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# Beyond the Accelerating Inflation Controversy: The Jerk and Jounce Price Variation

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**Abstract:** The aim of this paper is to re-interprete the accelerationist Phillips curve, by studying the effect of the higher-order derivatives of acceleration. We show that a complex dynamic behavior emerges when dealing with a jerk and jounce displacement of price settings, whose simple unfolding leads to a three dimensional vector field which generates a double-scroll chaotic attractor.
**Keywords:** New Keynesian model; Phillips curve; Jerk and jounce variation; Double scroll attractor.

## 1. Introduction

The higher derivatives of motion are rarely discussed. In real life, we normally experience the effect not only of acceleration, but also of jerk and jounce. Mathematically, jerk is the third-order derivative of our position with respect to time, whereas jounce is the fourth-order derivative. The classic example is a riding roller coaster which involves different types of motion during its cycling rounds. The feeling of a sudden change in acceleration is normally termed jerk, while the jounce is related to how often one feels the change, as the ride moves upwards and downwards.

The same applies also in economics to the standard macrodynamic framework that studies the trade-off between inflation and unemployment, namely the Phillips curve (Blanchard *et al.*, 2015). Normally, people expect inflation to increase as a result of an upward demand shift due to expansionary government policies. A high inflation rate today is likely to be followed by a high inflation rate tomorrow. But if the expected and the real inflation turn out to be equal, people do not expect a change in the level of inflation. As a result, the actual level of unemployment would fall short of its normal levels. Standard textbooks relate these movements around the natural (long run) equilibrium to an accelerating inflation rate. Indeed, as aggregate demand increases, more workers will be hired by firms in order to produce more output to meet the rising demand, and unemployment will decrease accordingly. However, due to the higher inflation, workers will claim for higher real wages to keep the pace of price change. This will diminish firms' profits and finally lead to a decrease in labor force employment. As a consequence, the rate of unemployment will move back to its natural rate, though inflation will be higher than its initial level. The wage-price spiral could emerge. Consequently, as pointed out by Lucas (1976) the attempt to decrease unemployment below its natural level will not produce any effect in real terms in the long run, unless policy changes are temporarily unanticipated, which leaves to the system only an accelerating level of inflation (Turnovsky, 2011).

The aim of this paper is to reinterpret the accelerationist Phillips curve, by studying the effect of the higher-order derivatives of acceleration. We show that a complex dynamic behavior emerges when dealing with a jerk and jounce displacement of price settings, whose simple unfolding leads to a three-dimensional vector field which generates a double-scroll chaotic attractor (Wang *et al.*, 2009).

The paper develops as follows. The second Section derives the continuous-time variant of the Phillips curve, and the associated output gap, joint with its dynamic properties. The third Section shows the emergence of the double-scroll chaotic attractor, using the transformation of the higher order inflation derivatives in terms of the Arneodo system. A brief conclusion reassesses the main findings of the paper, and a subsequent Appendix provides all the necessary proofs.

## 2. The Model

Consider the continuous-time version of the standard New Keynesian model developed by Werning (2011), where Households maximize the following separable utility function in consumption, *C*, and labor force, *N*, with elasticity $\varphi$, and discounted at a rate $\rho$,

$$\max \int_0^\infty \left( \log C - \frac{N^{1+\varphi}}{1+\varphi} \right) e^{-\rho t} dt$$
s.t.
$$\dot{B} = iB + WN - PC$$

under the budget constraint where $P$ is the level of prices, $W$ is the nominal wage, and $B$ is the amount of bonds issued at the nominal interest rate, $i$.

Define the inflation rate as $\pi = \frac{\dot{P}}{P}$. Standard Keynes-Ramsey optimization rule provides the following growth rate of the economy

$$\xi \equiv \frac{\dot{C}}{C} = i - \pi - \rho \qquad (1)$$

Assume now a competitive final goods producer that aggregates a continuum of intermediate inputs according to the following

$$Y = \left( \int_0^1 y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad (2)$$

with elasticity $\varepsilon \in (0,1)$, and where a continuum of monopolistically competitive intermediate goods firms indexed by $j \in [0,1]$ produce according to

$$y_j(t) = An_j \qquad (3)$$

Being $A$ the constant level of technology.
Following Rotemberg (1982), Schmitt-Grohé and Uribe (2004), and Fernández-Villaverde *et al.* (2014), we also assume that in presence of sticky prices, intermediate goods producers per period profits are

$$\Pi = p_j y_j - Wn_j \qquad (4)$$

though they have to pay a quadratic price adjustment cost

$$\Theta\left(\frac{\dot{p}_j}{p_j}\right) = \frac{\theta}{2}\left(\frac{\dot{p}_j}{p_j}\right)^2 PY \qquad (5)$$

where $\theta$ is the degree of stickiness.
Hence, the optimal control problem of optimizing firms reads

$$V(p_j) = \max_{p_j} \int_0^\infty e^{-\int_0^t i(s)ds} \left[ \Pi - \Theta\left(\frac{\dot{p}_j}{p_j}\right) \right] dt \qquad (6)$$

whose solution implies the following planar system of differential equations

$$\begin{aligned} &\theta \frac{\dot{p}_j}{p_j} \frac{P}{p_j} Y = \mu \\ &\dot{\mu} = i\mu - \left\{ \left[ (1-\varepsilon) + \varepsilon \frac{W}{Ap_j} \right] \left( \frac{p_j}{P} \right)^{-\varepsilon} + \theta \left( \frac{\dot{p}_j}{p_j} \right)^2 \frac{P}{p_j} \right\} Y \end{aligned} \qquad (7)$$

But since in a symmetric equilibrium $p_j = P$, then (7) reduces to

$$\begin{aligned} &\theta \pi Y = \mu \\ &\dot{\mu} = i\mu - \left[ (1-\varepsilon) + \varepsilon \frac{W}{AP} + \theta\pi \right] Y \end{aligned} \qquad (8)$$

and with a bit of mathematical computations, we can easily derive

$$\left( i - \pi - \frac{\dot{Y}}{Y} \right) \pi = \frac{\varepsilon - 1}{\theta} \left( \frac{\varepsilon}{\varepsilon - 1} \frac{W}{AP} - 1 \right) + \dot{\pi} \qquad (9)$$

Since in equilibrium aggregate consumption must be equal to aggregate output, and therefore in growth rate terms $\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \xi$, hence the equation for inflation in (9), i.e. the New Keynesian Phillips curve, can be also written as

$$\rho\pi = \frac{\varepsilon - 1}{\theta}\left(X^{1+\varphi} - 1\right) + \dot{\pi} \qquad (10)$$

where $X$ measures the output gap from the natural value of income, and $\frac{\dot{X}}{X} = i - \pi - r$ is the standard IS curve equation. Notice from (10) that inflation increases, and the economy overheats, when future output gaps are high. Using the log-linearization $x = \log X$, we close the model via the following three-dimensional system

$$\dot{x} = i - \pi - r \qquad \text{(IS)}$$

$$\dot{\pi} = \rho\pi - \eta x \qquad \text{(PC)}$$

$$i = i^* + \phi\pi + \phi_x x \qquad \text{(TR)}$$

where $\eta = \frac{(\varepsilon-1)(1+\varphi)}{\theta}$, and (TR) is the Taylor rule followed by the central bank (see Galí (2008); Carlin and Soskice (2009); Werning (2011)).[1]

Substituting (TR) into (IS), we finally obtain

$$\begin{aligned} \dot{x} &= i^* - r + (\phi - 1)\pi + \phi_x x \\ \dot{\pi} &= \rho\pi - \eta x \end{aligned} \qquad (S)$$

where $x$ and $\pi$ are the jump variables of the autonomous system. In equilibrium, $E^* \equiv (x^*, \pi^*)$, where

$$x^* = \frac{\rho}{\eta}\pi^*$$

$$\pi^* = \frac{i^* - r}{1 - \phi - \phi_x x^*}$$

which implies also that, a zero inflation target is always associated to a close-to-zero output gap.

Our aim is to prove that, in the vicinity of the optimal target, two cases can emerge, with completely different long-run dynamics. In particular, we show that: 1) if $\phi > 1$, the equilibrium is unique, and according to the Taylor principle in TR, this means that $i$ increases more than one-for-one with inflation rate, $\pi$; 2) if $\phi < 1$, the equilibrium is indeterminate, with periodic solutions that move the system dynamics off the saddle-path stable steady state. In the next section, this latter indeterminacy outcome will be further exploited to show the possibility of chaotic dynamics.

## 3. The Double Scroll Chaotic Dynamics

Consider the following linearization matrix associated with system ($S$) at the steady state, $E^*$, that is:

$$\mathbf{J} = \begin{bmatrix} \phi_{\mathbf{xx}} x^* + \phi_x & (\phi - 1) \\ -\eta & \rho \end{bmatrix}$$

Let the characteristic polynomial associated with **J** be

$$\mathrm{Det}(\lambda\mathbf{I} - \mathbf{J}) = \lambda^2 - \mathrm{Tr}\mathbf{J}\lambda + \mathrm{Det}\mathbf{J} \qquad (11)$$

where **I** is the identity matrix, and

$$\mathrm{Tr}\mathbf{J} = \rho + \left(\phi_{xx} x + \phi_x\right) \qquad (12a)$$

$$\mathrm{Det}\mathbf{J} = \rho\left(\phi_{xx} x + \phi_x\right) + \eta(\phi - 1) \qquad (12b)$$

are the Trace and Determinant of **J**, respectively.

In order to check whether system ($S$) may exhibit a set of periodic trajectories, the following must hold.

**Proposition 1** *If the autonomous system (S) has an equilibrium point* $E^*$ *with a pair of complex conjugate eigenvalues,* $\lambda_{1,2} = \tau \pm \omega i$, *associated with the Jacobian matrix,* **J**, *then the dynamic flow in the neighborhood of the equilibrium exhibits a repelling or attracting focus.*

[1] Recall that $i^*$ is the real interest rate, and $\phi = \phi(x)$ is the monetary policy sensitivity parameter, nonlinearly depending on the output gap (Gali, 2008).

**Proof.** Solving (11), we easily obtain that $\lambda_{1,2} = \frac{\mathrm{Tr}\mathbf{J} \pm \sqrt{\Delta}}{2}$, where $\Delta = (TrJ)^2 - 4DetJ$. Then, using (12a) and (12b), we have also that $\lambda_{1,2} = \tau \pm \omega i$, where $\tau = \frac{\rho + (\phi_{xx} x + \phi_x)}{2}$ and $\omega = \frac{4\eta(\phi - 1) - [\rho + (\phi_{xx} x + \phi_x)]^2}{2}$, given $[\rho + (\phi_{xx} x + \phi_x)]^2 - 4\eta(\phi - 1) < 0$, which is possible only if $\phi < 1$, since $\varepsilon \in (0,1)$. The latter implies that indeterminacy of equilibrium emerges. Moreover, a topological change from determinacy to indeterminacy occurs when $\phi$ is close to unity, at which Det**J** vanishes, and the structure of eigenvalues collapses to a repeated solution $\lambda_{1,2} = \tau$.

The tools of global analysis can be finally useful to investigate the stability of the equilibrium, when we move off the small neighborhood of the steady state. More precisely, we will demonstrate that the economy described by system (*S*), in presence of an accelerating inflation, may exhibit chaotic fluctuations around the natural rate of unemployment. We prove this by showing the set of necessary conditions for the emergence of a double scroll attractor, and determine the regions in the parameter space which imply the existence of such chaotic dynamics. The demonstration is not trivial and requires several steps to be accomplished.

Following Bella *et al.* (2013), the first step is to put system (*S*) in a form that resembles the nonlinear Arneodo system, which describes a family of three-dimensional autonomous differential equations of the following *jerk* function

$$\dddot{\pi} - \delta_1 \ddot{\pi} - \delta_2 \dot{\pi} - g(\pi) = 0 \qquad (13)$$

whose global structure is topologically equivalent to the original system (*S*), and where $g(\pi)$ is a third-degree polynomial. Moreover, assuming that the price variation is $x_1 = \dot{p} = \pi$, then price acceleration is $x_2 = \dot{x}_1 = \ddot{p} = \dot{\pi}$. Continuation of price derivatives over time provides a price jerk (i.e., an acceleration in inflation) given by $x_3 = \dot{x}_2 = \dddot{p} = \ddot{\pi}$, and finally a jerk in inflation (i.e., a jounce in price) at $\dot{x}_3 = p = \dddot{\pi}$. With these derivatives in hand, equation (14) can be put in the following three-dimensional hypernormal form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \qquad\qquad (P) \\ \dot{x}_3 &= \varepsilon_1 x_1 + \varepsilon_2 x_2 + \varepsilon_3 x_3 + s_1 x_1^2 + x_1^3 \end{aligned}$$

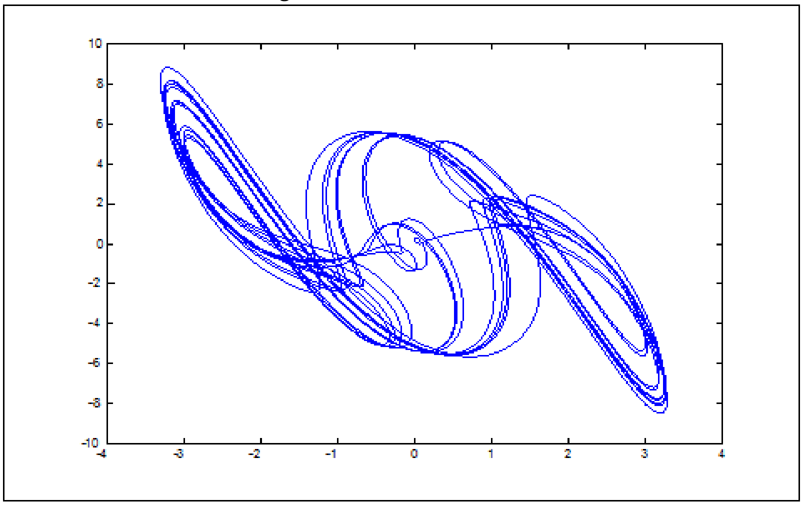
which in embryo contains several bifurcation singularities, and very rich associated dynamics, including the emergence of a complex chaotic attractor, crucially depending on the unfolding parameters $\varepsilon_1 = TrJ$, $\varepsilon_2 = -\Delta$, and $\varepsilon_3 = DetJ$, provided that $s_1 = (2\phi_{xx} + \phi_{xxx} x)$ is the term resulting from a second-order Taylor expansion in the equations of system *S* (see, (Gamero *et al.*, 1999)).

Interestingly, both the size and sign of $s_1$ provide very useful economic insights. In detail, $s_1$ represents the second order derivative of $\dot{x}$ with respect to *x*, which can be interpreted as a measure of the acceleration of speed adjustment of the output gap from its natural steady state level. Basically, this also suggests that, a negative sign of $s_1$ would imply an increase in magnitude in the output gap, which also translates in an accelerating inflation rate to adjust the economy after the output shock, as suggested by the standard assumptions of the New Keynesian model. Hence, if we slowly start applying a force in terms, for example, of an expansionary public spending, this may produce a small jerk on the inflation rate, and perhaps even a jounce, as we continuously apply more force on public budget to lower the unemployment rate (an increase the output gap) below its natural rate.

For the purpose of our paper, this means that only when $s_1 < 0$, and the standard non-accelerating inflation rate of unemployment (i.e., NAIRU) assumptions are confirmed, the double-scroll chaotic scenario is likely to emerge. We are able to show these differences numerically in the following example.

**Example 1:** *Let* $E^* \equiv (x^*, \pi^*) = (0,0)$. *Assume the hump-shaped function* $\phi = e^{-\frac{x^2}{2}}$, *which is continuous in all its derivatives and equal to unity at zero output gap,* $\phi(0) = 1$. *Consider also from Proposition 1 that, when* $\Delta = 0$ *, both* $\lambda_{1,2} = \frac{\mathrm{Tr}\mathbf{J}}{2}$ *and* $4\varepsilon_3 = \varepsilon_1$, *whereas* $s_1 = 0$. *If we deviate from the zero inflation equilibrium, an output gap is produced. Without any loss of generality, assume in fact* $i^* - r = 0.001$, *as in nowadays modern economies, set also a small social discount rate,* $\rho = 0.002$, *namely agents are more willing to anticipate future consumption. It follows consequently that* $(x^*, \pi^*) = (0.019, 0.02)$, *that is to say: an increase in inflation of 2%, due to an active public policy will increase the output gap of around the same amount. But this also would imply a jerk/jounce adjustment of inflation towards the equilibrium, i.e.* $s_1 = -0.007$. *Since* $\tau = 0.008$ *and* $\omega = 0$, *the structure of eigenvalues is still in the zone of topological change, then, the hyperbolic fixed point of S starts to exhibit a double scroll scenario (see Fig. 1).*

**Fig-1.** The double scroll attractor




It appears finally that the two-dimensional equilibrium manifold scrolls around a one-dimensional unstable branch, giving rise to the complicated transition from a stable/unstable Hopf bifurcating cycles towards the double scroll chaotic dynamics.

## 4. Conclusions

This paper innovates the literature regarding the dynamics in a continuous Phillips curve, by showing that a double scroll strange attractor emerges if higher order derivatives in the inflation rate are computed, giving rise to an Arneodo system of differential equations. In details, standard textbooks relate these movements around the natural (long run) equilibrium to an accelerating inflation rate. Indeed, as aggregate demand increases, more workers will be hired by firms in order to produce more output to meet the rising demand, and unemployment will decrease accordingly. However, due to the higher inflation, workers will claim for higher real wages to keep the pace of price change. This will diminish firms' profits and finally lead to a decrease in labor force employment. As a consequence, the rate of unemployment will move back to its natural rate, though inflation will be higher than its initial level. The wage-price spiral could emerge. Hence, the attempt to decrease unemployment below its natural level will produce economic effect in real terms only if an accelerating level of inflation is accepted, although this might trap the economy in a chaotic scenario.

## Appendix

Consider the continuous-time version of the standard New Keynesian model developed by Werning (2011), where Households maximize the following separable utility function in consumption, *C*, and labor force, *N*, with elasticity $\varphi$, and discounted at a rate $\rho$,

$$\max \int_0^\infty \left( \log C - \frac{N^{1+\varphi}}{1+\varphi} \right) e^{-\rho t} dt$$

s.t.

$$\dot{B} = iB + WN - PC$$

under the budget constraint where *P* is the level of prices, *W* is the nominal wage, and *B* is the amount of bonds issued at the nominal interest rate, *i*.

The problem can be solved via the current value Hamiltonian function

$$H_1 = \log C - \frac{N^{1+\varphi}}{1+\varphi} + \lambda(iB + WN - PC) \quad \text{(A.1)}$$

with the following necessary first order conditions

$$\frac{1}{C} = \lambda P \quad \text{(A.2a)}$$

$$N^{\varphi} = \lambda W \quad \text{(A.2b)}$$

$$\dot{\lambda} = \rho\lambda - \lambda i \quad \text{(A.2c)}$$

where $\lambda$ is the costate variable, namely the shadow-price of bonds issued to the market.

Define the inflation rate as $\pi = \frac{\dot{P}}{P}$. Then, using (A.2a) and (A.2c), the growth rate of the economy, $\xi = \frac{\dot{C}}{C}$, is

$\xi = i - \pi - \rho$ (A.3)

Assume now a competitive final goods producer that aggregates a continuum of intermediate inputs according to the following

$$Y = \left( \int_0^1 y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{(A.4)}$$

with elasticity $\varepsilon \in (0,1)$.

As it is standard in the literature, cost minimization assures the following demand for intermediate good *j*

$$y_j(p_j) = \left( \frac{p_j}{P} \right)^{-\varepsilon} Y \qquad \text{(A.5)}$$

given the aggregate price structure

$$P = \left( \int_0^1 p_j^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \qquad \text{(A.6)}$$

Assume also a continuum of monopolistically competitive intermediate goods producers $j \in [0,1]$.

Let production uses labor only to simplify the analysis. Then, we have also

$$y_j(t) = An_j \qquad \text{(A.7)}$$

Being *A* the constant level of technology.

In presence of sticky prices, intermediate goods producers per period profits are

$$\Pi = p_j y_j - Wn_j \qquad \text{(A.8)}$$

that is explicitly, by substituting (A.4) and (A.5),

$$\Pi = \left( \frac{p_j}{P} \right)^{-\varepsilon} \left[ p_j - \frac{W}{A} \right] Y \qquad \text{(A.9)}$$

though they have to pay a quadratic price adjustment cost

$$\Theta\left( \frac{\dot{p}_j}{p_j} \right) = \frac{\theta}{2} \left( \frac{\dot{p}_j}{p_j} \right)^2 PY \qquad \text{(A.10)}$$

where $\theta$ is the degree of stickiness.

Hence, the optimal control problem of optimizing firms reads

$$V(p_j) = \max_{p_j} \int_0^\infty e^{-\int_0^t i(s)ds} \left[ \Pi - \Theta\left( \frac{\dot{p}_j}{p_j} \right) \right] dt \qquad \text{(A.11)}$$

whose solution implies the maximization of the following current value Hamiltonian function

$$H_2 = p_j \left( \frac{p_j}{P} \right)^{-\varepsilon} Y - \frac{W}{A} \left( \frac{p_j}{P} \right)^{-\varepsilon} Y - \frac{\theta}{2} \left( \frac{\dot{p}_j}{p_j} \right)^2 PY + \mu \dot{p}_j \qquad \text{(A.12)}$$

and therefore

$$\theta \frac{\dot{p}_j}{p_j} \frac{P}{p_j} Y = \mu \qquad \text{(A.13a)}$$

$$\dot{\mu} = i\mu - \left\{ \left[ (1-\varepsilon) + \varepsilon \frac{W}{Ap_j} \right] \left( \frac{p_j}{P} \right)^{-\varepsilon} + \theta \left( \frac{\dot{p}_j}{p_j} \right)^2 \frac{P}{p_j} \right\} Y \qquad \text{(A.13b)}$$

But since in a symmetric equilibrium $p_j = P$, then (A.13a) and (A.13b) reduce to

$$\theta \pi Y = \mu \qquad \text{(A.13c)}$$

$$\dot{\mu} = i\mu - \left[ (1-\varepsilon) + \varepsilon \frac{W}{AP} + \theta \pi \right] Y \qquad \text{(A.13d)}$$

Time derivative of (A.13c) reads

$$\theta \dot{\pi} Y + \theta \pi \dot{Y} = \dot{\mu} \qquad \text{(A.14)}$$

Substituting (A.14) into (A.13d), we easily derive

$$\left(i-\pi-\frac{\dot{Y}}{Y}\right)\pi=\frac{\varepsilon-1}{\theta}\left(\frac{\varepsilon}{\varepsilon-1}\frac{W}{AP}-1\right)+\dot{\pi} \qquad \text{(A.15)}$$

Since in equilibrium aggregate consumption must be equal to aggregate output,

$$C=Y=AN \qquad \text{(A.16)}$$

but using FOCs in (A.2*i*), we have that $CN^{\varphi}=\frac{W}{P}$ and $P=\frac{\varepsilon}{\varepsilon-1}\frac{W}{A}$, thus

$$C=A\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\frac{1}{1+\varphi}} \qquad \text{(A.17)}$$

which in growth rate terms implies that $\frac{\dot{C}}{C}=\frac{\dot{Y}}{Y}=\xi$, and thus by (A.3)

$$\frac{\dot{Y}}{Y}=i-\pi-\rho \qquad \text{(A.18)}$$

Hence, the equation for inflation in (A.15) becomes

$$\rho\pi=\frac{\varepsilon-1}{\theta}\left(\frac{\varepsilon}{\varepsilon-1}\frac{W}{AP}-1\right)+\dot{\pi} \qquad \text{(A.19)}$$

Similarly, if we define the natural output as $Y^{n}$, and the natural real interest rate as $r=i-\pi$, then

$$\frac{\dot{Y}^{n}}{Y^{n}}=r-\rho$$

and also

$$P^{n}=\frac{\varepsilon}{\varepsilon-1}\frac{W^{n}}{A}$$

where $W^{n}/P^{n}$ is the natural wage rate.

Define the output gap as $X=Y^{n}/Y$. Hence

$$\frac{\dot{X}}{X}=\frac{\dot{Y}}{Y}-\frac{\dot{Y}^{n}}{Y^{n}}=i-\pi-r \qquad \text{(A.20)}$$

which is the standard *IS* curve.

Then, we can derive a new version of (A.15), in terms of output gap,

$$\rho\pi=\frac{\varepsilon-1}{\theta}\frac{W/P-W^{n}/P^{n}}{W^{n}/P^{n}}+\dot{\pi} \qquad \text{(A.21)}$$

but since we know that $CN^{\varphi}=\frac{W}{P}$,and that market clearing implies $C=Y$ and $N=Y/A$, then

$$\frac{W/P}{W^{n}/P^{n}}=\left(\frac{Y}{Y^{n}}\right)^{1+\varphi}=X^{1+\varphi} \qquad \text{(A.22)}$$

hence, substituting (A.22) into (A.21), we derive the relation between inflation and the output gap

$$\rho\pi=\frac{\varepsilon-1}{\theta}\left(X^{1+\varphi}-1\right)+\dot{\pi} \qquad \text{(A.23)}$$

Put together (A.20) and (A.23) to obtain the system

$$\frac{\dot{X}}{X}=i-\pi-r \qquad \text{(A.24a)}$$

$$\rho\pi=\frac{\varepsilon-1}{\theta}\left(X^{1+\varphi}-1\right)+\dot{\pi} \qquad \text{(A.24b)}$$

joint with the Taylor rule followed by the central bank

$$i=i^{*}+\phi\pi+\phi_{x}\log X \qquad \text{(A.25)}$$

to close the model.

Using the log-linearization by defining $x=\log X=\log Y-\log Y^{n}$, we can therefore approximate the following series

$$X^{1+\varphi}-1=e^{(1+\varphi)x}-1=(1+\varphi)x-1 \quad \text{(A.26)}$$

Using (A.26), the system in (A.24a), (A.24b) and (A.25) becomes

$$\dot{x}=i-\pi-r \quad \text{(A.27a)}$$

$$\dot{\pi}=\rho\pi-\eta x \quad \text{(A.27b)}$$

$$i=i^{*}+\phi\pi+\phi_{x}x \quad \text{(A.27c)}$$

where $\eta=\frac{(\varepsilon-1)(1+\varphi)}{\theta}$ (see Galí (2008); Werning (2011)).

Substituting (A.27c) into (A.27a), we finally obtain

$$\begin{aligned}\dot{x}&=i^{*}-r+(\phi-1)\pi+\phi_{x}x\\ \dot{\pi}&=\rho\pi-\eta x\end{aligned} \quad (S)$$

where $x$ and $\pi$ are the jump variables of the autonomous system.

## References


Bella, G., Mattana, P. and Venturi, B. (2013). The double scroll chaotic attractor in the dynamics of a fixed-price IS-LM model. *International Journal of Mathematical Modelling and Numerical Optimisation*, 4(1): 1-13.

Blanchard, O., Cerutti, E. and Summers, L. (2015). Inflation and activity: two explorations and their monetary policy implications. NBER Working Paper, 21726.

Carlin, W. and Soskice, D. (2009). *Teaching intermediate macroeconomics using the 3-equation model. In: Macroeconomic theory and macroeconomic pedagogy, ed. G. Fontana and M. Setterfield.* Palgrave Macmillan: Basingstoke.

Fernández-Villaverde, J., Guerrón-Quintana, P. and Rubio-Ramírez, J. F. (2014). Supply-side policies and the zero lower bound. *IMF Economic Review*, 62: 248-60. Available: http://www.nber.org/papers/w17543

Galí, J. (2008). *Monetary policy, inflation, and the business cycle: An introduction to the New Keynesian Framework.* Princeton University Press: Princeton.

Gamero, E., Freire, E., Rodriguez-Luis, A. J., Ponce, E. and Algaba, A. (1999). Hypernormal form calculation for triple-zero degeneracies. *Bulletin of the Belgian Mathematical Society*, 6: 357-68. Available: https://projecteuclid.org/download/pdf_1/euclid.bbms/1103065855

Lucas, R. (1976). *Econometric policy evaluation: A critique. In: The Phillips Curve and Labor Markets, ed. K. Brunner and A.H. Meltzer.* Carnegie-Rochester Conferences on Public Policy: Amsterdam.

Rotemberg, J. (1982). Sticky prices in the United States. *Journal of Political Economy*, 90(6): 1187-211.

Schmitt-Grohé, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2): 198-230.

Turnovsky, S. J. (2011). Stabilization theory and policy: 50 years after the phillips curve. *Economica*, 78(309): 67-88.

Wang, J., Chen, Z. and Yuan, Z. (2009). Existence of a new three-dimensional chaotic attractor. *Chaos, Solitons and Fractals*, 42( 5): 3053-57.

Werning, I. (2011). Managing a liquidity trap: Monetary and fiscal policy. NBER Working Paper, 17344.