-2. Differentiated Individual deposits data (DPARTICULIERS series) 2007-2014



This graph suggests that the series DPARTIUCILERS is stationary, which can be checked by applying the ADF test.

➢ **Augmented Dickey Fuller Test for the series DPARTICULIERS**

Table-4. ADF-Test for DPARTICULIERS series

File Edit Object View Proc Quick Options Add-ins Window Help

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats | Ident

Augmented Dickey-Fuller Unit Root Test on DPARTICULIERS

Null Hypothesis: DPARTICULIERS has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.59173	0.0000
Test critical values:	1% level	-2.589795	
	5% level	-1.944286	
	10% level	-1.614487	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DPARTICULIERS)
Method: Least Squares
Date: 06/01/15 Time: 11:10
Sample (adjusted): 2007M03 2014M12
Included observations: 94 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DPARTICULIERS(-1)	-1.096572	0.103531	-10.59173	0.0000

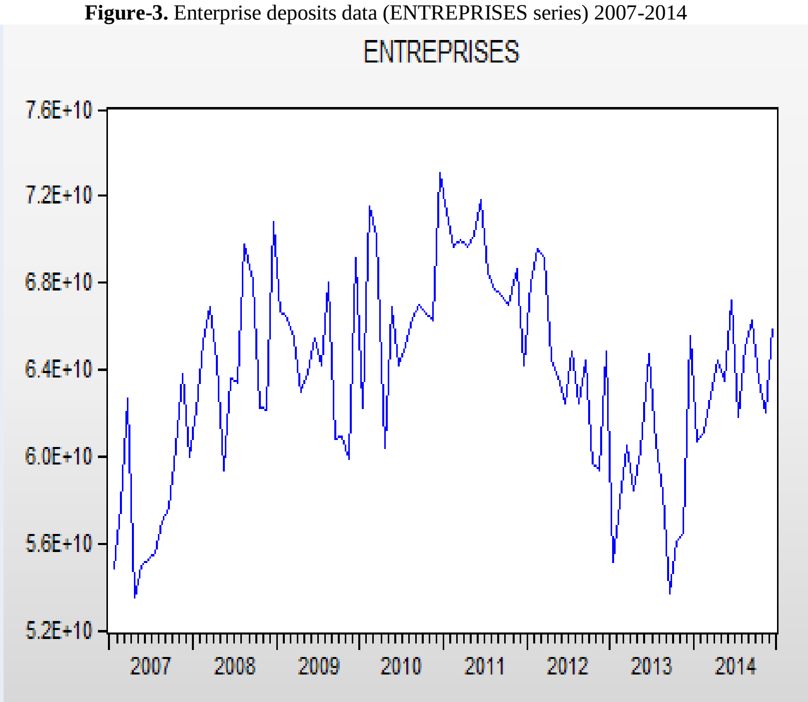
R-squared	0.546746	Mean dependent var	4840012.
Adjusted R-squared	0.546746	S.D. dependent var	1.61E+09
S.E. of regression	1.09E+09	Akaike info criterion	44.45895
Sum squared resid	1.10E+20	Schwarz criterion	44.48601
Log likelihood	-2088.571	Hannan-Quinn criter.	44.46988
Durbin-Watson stat	1.981851		

The observed t-Statistic is equal to -10.59173. It is smaller than the critical value -1.944286 to 5%, so we accept the hypothesis of the stationarity of the series DPARTICULIERS at the 5% significance. We conclude that the series PARTICULIERS is not-stationary and integrated of order 1.

4.1.2. Studies of Enterprise Deposits Time Series

The second time series that we analyze is the enterprise deposits series. The data are spread over eight years, from January 2007 to December 2014. Thus we have 96 monthly observations. We denote this series by ENTREPRISES.

Figure-3. Enterprise deposits data (ENTREPRISES series) 2007-2014



The graph suggests that these series is non-stationary, which can be checked by the augmented Dickey-Fuller test.

➢ Augmented Dickey Fuller Test for the series ENTREPRISES

Table-5. ADF-Test for ENTREPRISES series

Series: ENTREPRISES Workfile: ENTREPRISES::Entreprises\

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats

Augmented Dickey-Fuller Unit Root Test on ENTREPRISES

Null Hypothesis: ENTREPRISES has a unit root
Exogenous: None
Lag Length: 2 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.092925	0.7098
Test critical values:	1% level	-2.590065	
	5% level	-1.944324	
	10% level	-1.614464	

*MacKinnon (1996) one-sided p-values.

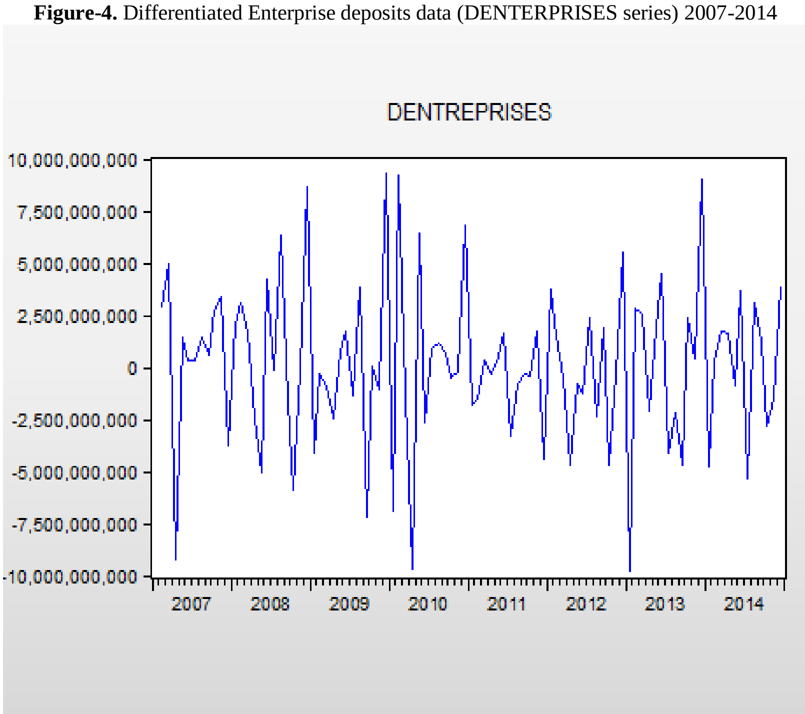
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(ENTREPRISES)
Method: Least Squares
Date: 04/05/16 Time: 22:17
Sample (adjusted): 2007M04 2014M12
Included observations: 93 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ENTREPRISES(-1)	0.000516	0.005555	0.092925	0.9262
D(ENTREPRISES(-1))	-0.520934	0.100165	-5.200743	0.0000
D(ENTREPRISES(-2))	-0.278011	0.099898	-2.782945	0.0066

R-squared	0.235374	Mean dependent var	34825105
Adjusted R-squared	0.218382	S.D. dependent var	3.87E+09
S.E. of regression	3.42E+09	Akaike info criterion	46.77629
Sum squared resid	1.05E+21	Schwarz criterion	46.85798
Log likelihood	-2172.097	Hannan-Quinn criter.	46.80927
Durbin-Watson stat	1.983022		

The observed t-Statistic is equal to 0.092925. It exceeds the critical value -1.944324 to 5%, so we accept the null hypothesis of non-stationarity of the series at the 5% significance.
We deduce that the series ENTERPRISES is not-stationary to the 5% significance.
We now stationarize the series ENTREPRISES and check the stationarity of the differentiated series by the ADF test. The latter series will be denoted DENTREPRISES.

Figure-4. Differentiated Enterprise deposits data (DENTERPRISES series) 2007-2014



This graph suggests that the series ENTERPRISE is stationary. This can be checked by the ADF test.

➢ **Augmented Dickey Fuller Test for series DENTREPRISES**

Table-6. ADF-Test for DENTREPRISES series

Series: DENTREPRISES Workfile: ENTREPRISES::Entreprises\

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats | I

Augmented Dickey-Fuller Unit Root Test on DENTREPRISES

Null Hypothesis: DENTREPRISES has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.83495	0.0000
Test critical values:	1% level	-2.590065	
	5% level	-1.944324	
	10% level	-1.614464	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DENTREPRISES)
Method: Least Squares
Date: 04/05/16 Time: 22:22
Sample (adjusted): 2007M04 2014M12
Included observations: 93 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DENTREPRISES(-1)	-1.797958	0.165941	-10.83495	0.0000
D(DENTREPRISES(-1))	0.277557	0.099234	2.797007	0.0063

R-squared	0.730097	Mean dependent var	-10987143
Adjusted R-squared	0.727131	S.D. dependent var	6.51E+09
S.E. of regression	3.40E+09	Akaike info criterion	46.75488
Sum squared resid	1.05E+21	Schwarz criterion	46.80934
Log likelihood	-2172.102	Hannan-Quinn criter.	46.77687
Durbin-Watson stat	1.982709		

The observed t-Statistic is equal to -10.83495. It is smaller than the critical value -1.944324 to 5%, so we accept the hypothesis of the stationarity of the series DENTREPRISES at the 5% significance. We conclude that the series ENTREPRISES is not-stationary integrated of order 1.

4.2. Modeling the Individual Deposits

4.2.1. Box Jenkins Method (ARMA) for the individual deposits

Identification

The correlogram of the series DPARTICULIERS is given below:

Table-7. Correlogram of DPARTICULIERS series

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.397	-0.397	15.451	0.000
		2	-0.281	-0.247	15.701	0.000
		3	0.026	-0.122	15.771	0.001
		4	-0.058	-0.139	16.115	0.003
		5	-0.031	-0.154	16.211	0.006
		6	0.238	0.178	22.066	0.001
		7	-0.110	0.093	23.329	0.001
		8	-0.025	0.041	23.394	0.003
		9	0.051	0.077	23.671	0.005
		10	0.026	0.140	23.744	0.008
		11	-0.188	-0.143	27.620	0.004
		12	0.275	0.117	36.002	0.000
		13	-0.018	0.178	36.040	0.001
		14	-0.153	-0.041	38.688	0.000
		15	0.093	-0.009	39.676	0.001
		16	-0.071	-0.075	40.270	0.001
		17	-0.002	-0.007	40.270	0.001
		18	0.213	0.131	45.721	0.000
		19	-0.176	-0.070	49.458	0.000
		20	-0.163	-0.259	52.738	0.000
		21	0.198	-0.019	57.601	0.000
		22	-0.137	-0.188	59.976	0.000
		23	0.061	-0.066	60.450	0.000
		24	0.073	-0.017	61.146	0.000
		25	0.017	0.102	61.185	0.000
		26	-0.129	0.074	63.420	0.000
		27	0.019	-0.058	63.467	0.000
		28	0.018	0.075	63.510	0.000
		29	-0.015	0.116	63.540	0.000
		30	0.105	0.056	65.107	0.000

The confidence interval is given by par $\left[\frac{-1.96}{\sqrt{T}}, \frac{1.96}{\sqrt{T}}\right] = [-0.20, 0.20]$. The two first AC and PAC of the table belong to the confidence interval, which suggests that $p_{max} = 2$ and $q_{max} = 2$. We test then the models : AR(1), MA(1), AR(2), MA(2), ARMA(1,1), ARMA(1,2), ARMA(2,1), ARMA(2,2).

Estimation

We estimate all the previous models and we select only those whose the Student Statistic satisfies the condition: $|t\ de\ Student| > 1.96$ for all the coefficients. The only models retained are ARMA(1,2) and ARMA(2,2).

Validation

To validate the two models, we need to ensure that their residuals are white noises. This is the case if the p-value of the correlogram of residuals exceed 5%.

Table-8. Correlogram of residuals series

Equation: EQ8 Workfile: DAV::DAV\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals

Date: 06/01/15 Time: 12:04
Sample: 2007M04 2014M12
Included observations: 93
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.185	-0.185	3.3033	
		2	-0.024	-0.061	3.3594	
		3	-0.057	-0.076	3.6787	
		4	0.055	0.029	3.9830	
		5	-0.070	-0.062	4.4728	0.694
		6	0.131	0.112	6.2154	0.763
		7	-0.129	-0.090	7.9171	0.799
		8	0.065	0.032	8.3566	0.849
		9	-0.050	-0.030	8.6182	0.813
		10	-0.133	-0.176	10.499	0.422
		11	-0.019	-0.059	10.540	0.150
		12	0.290	0.250	19.710	0.182
		13	0.005	0.131	19.713	0.175
		14	-0.215	-0.220	24.898	0.205
		15	0.082	0.047	25.664	0.212
		16	-0.080	-0.068	26.399	0.536
		17	0.143	0.120	28.786	0.628
		18	-0.027	-0.019	28.873	0.699
		19	0.011	0.018	28.889	0.723
		20	0.009	0.051	28.898	0.150
		21	0.036	-0.013	29.055	0.182
		22	-0.134	0.014	31.295	0.175
		23	0.056	-0.018	31.691	0.205
		24	0.140	0.083	34.207	0.628
		25	-0.019	-0.043	34.257	0.699

Equation: EQ7 Workfile: DAV::DAV\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals

Date: 06/01/15 Time: 12:04
Sample: 2007M03 2014M12
Included observations: 94
Q-statistic probabilities adjusted for 3 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.015	0.015	0.0211	
		2	-0.067	-0.068	0.4678	
		3	-0.082	-0.080	1.1373	
		4	0.026	0.024	1.2075	0.536
		5	-0.021	-0.033	1.2530	0.628
		6	0.143	0.142	3.3431	0.699
		7	-0.049	-0.055	3.5917	0.723
		8	0.092	0.113	4.4881	0.786
		9	-0.014	-0.004	4.5095	0.637
		10	-0.105	-0.110	5.6860	0.717
		11	0.039	0.075	5.8536	0.694
		12	0.297	0.262	15.561	0.763
		13	-0.012	-0.015	15.575	0.799
		14	-0.258	-0.276	23.105	0.849
		15	-0.040	0.020	23.284	0.813
		16	-0.123	-0.154	25.045	0.422
		17	0.055	0.014	25.397	0.150
		18	-0.069	-0.148	25.966	0.182
		19	-0.047	-0.055	26.237	0.175
		20	-0.015	-0.012	26.264	0.205
		21	0.004	-0.045	26.266	0.212
		22	-0.133	0.008	28.487	0.536
		23	0.052	0.002	28.832	0.628
		24	0.142	0.107	31.422	0.699
		25	-0.055	-0.064	31.817	0.723

The p-value column suggest that the residuals of the two estimated models are white noises. To choose between the two models, we use the AIC criterion for the best model.

AIC selection criterion of the best model

We will retain the model with the smallest AIC. We remark that:
AIC-ARMA(2,2) = 44.31 < AIC-ARMA(1,2) = 44.37
We retain the ARMA(2,2) model which can be written in the form:

$$\boldsymbol{X_t = 1,75X_{t-1} - 0,77X_{t-2} + \varepsilon_t - 1,92\varepsilon_{t-1} + 0,94\varepsilon_{t-2} + 1.51 \times 10^8} \quad (3)$$

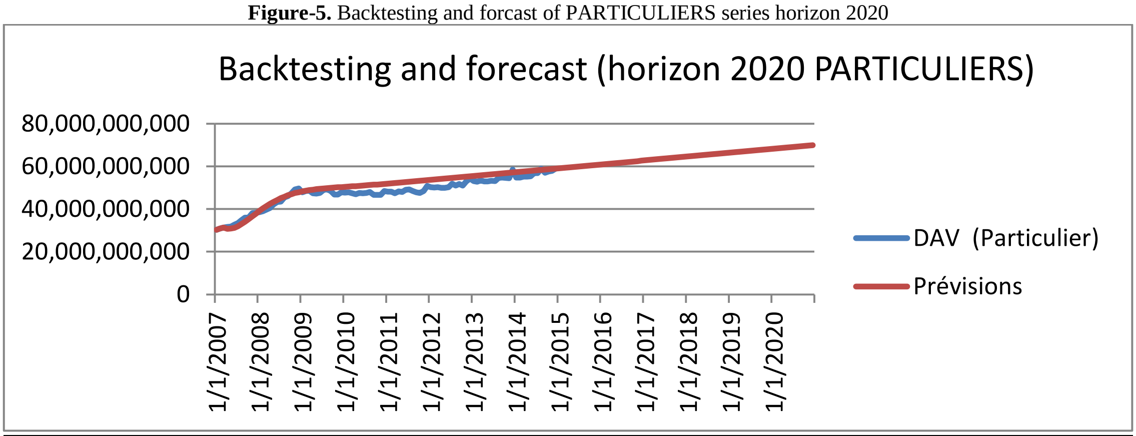
where X_t represent the process of DPARTICULIERS and ε_t is a white noise.
Since $X_t = \text{PARTICULIERS}_t - \text{PARTICULIERS}_{t-1}$, the model can finally be written :

$$\begin{aligned}\mathbf{PARTICULIERS}_t &= \mathbf{2,75 \times PARTICULIERS}_{t-1} - \mathbf{0,77 \times PARTICULIERS}_{t-2} + \boldsymbol{\varepsilon_t} - \mathbf{1,92} \times \boldsymbol{\varepsilon_{t-1}} \\ &+ \mathbf{0,94} \times \boldsymbol{\varepsilon_{t-2}} + \mathbf{1.51 \times 10^8}\end{aligned} \quad (4)$$

Back-Testing

We need to backtest the forecast model retained to ensure the performance of its predictive power. The results of the back-testing is given below.

Figure-5. Backtesting and forcast of PARTICULIERS series horizon 2020



Future Forcasts

The forecast of monthly future values of the individual deposits for the four future years 2015, 2016, 2017 and 2018 is given in the following table.

Table-9. Forecast of monthly future values of the individual deposits 2015-2018

Date	Forecasts of individual deposits (MAD)	Date	Forecasts of individual deposits (MAD)
31/01/2015	59 163 683 636	31/01/2017	62 795 767 861
28/02/2015	59 314 997 266	28/02/2017	62 947 112 299
31/03/2015	59 466 314 507	31/03/2017	63 098 456 617
30/04/2015	59 617 635 236	30/04/2017	63 249 800 809
31/05/2015	59 768 959 265	31/05/2017	63 401 144 874
30/06/2015	59 920 286 360	30/06/2017	63 552 488 815
31/07/2015	60 071 616 259	31/07/2017	63 703 832 637
31/08/2015	60 222 948 685	31/08/2017	63 855 176 348
30/09/2015	60 374 283 354	30/09/2017	64 006 519 957
31/10/2015	60 525 619 988	31/10/2017	64 157 863 472
30/11/2015	60 676 958 319	30/11/2017	64 309 206 904
31/12/2015	60 828 298 094	31/12/2017	64 460 550 262
31/01/2016	60 979 639 077	31/01/2018	64 611 893 556
29/02/2016	61 130 981 055	28/02/2018	64 763 236 795
31/03/2016	61 282 323 835	31/03/2018	64 914 579 987
30/04/2016	61 433 667 245	30/04/2018	65 065 923 141
31/05/2016	61 585 011 137	31/05/2018	65 217 266 263
30/06/2016	61 736 355 382	30/06/2018	65 368 609 361
31/07/2016	61 887 699 871	31/07/2018	65 519 952 438
31/08/2016	62 039 044 512	31/08/2018	65 671 295 502
30/09/2016	62 190 389 230	30/09/2018	65 822 638 554
31/10/2016	62 341 733 966	31/10/2018	65 973 981 600
30/11/2016	62 493 078 672	30/11/2018	66 125 324 641
31/12/2016	62 644 423 312	31/12/2018	66 276 667 681

4.2.2. The Stochastic Model of Jarrow and Van Deventer for the Individual Deposits

The stochastic model of Jarrow and Van Deventer is based on the assumption that the behavior of depositors is influenced not only by the trend of the market rate, but also by its change to a monthly lag. The model is written in the form:

$$\boldsymbol{log(PARTICULIERS_t) = c_1 \times log(PARTICULIERS_t) + c_2 \times t} \quad (5)$$

$$\boldsymbol{+c_3 R_t + c_4 (R_t - R_{t-1}) + \varepsilon_t}$$

where ε_t is a Gaussian white noise and R_t is the weighted average rate of the secondary market.

We will denote: $LOGPARTICULIERS_t = log(PARTICULIERS_t)$.

The ordinary least squares gives the following results:

Table-10. Results of the Jarrow and Van Deventer Model for the individual deposits

Equation: UNTITLED Workfile: MODELE STOCHASTIQUE::mo...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Akaike info criterion

Dependent Variable: LOGPARTICULIERS
Method: Least Squares
Date: 06/09/15 Time: 15:47
Sample (adjusted): 2007M02 2014M12
Included observations: 95 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.044769	0.674740	3.030453	0.0032
LOGPARTICULIERS(-1)	0.916581	0.027635	33.16782	0.0000
TMP	0.014703	0.295724	0.049720	0.9605
DTMP	-0.700789	0.428920	-1.633848	0.1058
@TREND	0.000251	0.000158	1.587999	0.1158

R-squared	0.983439	Mean dependent var	24.58527
Adjusted R-squared	0.982703	S.D. dependent var	0.151245
S.E. of regression	0.019892	Akaike info criterion	44.94583
Sum squared resid	0.035611	Schwarz criterion	-4.811420
Log likelihood	239.9272	Hannan-Quinn criter.	-4.891521
F-statistic	1336.088	Durbin-Watson stat	2.336269

The R-squared is equal to 0.98, so the model explains 98% of the variation of the logarithm deposits. Only the constant and the variable $LOGPARTICULIERS_{t-1}$ are significant. So we retain the model:

$$\boldsymbol{LOGPARTICULIERS_t = 0,91 \times LOGPARTICULIERS_{t-1} + 2,04} \qquad (6)$$

On the basis of Akaike criterion, we remark that the ARMA(2,2) is better than the stochastic model in the case of individuals deposits. We thus use the ARMA(2,2) for forecasting the individual deposits and for elaborating of their flowing convention.

4.3. Modeling the Enterprise Deposits

4.3.1. The Box Jenkins Model (ARMA) for the Enterprise Deposits

Identification

The correlogram of the DENTREPRISES series is given below:

Table-11. Correlogram of DENTREPRISES series

Series: DENTREPRISE Workfile: ENTRE::Entre\

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats | Ide

Correlogram of DENTREPRISE

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.394	-0.394	15.226	0.000
		2	-0.228	-0.269	15.735	0.000
		3	0.017	-0.159	15.763	0.001
		4	-0.063	-0.186	16.167	0.003
		5	-0.006	-0.172	16.170	0.006
		6	0.184	0.097	19.688	0.003
		7	-0.102	0.027	20.769	0.004
		8	-0.052	-0.040	21.051	0.007
		9	0.042	-0.004	21.237	0.012
		10	0.073	0.128	21.807	0.016
		11	-0.146	-0.072	24.145	0.012
		12	0.170	0.088	27.369	0.007
		13	-0.010	0.137	27.381	0.011
		14	-0.107	0.020	28.671	0.012
		15	0.070	0.037	29.240	0.015
		16	-0.053	-0.047	29.569	0.020
		17	-0.036	-0.054	29.719	0.028
		18	0.228	0.174	35.945	0.007
		19	-0.182	-0.041	39.957	0.003
		20	-0.155	-0.256	42.905	0.002
		21	0.199	-0.006	47.860	0.001
		22	-0.140	-0.194	50.330	0.001
		23	0.059	-0.139	50.777	0.001
		24	0.052	-0.112	51.125	0.001

The confidence interval is given by par $\left[\frac{-1.96}{\sqrt{T}}, \frac{1.96}{\sqrt{T}}\right] = [-0.20, 0.20]$. The two first AC and PAC of the table belong to the confidence interval, which suggests that $p_{max} = 2$ and $q_{max} = 2$. We test then the models : AR(1), MA(1), AR(2), MA(2), ARMA(1,1), ARMA(1,2), ARMA(2,1), ARMA(2,2).

Estimation

We estimate all the previous models and we select only those whose the Student Statistic satisfies the condition: $|t\ de\ Student| > 1.96$ for all the coefficients. The only model retained is ARMA(1,2).

Table-12. Estimation of DENTREPRISES by ARMA(1,2) model

Equation: EQ4 Workfile: ENTRE::Entre\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: DENTREPRISE
Method: Least Squares
Date: 11/05/15 Time: 10:55
Sample (adjusted): 2007M03 2014M12
Included observations: 94 after adjustments
Convergence achieved after 18 iterations
MA Backcast: 2007M01 2007M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	87181036	2.41E+08	11.19535	0.0000
AR(1)	0.928337	0.165006	5.626067	0.0000
MA(1)	-1.580932	0.203032	-7.786621	0.0000
MA(2)	0.599547	0.150039	3.995950	0.0001

R-squared	0.271762	Mean dependent var	87181036
Adjusted R-squared	0.247488	S.D. dependent var	3.88E+09
S.E. of regression	3.37E+09	Akaike info criterion	46.75466
Sum squared resid	1.02E+21	Schwarz criterion	46.86289
Log likelihood	-2193.469	Hannan-Quinn criter.	46.79838
F-statistic	11.19535	Durbin-Watson stat	1.902852
Prob(F-statistic)	0.000003		

Inverted AR Roots	.93	
Inverted MA Roots	.95	.63

Validation

To validate the model retained, we need to ensure that its residual is a white noise. This is the case if the p-value of the correlogram of the residuals exceed 5%.

Table-13. Correlogram of residuals for ARMA(1,2) model

Equation: EQ4 Workfile: ENTRE::Entre\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals

Date: 11/05/15 Time: 11:08
Sample: 2007M03 2014M12
Included observations: 94
Q-statistic probabilities adjusted for 3 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.022	0.022	0.0452	
		2	-0.057	-0.058	0.3691	
		3	-0.037	-0.035	0.5078	
		4	-0.061	-0.063	0.8836	0.347
		5	0.044	0.043	1.0790	0.583
		6	0.180	0.172	4.4177	0.220
		7	-0.048	-0.056	4.6567	0.324
		8	-0.059	-0.041	5.0198	0.413
		9	0.033	0.049	5.1372	0.526
		10	0.068	0.082	5.6378	0.583
		11	-0.052	-0.079	5.9324	0.655
		12	0.137	0.122	7.9990	0.534
		13	-0.014	0.007	8.0217	0.627
		14	-0.113	-0.094	9.4514	0.580
		15	-0.003	-0.024	9.4523	0.664
		16	-0.063	-0.075	9.9126	0.701
		17	-0.013	0.005	9.9312	0.767
		18	0.096	0.036	11.035	0.750
		19	-0.247	-0.269	18.402	0.301
		20	-0.237	-0.211	25.233	0.090
		21	0.041	0.046	25.445	0.113
		22	-0.120	-0.197	27.252	0.099

The p-value column suggest that the residuals of the estimated model is a white noise. The model ARMA(1,2) can be written :

$$\boldsymbol{Y_t = 0,92 \times Y_{t-1} + \varepsilon_t - 1,58 \times \varepsilon_{t-1} + 0,59 \times \varepsilon_{t-2} + 8,71 \times 10^7} \quad (7)$$

where Y_t represents the process DENTREPRISES and ε_t is a white noise.
Using $Y_t = \text{ENTREPRISES}_t - \text{ENTREPRISES}_{t-1}$, the model can finally be written :

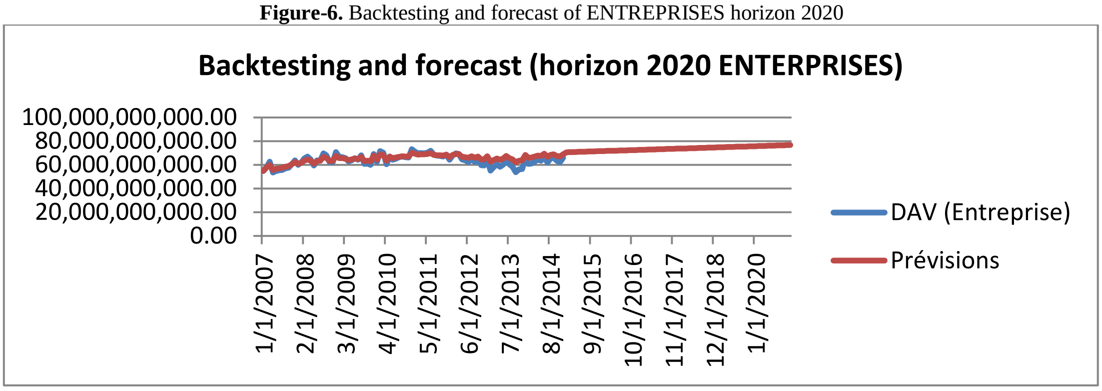
$$\boldsymbol{\text{ENTREPRISES}_t = 1,92 \times \text{ENTREPRISES}_{t-1} - 0,92 \times \text{ENTREPRISES}_{t-2} + \varepsilon_t - 1,58 \times \varepsilon_{t-1} + 0,59 \times \varepsilon_{t-2} + 8,71 \times 10^7} \quad (8)$$

This model will be used in backtesting and forecasting future values of enterprise deposits.

Back-Testing

We need to backtest the model to ensure the performance of its predictive power.

Figure-6. Backtesting and forecast of ENTREPRISES horizon 2020



Future Forecasts of Enterprise Deposits

The monthly forecast of future values of the enterprise deposits for the four years 2015, 2016, 2017 and 2018 is given in the following table.

Table-14. Forecast of enterprise deposits 2015-2018

Date	Forecast of enterprise deposits (MAD)	Date	Forecast of enterprise deposits (MAD)
31/01/2015	70 573 482 650	31/01/2017	72 628 915 659
28/02/2015	70 659 345 065	28/02/2017	72 714 424 412
31/03/2015	70 745 177 022	31/03/2017	72 799 928 053
30/04/2015	70 830 980 706	30/04/2017	72 885 426 948
31/05/2015	70 916 758 141	31/05/2017	72 970 921 437
30/06/2015	71 002 511 210	30/06/2017	73 056 411 836
31/07/2015	71 088 241 658	31/07/2017	73 141 898 438
31/08/2015	71 173 951 106	31/08/2017	73 227 381 515
30/09/2015	71 259 641 059	30/09/2017	73 312 861 320
31/10/2015	71 345 312 915	31/10/2017	73 398 338 087
30/11/2015	71 430 967 970	30/11/2017	73 483 812 034
31/12/2015	71 516 607 428	31/12/2017	73 569 283 363
31/01/2016	71 602 232 407	31/01/2018	73 654 752 262
29/02/2016	71 687 843 945	28/02/2018	73 740 218 904
31/03/2016	71 773 443 004	31/03/2018	73 825 683 452
30/04/2016	71 859 030 480	30/04/2018	73 911 146 056
31/05/2016	71 944 607 201	31/05/2018	73 996 606 854
30/06/2016	72 030 173 940	30/06/2018	74 082 065 977
31/07/2016	72 115 731 411	31/07/2018	74 167 523 544
31/08/2016	72 201 280 278	31/08/2018	74 252 979 667
30/09/2016	72 286 821 158	30/09/2018	74 338 434 449
31/10/2016	72 372 354 624	31/10/2018	74 423 887 987
30/11/2016	72 457 881 206	30/11/2018	74 509 340 369
31/12/2016	72 543 401 398	31/12/2018	74 594 791 678

4.3.2. Stochastic Model of Jarrow and Van Deventer for Enterprise Deposits

The stochastic model of Jarrow and Van Deventer can be written:

$$\boldsymbol{log(ENTREPRISES_t) = c_1 \times log(ENTREPRISES_t) + c_2 \times t} \quad (9)$$

$$\boldsymbol{+c_3 R_t + c_4 (R_t - R_{t-1}) + \varepsilon_t}$$

where ε_t is a Gaussian white noise and R_t is the weighted average rate of the secondary market.
We denote: $LOGENTRE_t = log(ENTREPRISES_t)$.

The ordinary least squares gives the following results:

Table-15. Results of Jarrow and Van Deventer model for enterprise deposits

Equation: EQSTO Workfile: ENTRE::entre\

View Proc Object Print Name Freeze Estimate Forecast Stats Resids

Dependent Variable: LOGENTRE
Method: Least Squares
Date: 11/05/15 Time: 15:29
Sample (adjusted): 2007M02 2014M12
Included observations: 95 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	9.873304	1.978994	4.989053	0.0000
LOGENTRE(-1)	0.604535	0.079272	7.626091	0.0000
@TREND	6.81E-06	0.000207	0.032868	0.9739
TMP	-0.884694	0.818215	-1.081249	0.2825
DTMP	-2.174763	1.168491	-1.861172	0.0660

R-squared	0.452858	Mean dependent var	24.87649
Adjusted R-squared	0.428541	S.D. dependent var	0.071568
S.E. of regression	0.054102	Akaike info criterion	239.9272
Sum squared resid	0.263430	Schwarz criterion	-2.810291
Log likelihood	144.8735	Hannan-Quinn criter.	-2.890392
F-statistic	18.62278	Durbin-Watson stat	2.358932

The R-square is equal to 0.45, thus the model explains only 45% of the change of the logarithm of enterprise deposits.
Only the constant and $LOGENTRE_{t-1}$ variable are significant. The model is reduced to:

$$\boldsymbol{LOGENTRE_t = 0,60 \times LOGENTRE_{t-1} + 9,87} \quad (10)$$

On the basis of Akaike criteria, we remark that the ARMA (1,2) model is better than the stochastic model for the enterprise deposits. Thus the ARMA (1,2) model is used to forecast enterprise deposits and to elaborate the flowin convention.

5. Modeling Flowing Conventions of Deposits

The flowing function in liquidity of the production (Paul *et al.*, 2003) gives the probability that a new production of one dirham entering the balance sheet at a time t will be still present at a later date T. If we denote by $PN(t)$ the new production appeared at t and $PN(t,T)$ the amount of this production still present at time T, the flowing function of the production will be defined by the following relationship:

$$S(t,T) = \frac{PN(t,T)}{PN(t)} \quad (11)$$

This flowing function defines the flowing convention in liquidity of the production and it has the following properties:

- ✓ $S(t,t) = 1$: A dirham entering the balance sheet at time t is still in the balance sheet at time t;
- ✓ $S(t,+\infty) = 0$: The production disappears sooner or later from the balance sheet. Precisely, we impose a time limit, not infinite, of the presence on the balance sheet.

The flowing seems in this case quite simple, but only for contractual products where the theoretical flowing convention corresponds to the flowing as implied by the terms of the contract, though in reality, the customer has several options (for example the prepayment option) that will change the flowing convention.

Unlike contractual products, the definition of an effective flowing remains fairly problematic for deposits, for which there is no contractual flowing. They are subject to a rather complex modeling.

In a global context, we can assume that the flowing functions of the production depend on:

- ✓ the date t of the entry of the production in the balance sheet;
- ✓ the time elapsed between the date t of the entry on the balance sheet and the date considered T;
- ✓ other variables such as market rate between t and T.

The most common practice assumes that only the time between dates t and T affects the probability that a dirham entered in the balance sheet at time t will still be present at time T. This means that past and future new productions elapse in the same way and that their entry date in the balance sheet is irrelevant.

There are several types of flowing conventions that the bank may choose to adopt for its deposits. These depend on the degree of prudence necessary to apprehend the liquidity risk and on the investment and expansion policy of bank.

In continuous time, we also define the flowing rate (or velocity) $\lambda(t,T)$ at a given time t by:

$$\lambda(t,T) = -\frac{\partial lnS(t,T)}{\partial T} \tag{12}$$

Therefore, the flowing function ca be written as:

$$S(t,T) = exp(-\int_t^T \lambda(t,s)ds) \tag{13}$$

If we assume the flowing rate is constant, we get:

$$S(t,T) = exp(-\lambda(T-t)) \text{ where } \lambda > 0 \tag{14}$$

We remark that this formulation complies with the properties mentioned above which are: $S(t,t) = 1$ and $S(t,+\infty) = 0$.

Thus if the flowing rate is constant we obtain:

$$D(t,T) = D(t) \times exp(-\lambda(T-t)) \tag{15}$$

Where $D(t) = PN(t)$ is the stock of deposits at date t and $D(t,T) = PN(t,T)$ is the stock of deposits which is still present at date T.

To estimate the flowing rate λ, we rely on the series of the deposits and on the rate of growth of deposits:

$$\lambda_h = \frac{D_{h+1} - D_h}{D_h} \tag{16}$$

Assuming that the exit rate and the entry rate are equal, we define the flowing rate λ as the geometric mean of λ_h:

$$\lambda = \sqrt[n]{\prod_{h=1}^{N}(1 + |\lambda_h| - 1)} \tag{17}$$

Indeed, the geometric mean is less sensitive than the arithmetic mean upon the great values of a data series.

To simplify, we will assume in the following that $t = 0$.

5.1. Modeling Flowing Convention of Individual Deposits

We consider that at the origin date $t = 0$, the month of December 2014, the bank stops its activity of collecting the individual deposits. The flowing function can be written:

$$S(0,T) = exp(-\lambda T) \tag{18}$$

Using

$$\lambda_h = \frac{D_{h+1} - D_h}{D_h} \quad \text{and} \quad \lambda = \sqrt[n]{\prod_{h=1}^{N}(1 + |\lambda_h| - 1)}$$

we obtain

$$\lambda = 0.0079692 \ \ (0.8\%)$$

The flowing of individual deposits is obtained from the formula:

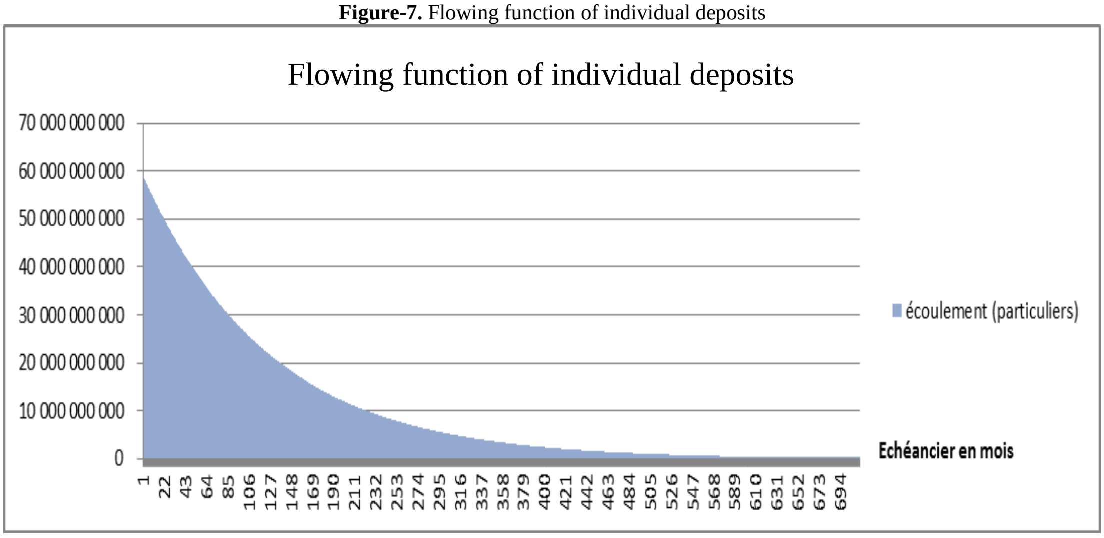
$$D(0,T) = D(0) \times exp(-\lambda T) \tag{19}$$

$D(0)$ being the individual deposits at time 0, the month of December 2014 when the bank stops its collection activity for individual deposits. We have here:

$$D(0) = 58\ 947\ 362\ 241\ \text{MAD} \tag{20}$$

The graph below illustrates the flowing function of individual deposits:

Figure-7. Flowing function of individual deposits

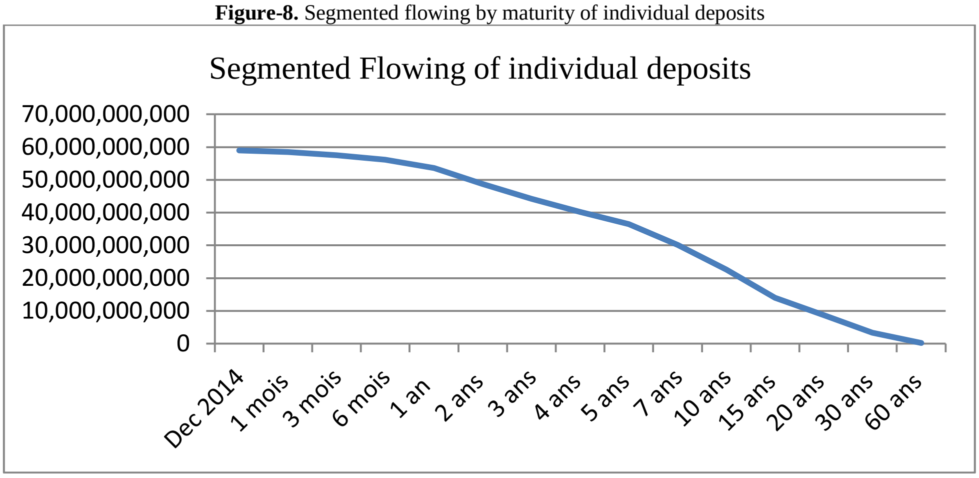


The flowing conventions adopted by banks respect segmentation by maturity categories and the final segmented flowing of individual deposits is given in the following table.

Table-16. Segmented flowing by maturity of individual deposits

Maturity	Individual deposit (MAD)	Flowing percentage %	Withdrawl (MAD)	Withdrawl percentage
Déc 2014	**58 947 362 241**	0,0%	0	0,00
1 mois	58 479 462 861	0,8%	467 899 380	0,79%
3 mois	57 554 776 586	2,4%	924 686 275	1,57%
6 mois	56 195 089 686	4,7%	1 359 686 900	2,31%
1 an	53 571 321 681	9,1%	2 623 768 006	4,45%
2 ans	48 685 579 770	17,4%	4 885 741 911	8,29%
3 ans	44 245 420 930	24,9%	4 440 158 840	7,53%
4 ans	40 210 207 674	31,8%	4 035 213 256	6,85%
5 ans	36 543 008 681	38,0%	3 667 198 992	6,22%
7 ans	30 181 461 842	48,8%	6 361 546 839	10,79%
10 ans	22 653 965 041	61,6%	7 527 496 801	12,77%
15 ans	14 043 784 314	76,2%	8 610 180 728	14,61%
20 ans	8 706 108 511	85,2%	5 337 675 803	9,05%
30 ans	3 345 830 421	94,3%	5 360 278 089	9,09%
60 ans	189 908 094	99,7%	3 155 922 327	5,35%
150 ans	34 727	100%	189 873 367	0,32%
		amount	**58 947 327 515**	**100%**

Figure-8. Segmented flowing by maturity of individual deposits



The previous table shows that the majority of individual deposits (61.6%) elapses during the first 10 years, and almost all of the deposits will elapse in 60 years.

5.2. Modeling Flowing Conventions of Enterprise Deposits

We consider that at the origin date $t = 0$, the month of December 2014, the bank stops its activity of collecting the individual deposits. The flowing function can be written:

$$S(0,T) = exp(-\lambda T)$$

Using

$$\lambda_h = \frac{D_{h+1} - D_h}{D_h} \quad \text{and} \quad \lambda = \sqrt[n]{\prod_{h=1}^{N}(1 + |\lambda_h| - 1)}$$

we obtain

$$\lambda = 0{,}026385 \ (2.63\%)$$

The flowing of individual deposits is obtained from the formula:

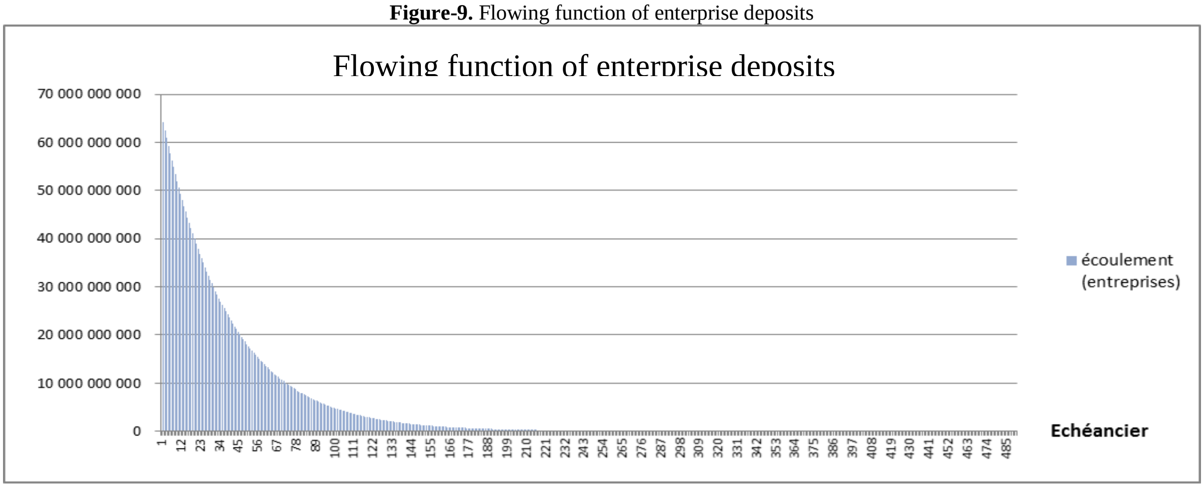
$$D(0,T) = D(0) \times exp(-\lambda T)$$

$D(0)$ being the enterprise deposits at time 0, the month of December 2014 when the bank stops its collection activity for enterprise deposit. We have here:

$$D(0) = 65\ 928\ 492\ 751 \text{ MAD} \qquad (21)$$

The graph below illustrates the flowing function of enterprise deposits:

Figure-9. Flowing function of enterprise deposits

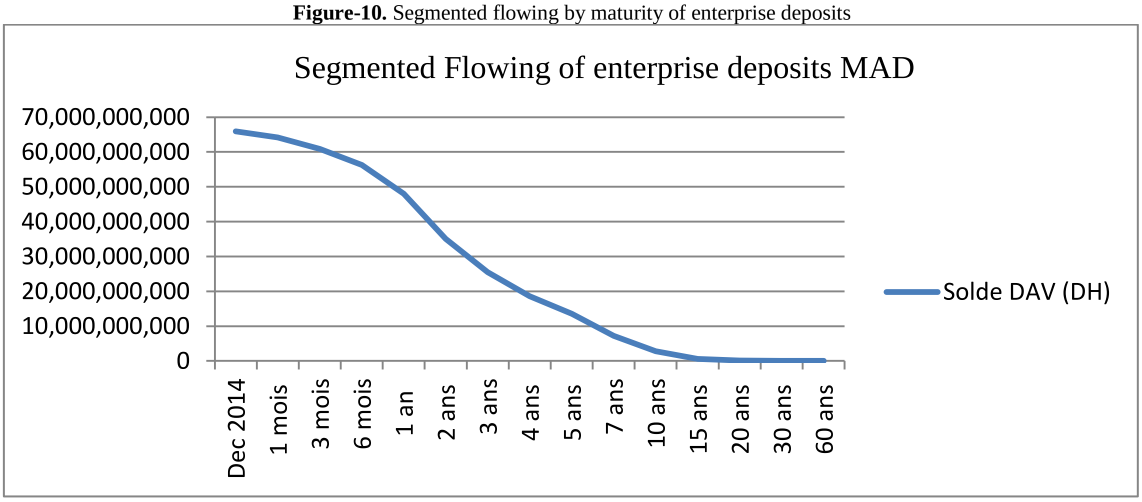


The flowing conventions adopted by banks respect a segmentation by maturity categories and the final segmented flowing of enterprise deposits is given in the following table.

Table-17. Segmented flowing by maturity of enterprise deposits

Maturity	Enterprise Deposits (MAD)	Flowing Percentage %	Withdrawls (MAD)	Withdrawl Percentage
Déc 2014	**65 928 492 751**	0,0%	0	0,00
1 mois	64 211 701 955	2,6%	1 716 790 796	2,60%
3 mois	60 911 072 922	7,6%	3 300 629 033	5,01%
6 mois	56 275 498 646	14,6%	4 635 574 276	7,03%
1 an	48 035 858 485	27,1%	8 239 640 161	12,50%
2 ans	34 999 187 819	46,9%	13 036 670 666	19,77%
3 ans	25 500 598 649	61,3%	9 498 589 170	14,41%
4 ans	18 579 874 905	71,8%	6 920 723 744	10,50%
5 ans	13 537 397 935	79,5%	5 042 476 970	7,65%
7 ans	7 186 542 770	89,1%	6 350 855 165	9,63%
10 ans	2 779 695 625	95,8%	4 406 847 145	6,68%
15 ans	570 767 573	99,1%	2 208 928 052	3,35%
20 ans	117 198 307	99,8%	453 569 266	0,69%
30 ans	4 941 348	100,0%	112 256 960	0,17%
60 ans	370	100,0%	4 940 978	0,01%
150 ans	0	100%	370	0,00%
		Total	**65 928 492 751**	**100%**

Figure-10. Segmented flowing by maturity of enterprise deposits



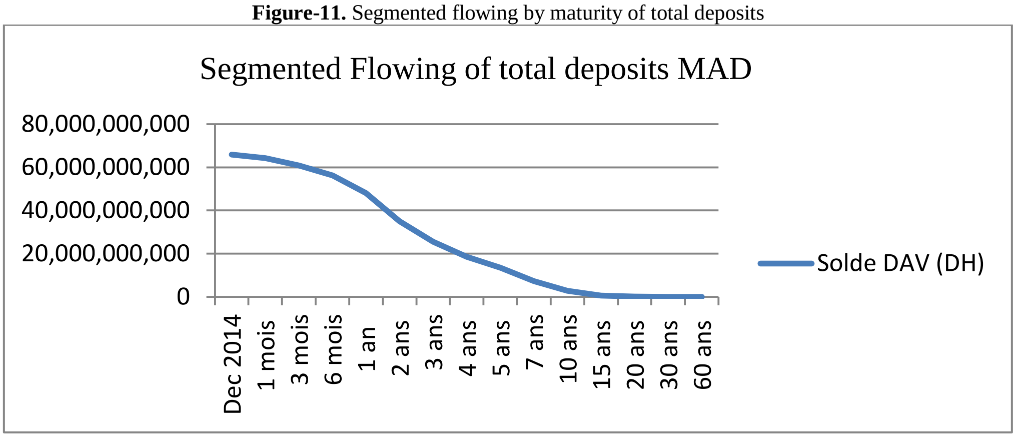
The previous table shows that the majority of individual deposits (61.3%) elapses during the first 3 years, and almost all of the deposits will elapse in 20 years.

5.3. Modeling Flowing Conventions of Total Deposits (Individual+Enterprises)

Table-18. Segmented flowing by maturity of total deposits

Maturity	Total deposits (MAD)	Flowing Percentage %	Withdrawls (MAD)	Witdrawl Percentage
Déc 2014	124 875 854 992	0,0%	0	0,00
1 mois	122 691 164 816	1,7%	2 184 690 176	1,75%
3 mois	118 465 849 509	5,1%	4 225 315 307	3,38%
6 mois	112 470 588 333	9,9%	5 995 261 176	4,80%
1 an	101 607 180 166	18,6%	10 863 408 167	8,70%
2 ans	83 684 767 589	33,0%	17 922 412 577	14,35%
3 ans	69 746 019 579	44,1%	13 938 748 010	11,16%
4 ans	58 790 082 578	52,9%	10 955 937 000	8,77%
5 ans	50 080 406 616	59,9%	8 709 675 962	6,97%
7 ans	37 368 004 612	70,1%	12 712 402 004	10,18%
10 ans	25 433 660 666	79,6%	11 934 343 946	9,56%
15 ans	14 614 551 887	88,3%	10 819 108 779	8,66%
20 ans	8 823 306 818	92,9%	5 791 245 069	4,64%
30 ans	3 350 771 769	97,3%	5 472 535 049	4,38%
60 ans	189 908 464	99,8%	3 160 863 305	2,53%
150 ans	34 727	100%	189 873 738	0,15%
		somme	**124 875 820 265**	**100%**

Figure-11. Segmented flowing by maturity of total deposits



6. Conclusion

The most critical challenges confronting financial institutions when managing liquidity risk is so-called non-maturity accounts. Liquidity risk is thus one of the major risks inherent in the banking business. It occurs when the bank is unable to meet its commitments at the time of their occurrence. These accounts have a non-contractual maturity, and their risk management is complicated by the embedded options that depositors may exercise. Liquidity risk is not the risk that there are massive withdrawals, but the risk they are unanticipated. In this paper, we have applied two models to assess non-maturity deposits of a Moroccan commercial bank: ARMA time series model and the Stochastic model of Jarrow and Van Deventer. We have tackled separately the individual deposits and the enterprise deposits in order to compare their decay rate in tile. We have been able to select the best models by means of a selection criteria. Through a back-testing and a forecast of future deposits we have obtained forecasts of four years of non-maturity future deposits. We conclude our study by modeling the decay rates of non-maturity deposits by assessing a flow function of these latter.

Our results have contributed to the modeling of non-maturity deposits of a Moroccan bank and consequently we have provided to the bank decision-makers a tool of liquidity risk management in a perspective of asset liability management.

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