

## Measures of the Output Gap in Turkey: An Empirical Assessment of Selected Methods

Ilyas Şıklar\*

Anadolu University, Department of Economics, Turkey

Suzan Şahin

Anadolu University, Department of Economics, Turkey

### Abstract

The output gap indicating the difference between the actual and potential levels of output is a critical factor for estimating the inflationary pressures in an economy. If the main target of a central bank is ensuring and maintaining the price stability, estimating the output gap with a minimum error is crucial for the efficiency of the monetary policy. In this study, we estimated the output gap in Turkey for the 2002-2014 period by using four different methods. Two of these estimation methods are purely statistical (Linear Trend and Hodrick-Presscot (HP) Filtering) while the others are integrated with the relations suggested by the economic theory (multivariate structural model and structural autoregressive (SVAR) model). By using empirical decision criteria common in the literature, we conclude that SVAR model produces the most reliable output gap estimates to explain inflationary pressures in Turkey. However, we also found that the Hodrick-Presscot filtering method is the second best methodology in the output gap estimation process.

**Keywords:** Output gap; Potential GDP; HP filter; SVAR model; Linear trend model; Multivariate structural model.



CC BY: [Creative Commons Attribution License 4.0](https://creativecommons.org/licenses/by/4.0/)

### 1. Introduction

The output gap is defined as the difference between the actual output and the potential output. The calculation of the output gap with a minimum error is important for the effectiveness of the monetary policy. If the real output is higher than the potential, that is, if the output gap is positive, the resulting demand pressure may be at a level that would lead to inflation. Therefore, the output gap is considered as a sign of demand-side inflationary pressure by the policy makers.

'Potential output' is a concept which refers to a long-term process, while 'output gap' refers to a short or medium-term process. Demand-side shocks affect output gap by effecting real output, while supply-side shocks affect potential output. While the Gross Domestic Product (GDP), a component of the output gap, refers to realized output, exact calculation of the other component (i.e. the potential output) is difficult both in theory and in practice. Therefore, various methods have been developed to estimate the potential output. These methods have a wide spectrum ranging from univariate statistical filters to multivariate structural models. Because of their nature, statistical models can be established by using less information compared to theory-based structural models. Their prominent characteristic therefore is the ease of application. Despite statistical models are highly preferred, the choice of the correction parameters or initial values used in the univariate statistical filter methods result in different estimation outcomes. Another criticism claims that these methods are not based on economic theory. Structural models are both multivariate and stemmed from the economic theory. Furthermore, they contain additional information on growth dynamics. For these reasons, they are advocated as alternative calculation methods against the criticisms on statistical models. This is why structural models in the literature are more common. Structural models give an opportunity to obtain output gap estimates which are suitable for stationary inflation assumptions.

The main aim of the present study is to determine the estimation method that best predicts the effect of output gap on inflation. In this context, four of the most commonly used methods are chosen; two statistical methods and two structural methods. The estimates are made by using the quarterly data obtained for Turkey in 2002-2014 period. The statistical methods are Hodrick Prescott (HP) Filter model and Linear Trend model, and the structural ones are Multivariate Structural Model and Structural Autoregressive (SVAR) model. In our study, output is represented by the GDP in constant prices. In this context, quarterly frequency series including 2002-2014 years are used. The base year for this time-series is 2007, as determined by the Turkish Statistical Institute. Although the new GDP data for the quarterly frequency of our country are accessible since 1998, the reason why we focus on the period between 2002-2014 in this study is to exclude the financial crisis experienced in 2001 from the review period. Thus, the possibility a structural break due to the crisis is excluded from the model. A data used in the study have been collected from the Electronic Data Delivery System of the Central Bank of the Republic of Turkey (CBRT).

The rest of the paper is organized as follow: Part 2 shortly reviews the definition and importance of potential output and output gap while Part 3 essentially deals with output gap modeling and estimation methodologies. Part 4 empirically evaluates the forecasting performance of the output gap estimates and finally Part 5 concludes the study.

## 2. The Definition and Importance of Potential Output and Output Gap

The term potential output was coined by M. Arthur Okun in 1962, and defined as the level of output under full employment conditions. According to another definition by Okun, potential output shows the level of the use of production factors which does not cause inflationary pressures. In its recent use, potential output refers to the level of sustainable real GDP. Potential output is also a key concept for economic policy since it is used to measure the standards of sustainable living (Horn *et al.*, 2007).

Various methods can be used to calculate the potential output. Although the literature continues to change and develop on this topic, the following methods are worth mentioning (the first four of them are predominantly statistical, and the others are predominantly economic measures): Deterministic Trend, Univariate Filtering, Unobserved Components, Multiple Filtering, Structural VAR (Blanchard-Quah), Production Function, and Macroeconomic Models (Yavan and ve Türker Kaya, 2007).

The output gap is a critical concept used in inflation targeting monetary policy. The inflation targeting is based on the assumption that prices and wages are sticky in short-term. When determining inflation target, central banks use short-term interest rate as monetary policy instrument. This interest rate influences the national output gap and the change in output gap also affects the inflation rate (Çiçek, 2009).

## 3. The Output Gap Modeling and Estimation for Turkey

As mentioned earlier, output gap is defined as the difference between actual output and the potential output.

$$gap_t = y_t - y_t^T$$

Here,  $gap$  is the output gap;  $y$  is the actual output and  $y^T$  is the potential output. In such formulation, a positive value for the gap indicates excessive demand, and a negative value indicates overcapacity. The presence of the output gap indicates that there is a temporary deviation from the potential output level. The level of potential output should be estimated since it is an unobservable economic variable. Because there are various definitions and methods on this issue, various results can be obtained.

According to Brouwer and Ericsson (1995), Debelle and Vickery (1997), output gap is an economic variable that contains highly valuable information on fluctuations in prices and wages. However, from an economic policy perspective, the trend or potential component of the mentioned variables must be defined in terms of a constant inflation rate. This cannot be done with the estimation of the output gap with pure statistical techniques, but it can be applied in multivariate models and structural models. In other words, structural models make it possible to obtain output forecasts consistent with constant inflation assumptions.

Figure 1 below shows the course followed by the logarithmic level of real GDP during the review period, while Figure 2 indicates the quarterly and annual (four-quarter) economic growth rates in the same period.

Figure-1. Real GDP (logarithmic level)

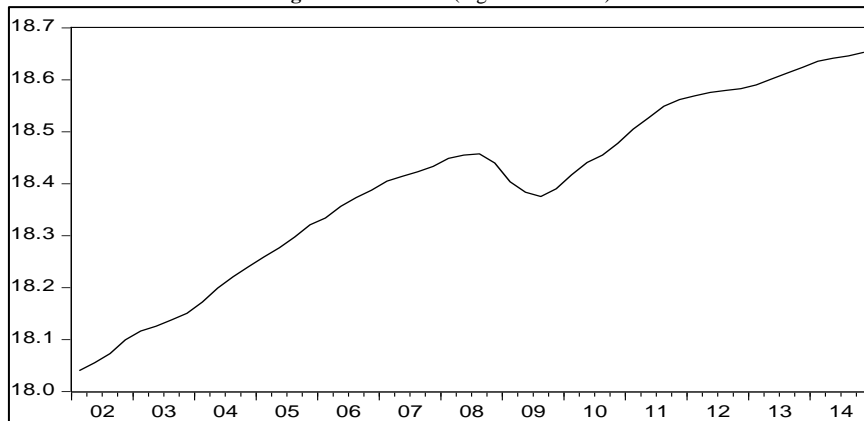
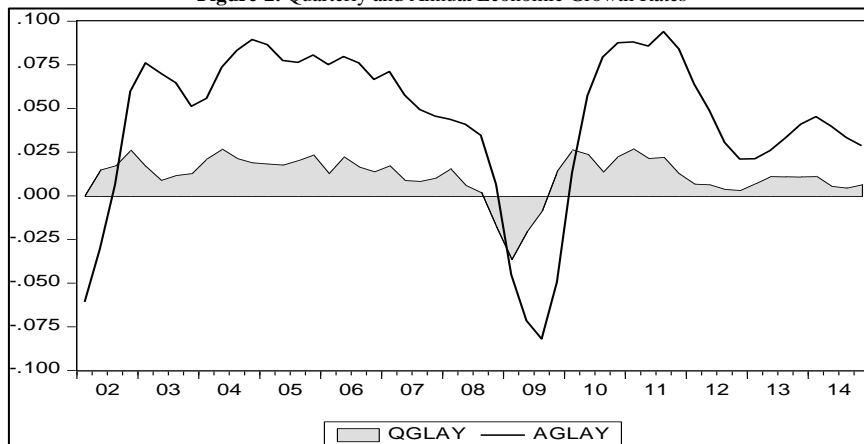


Figure-2. Quarterly and Annual Economic Growth Rates



Note: QGLAY represents quarterly and the AGLAY represents annual growth rate

According to the graphs, the economy experienced a serious recession in 2008-2010 due to the global crisis in 2008. On the other hand, the existence of a contraction period observed in 2012 can also be observed. The contraction in 2008, however, was short-term, and the contraction in 2012 appears to be longer. Our general expectation is that these two sub-periods resulted in negative output gaps with similar characteristics during the entire review period. In the following section, we will look through the methods that we use to estimate output gap and the results they generate.

### 3.1. Estimation of Output Gap by Statistical Techniques

There are a number of methods that can be used when the output gap is estimated by univariate statistical time series techniques. These methods range from simple to complex in various scales, from the simple trend method to ARIMA (Box-Jenkins) model, from the simple Hodrick-Prescott filtering to multi-variate Kalman filtering. Since our goal is not to compare the performance of univariate time series techniques with structural (multivariate) models, we prefer the most common of the above-mentioned techniques to compare the output gap bias estimates of structural models. Hence, the output gap will be estimated by using the linear time trend and Hodrick-Prescott (HP) filtering methods.

#### 3.1.1. Linear Time Trend Model

The simplest way to estimate the output gap is to estimate potential output level with linear trend. Estimated trend equation with the use of the logarithmic real GDP series for the quarterly data is as follow:

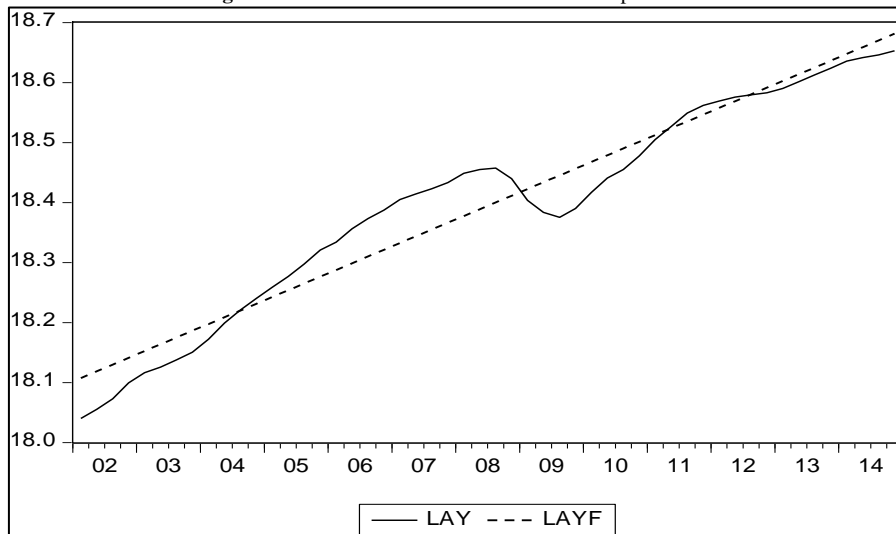
$$y_t = 17,93 + 0,01(trend)$$

$$(0,02) \quad (0,001)$$

$$Adj R^2 = 0,94$$

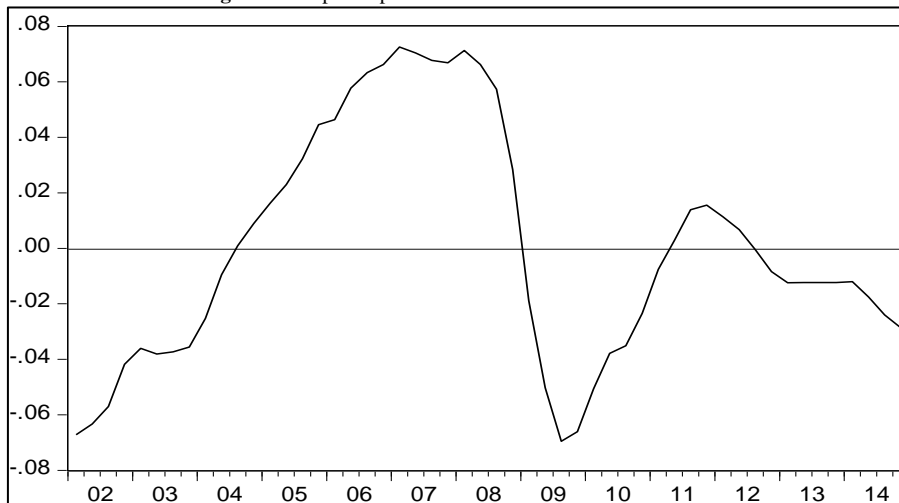
Here, the values in parentheses express the standard error values and  $Adj R^2$  is the determination coefficient adjusted for degrees of freedom. According to the equation, the trend growth rate of output is estimated to be around 4.5 percent annually in the 13-year-period. It is possible to see the potential and actual output values obtained as a result of this trend equation in Figure 3 while the output gap values obtained from the relation expressed by this equation are given in Figure 4.

Figure-3. Actual and Linear Trend Potential Output Values



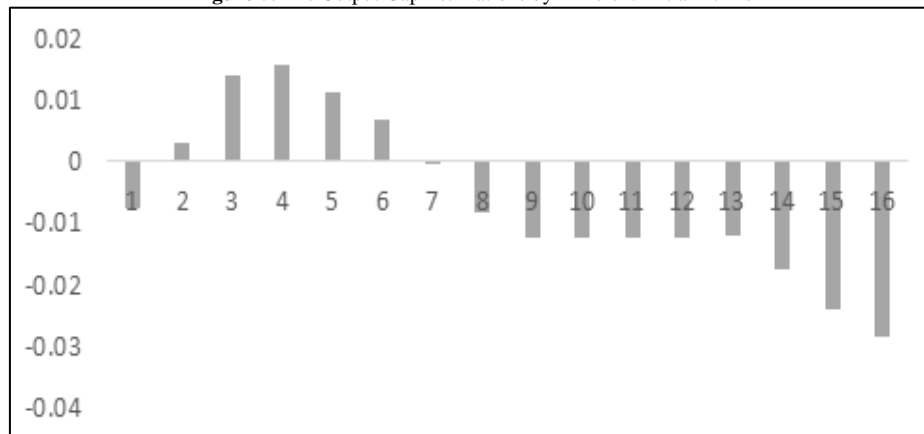
Note: LAY refers to real GDP and the LAYF refers to potential GDP

Figure-4. Output Gap Estimation with Linear Trend Method



This model can be criticized because of the fact that it is based on the estimation period of the forecasting dimension. For example, the trend equations obtained when we move the initial period of the model from the first quarter of 2002 to the fourth quarter of 2005 by moving one quarter onwards at a time, and the estimated output gap values for the last quarter of 2014 can be seen in Figure 5.

Figure-5. The Output Gap Estimations by Different Initial Terms



As can be seen in the Figure 5, when the whole period is taken into account, the potential output is larger than the actual output in the last quarter of 2014, relative to the respective trend values and there is negative output gap. However, when we take the next quarter as the initial period, the deficit is almost zero, and even actual output is greater than potential output. This situation continues to increase until the beginning of the third quarter of 2003 and the estimated output gap values for the last quarter of 2014 turn out to be positive. However, when the beginning of the forecast period is shifted further, the situation is reversed. Moreover, the model is causing us to estimate again negative output gap values for the last quarter of 2014. So, in the prediction of the linear trend models, the choice of the starting and ending points of the forecast period is critical. This is important because we point out that starting the forecast period in 2002 is a good choice. On the other hand, the assumption that the potential output grows at a constant rate is not a presumptive assumption. It is more realistic to accept that in a developing economy like Turkey factors that affect potential output may change over time in a country where significant structural reforms are at stake. This fact is evident in the characteristics of the predicted output gap time series. If the output is determined by a deterministic trend, residual terms obtained as a result of eliminating this trend from the time series are expected to have stationary time series characteristics. However, if the output-related time series is an integrated series at the first order, in other words, if it follows a stochastic trend, then the residual series obtained by the elimination of linear trend off will be a series of non-stationary time periods. In this case, the assumption that the output gap is a mean reverting variable is violated. The literature on whether output follows a deterministic trend or not, on whether it involves structural breaks or not, and on whether it has a stochastic trend or not, is so wide that it prevents a definitive judgment (Diebold and Senhadji, 1996).

Table 1 below shows some statistics on real GDP in Turkey between 2002 and 2014, the review period. According to the first two rows of the table, output follows a stochastic trend. However, the level of probability that affects this decision is very low. In the third and fourth lines of the table, it can be seen that the output gap time series estimated through linear trends are not stable at both the level and the first difference. These results are similar to those obtained by Hodrick and Prescott (1997), and emphasize the need to apply various purification techniques.

Table-1. Unit Root Test Results for Output and Output Gap

Variable	Constant	t-ratio	Trend	t-ratio	Lag	t-ratio (ADF)	Marginal Significance Level
y	1,426	2,058	0,001	1,865	-0,079	-2,047	0,562
Δy	0,008	3,440	-0,000	-1,200	-0,500	-3,780	0,026
gap	0,007	2,057	-0,000	-2,004	-0,105	-2,483	0,335
Δgap	0,003	0,919	-0,000	-0,979	-0,441	-2,574	0,293

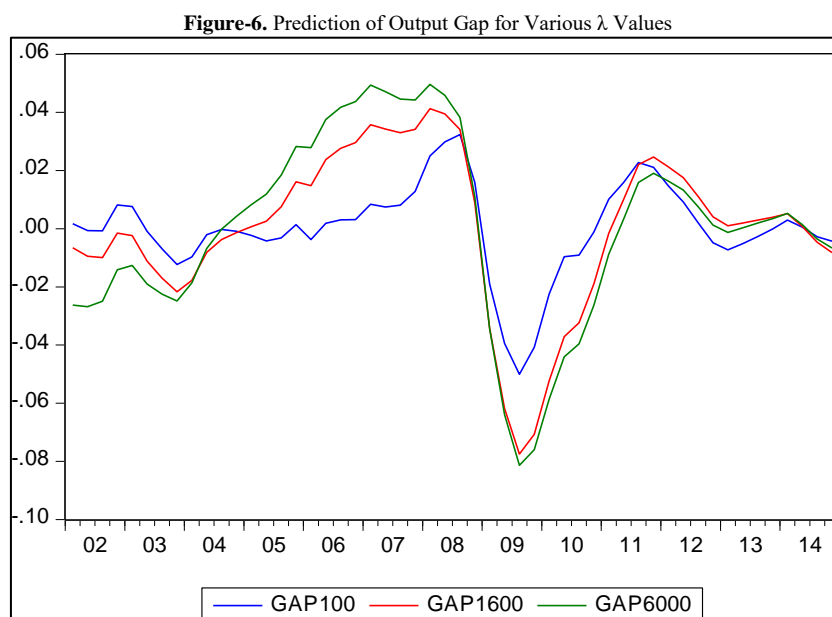
### 3.1.2. Hodrick – Prescott (HP) Filter

This purification technique, proposed by Hodrick and Prescott (1997), considers the existence of linear trends in the time series as a special case. In the HP filtration technique, the potential component of output is obtained by minimizing the following loss function:

$$L = \sum_{t=1}^S (y_t - y_t^T)^2 + \lambda \sum_{t=2}^{S-1} (\Delta y_{t+1}^T - \Delta y_t^T)^2$$

where  $S$  indicates the magnitude of the sample size, and  $\lambda$  expresses the weight of the potential output growth. Changing this weight influences how potential output reacts to changes in actual output. According to the equation, as the weight approaches infinity, the loss function is minimized by reducing the variations in the potential output. This means that the potential output growth remains constant, in other words, the linear trend growth rate is

achieved. On the other hand, when the weight determined by the  $\lambda$  parameter is zero, the loss function is minimized by abolishing the difference between potential and actual outputs. This means that the potential output is equal to the actual output. This can be seen in Figure 6 below. When the values of  $\lambda$  are defined as 6000, 1600 and 100, according to the figure, the smaller the parameter value, the smaller the estimated output gap values we get.



The most important advantage of the HP filtering technique is that a wide range of values for  $\lambda$  can make output gap estimations stationary. On the other hand, this method also allows the trend to change over time. Thus, the forecasting power is increased in the estimation of the output gap. What can be criticized in HP filtering technique is that the value to be set for the  $\lambda$  parameter can be changed arbitrarily. For example, in Figure 6, considering the estimation for the last quarter of 2002, output is over the potential at low weight (positive output gap), and below the potential at high weight (negative output gap). In fact, in different estimates we make, when the  $\lambda$  value is determined in the range of 100 - 1500, the output gap is positive, and when the  $\lambda$  value is over 1500, the output gap is negative. Therefore, this method is not a useful method for determining the absolute value of an output gap at a certain date.

With the HP filtering technique,  $\lambda$  value does not only influence the size of the gap, but it also influences the relative value of the gap, and the timing of the troughs and peaks observed in output. For example, according to Figure 6, the high  $\lambda$  value in 2007 points to a very high positive output gap rate compared to 2004, while the low  $\lambda$  value indicates that this positive output gap rate cannot change much compared to 2004. However, this situation is completely reversed in 2008. In this case, it can be said that the turning points in the output change with the value determined for the weight ( $\lambda$ ).

If the selection of the weight parameter is decisive in terms of the results, there should be a distinct and clear criterion for the selection of the value of the parameter so that the method can be useful. According to the criterion set by Hodrick and Prescott (1997), the appropriate  $\lambda$  value determines the relative magnitude of the variances of shocks occurring in the temporary and permanent components of output. In the study mentioned, this value is determined as 1600 for the real GDP time series for the U.S. Guay and St. Amant (1996) present Monte Carlo evidence for the  $\lambda$  parameter, based on the frequency of the data, to be determined as 100 for the annual data, 1600 for the quarterly data and 14400 for the monthly data. For this reason, many empirical studies use these suggested values for the HP filtering technique, taking the frequency of the data into account. While the performance of the output gap estimates is evaluated in the following sections of our paper, the output gap values produced by the proposed value of  $\lambda = 1600$  for the quarterly frequency data will be taken as the output gap values generated by the HP filtering method.

### 3.2. Estimation of Output Gap by Structural Techniques

Structural methods for output gap estimation can be considered as the methods based on the theory of economics. As can be seen above, the linear trend and HP filtering techniques we have discussed are pure statistical techniques. Other numerical and structural information that can be obtained from economics is not used in the application of these methods. The output values for quarterly frequency are enough to estimate it. In the case of structural forecasting methods, the estimated potential output is influenced by possible economic factors. We will use the two most common methods in the literature for this purpose. These methods are multivariate structural method and structural vector autoregressive model method.

#### 3.2.1. Multivariate Structural Model

It is possible to mention some economic indicators (such as capacity utilization rate, electricity consumption) and economic relations (such as the Phillips Curve and the Okun Law) that contain information about the supply side

of the economy and the business cycle conditions. As a matter of fact, **Laxton and Tetlow (1992)** enhance the HP filtering technique to cover economic information in order to estimate output gap. Accordingly, potential output is defined as a time series that minimizes the loss function as follow:

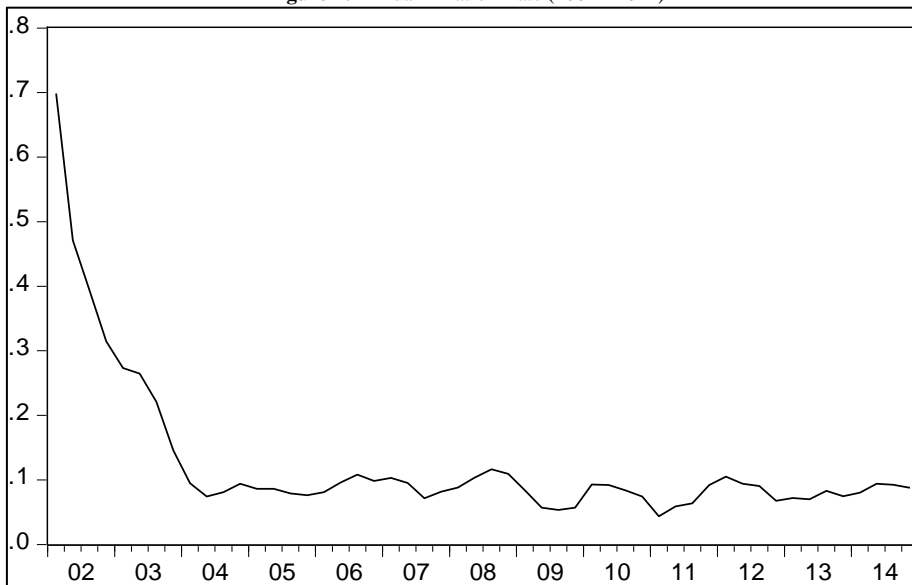
$$L = \sum_{t=1}^S (y_t - y_t^T)^2 + \lambda \sum_{t=2}^{S-1} (\Delta y_{t+1}^T - \Delta y_t^T)^2 + \sum_{t=1}^S \mu_t \varepsilon_{\pi,t}^2 + \sum_{t=1}^S \beta_t \varepsilon_{u,t}^2 + \sum_{t=1}^S \varphi_t \varepsilon_{cu,t}^2$$

where, in addition to the variables previously defined  $\varepsilon$  represents the error terms obtained from regression equations. The terms  $\pi$ ,  $u$ , and  $cu$  in the subscripts of the error terms refer to the Phillips Curve Equation, the Okun's Law Equation, and the Capacity Utilization Equation, respectively. Therefore, the corresponding error term is used in the loss function of the error terms obtained by estimating these equations. On the other hand, the parameters  $\mu$ ,  $\beta$  and  $\varphi$  in the loss function should be considered as time varying weights. In deriving these error terms, the following equations are used:

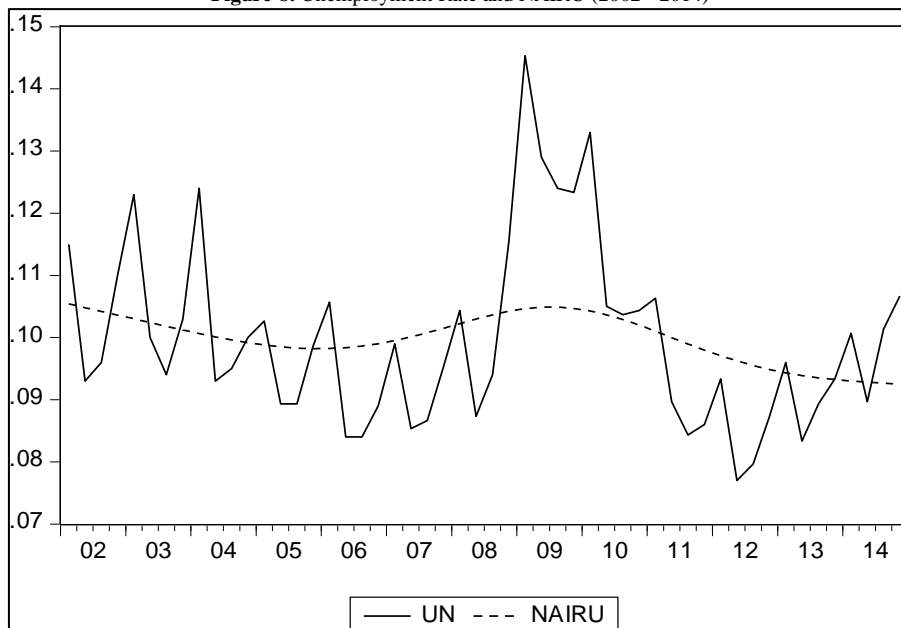
Phillips Curve:  $\pi_t = \pi_t^e + A(L)(y_t - y_t^T) + \varepsilon_{\pi,t}$   
 Okun's Law:  $u_t = nairu_t - B(L)(y_t - y_t^T) + \varepsilon_{u,t}$   
 Capacity Utilization:  $cu_t = cu_t^T + C(L)(y_t - y_t^T) + \varepsilon_{cu,t}$

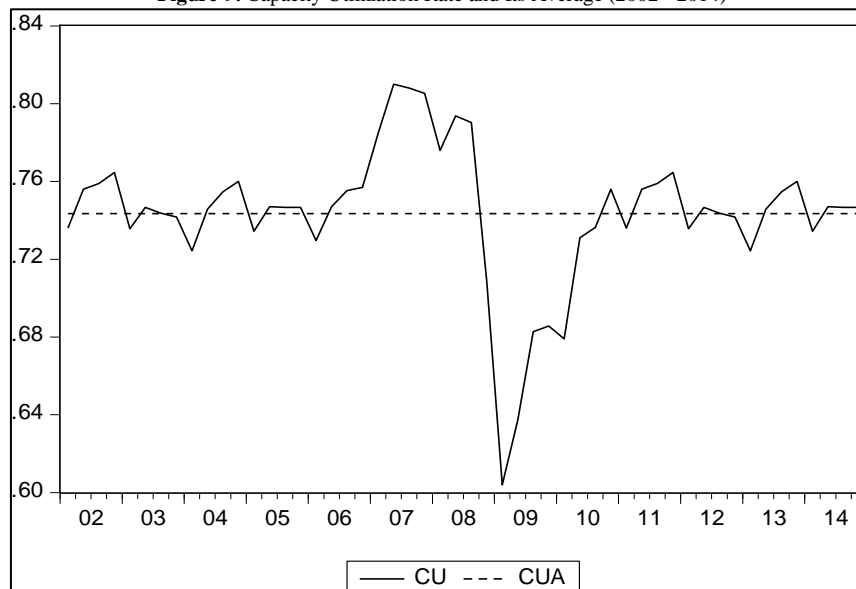
When actual output is larger than potential output (assuming a positive output gap), according to Phillips Curve, the realized inflation will be higher than the anticipated inflation. According to Okun's Law equation, actual unemployment rate is lower than NAIURU value, i.e. unemployment rate which does not accelerate inflation (equilibrium unemployment rate), when actual output is higher than potential output. According to the equation of capacity utilization, which is regarded as an indicator of the supply side of the economy, the capacity utilization in the economy is above the trend when the actual output is greater than the potential output. The behavior of the variables involved in these equations are shown in **Figures 7 8 and 9**.

**Figure-7.** Annual Inflation Rate (2002 - 2014)



**Figure-8.** Unemployment Rate and NAIURU (2002 - 2014)



**Figure-9.** Capacity Utilization Rate and Its Average (2002 - 2014)

The multivariate model included in the equations above allows to determine the following items:

- (1) the weighted average of actual output deviations from potential output,
- (2) changes in the rate of increase in potential output, and
- (3) the potential level of output by minimizing the errors caused by the three predetermined structural relations (Phillips Curve, Okun's Law and capacity utilization).

The determination of the output gap based on the information listed above allows for more realistic estimates of potential output. It is also expected that the output gap values obtained will become more reliable.

In order to estimate the output gap by using this method, it is necessary to estimate the Phillips Curve, the Okun's Law and the capacity utilization equations. To obtain the initial values in these equations, potential output is based on the HP filtering technique and  $\lambda = 1600$ . Another problem in the estimation of the Phillips curve is the formation of inflation expectations. We will assume that the expectations are adaptive and determined by the past inflation rates. Since the model is estimated by using quarterly frequency data, taking the annual inflation rates in the past year as a basis can be considered as a sufficient in the formation of expectations. In the light of these explanations, the estimated Phillips Curve equation is as follow:

$$\pi_t = 0,32 \pi_{t-1} + 0,11 \pi_{t-2} + 0,07 \pi_{t-3} + 0,36 \pi_{t-4} + 0,11(y_t - y_t^T)$$

$$(0,11) \quad (0,12) \quad (0,09) \quad (0,09) \quad (0,09)$$

$$RSS = 0,01385$$

The values given in parentheses below the coefficient estimates in the above equation represent the standard error of the coefficient estimated and RSS represents the residual sum of squares. Since this equation will be used in forecasting potential output, the result shows that current inflation should be considered in estimating the current output gap.

The NAIRU value, in the the Okun's Law equation has been determined by using the long-term trend value as indicated by [Debelle and Vickery \(1997\)](#). [Figure 8](#) which was given earlier shows the course of this long-term value. Accordingly, the estimate for the Okun's Law equation is as follow:

$$(u_t - nairu_t) = -0,32 (y_t - y_t^T)$$

$$(0,0,5)$$

$$RSS = 0,004734$$

According to this equation, unemployment will decrease if the current demand in the economy is strong compared to potential output. On the contrary, if there is a negative output gap, there will be an upward trend in unemployment.

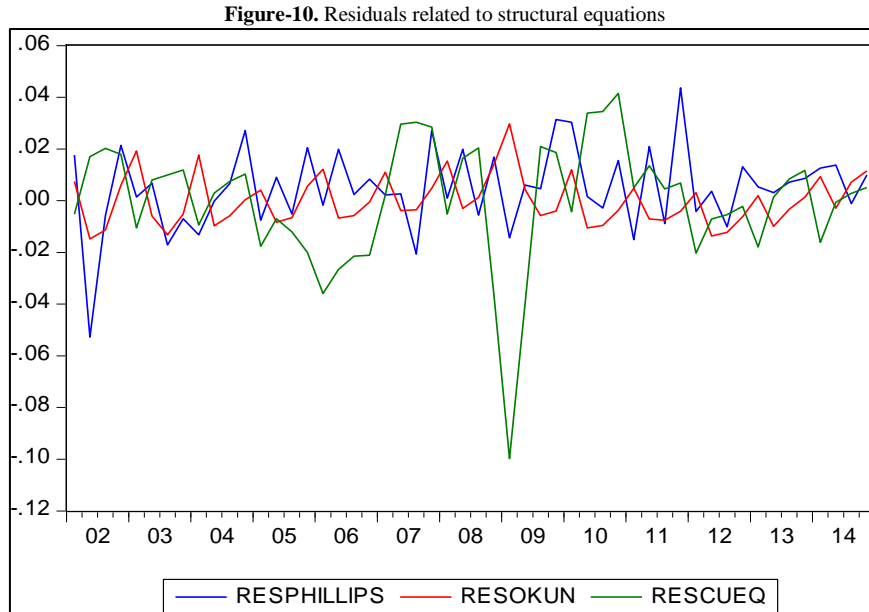
Capacity utilization included in the model as an indicator of the supply side of the economy is the manufacturing industry capacity utilization rate determined by the questionnaire of the Turkish Statistical Institute. The trend of these data and the comparison of them with the whole period average can be seen in [Figure 9](#). First of all, it is seen that 2008 - 2009 global crisis caused a serious deviation in capacity utilization rate. Given that the capacity utilization rate, which declined to 60 per cent at the beginning of 2009, was about 74.5 per cent of the survey period average, the size of the slip emerges. On the other hand, the creation of these data through the questionnaire method also implies a number of problems. For example, whether firms make a clear distinction about labor and capital constraints, or whether the conditions that firms define as "normal" vary depending on the situation in the business cycle are two of these problems. Despite these disadvantages, the most important data we can use for the supply side of the economy is the capacity utilization rate, and it is among the leading indicators of the business cycle. The estimated equation for capacity utilization in this study is as follow:

$$cu_t = -0,01 + 0,90 (y_t - y_t^T)$$

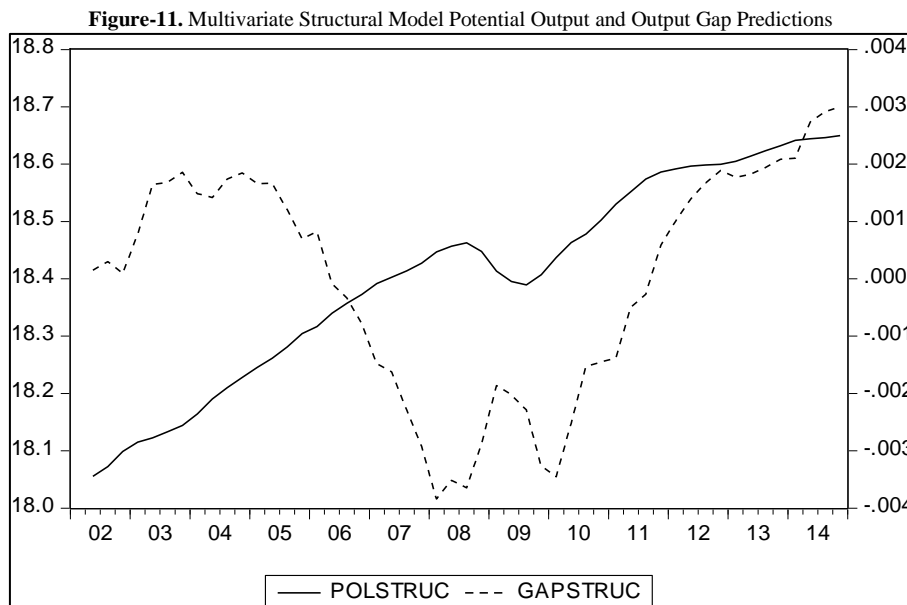
$$(0,003) \quad (0,12)$$

$$RSS = 0,028200$$

This equation fits the theoretical expectations indicating that capacity utilization carries information that should be considered about the current output gap in the economy. Residual terms obtained from the above equations can be seen in Figure 10 below.



By using these residual series, the previously given multivariate loss function can be minimized for solving potential output. The basic properties of the successive minimization technique used can be explained as follow: First, residual terms obtained by predicting the above structural equations will form the basic variables to be used in estimating the loss function. By using these terms in the first stage, the potential output value is estimated, and the initial output gap is calculated. Then structural equilibrium is re-estimated by including output gap, so that the variability of potential output is minimized. This procedure continues as the coefficient for the output gap variable decreases, and when the coefficient increases, the previous regression equation is considered to be the equation that fulfills the minimization condition. The potential output values obtained from the estimation of this last equation are the basic values used in the calculation of the output gap. Given the number of explanatory variables used and the size of the data set, the methodology described above requires the estimation of 74 consecutive regression equations and in the 73th estimation minimization condition is provided. The Figure 11 below shows the potential output values obtained from the last equation providing the minimization condition and the calculated output gap values accordingly.



Compared to the previously calculated output gap values, it is seen that there is a serious contraction in the output gap values calculated with this method. The averages of the output trend estimates for the linear trend, and HP filtering techniques that were performed before are 0.001317 and 0.000129, respectively. The average of the output gap values obtained from the multivariate structural model is very close to zero (-8,34E-14). On the other hand, the standard deviations of the output gap values produced by linear trend and HP filtering methods are



0.041653 and 0.027280, respectively, whereas the standard deviation value obtained from the multivariate structural model is 0.001993. In this case, it can be said that the use of additional economic information makes it more realistic to estimate the output gap. Moreover, the output gap values support the results obtained by the Phillips Curve and Okun's Law equations.

### 3.2.2. Structural Vector Auto-Regression (SVAR) Model

In this part of the study, the potential output and output gap values will be estimated by using the three-variable structural vector autoregressive (SVAR) model proposed by Bjørnland *et al.* (2006). It may be considered as a necessity to incorporate the long-term constraints proposed by Blanchard and Quah (1989), which are the main features of the SVAR model, into the model. The mentioned authors apply the long-run constraints to the bivariate VAR model to see the consequences of long-lasting permanent shocks and short-term transitory shocks. If the GDP time series data that is used to represent output have a high frequency (for instance, quarterly data like in our analysis), the short-term is accepted as a period through which amount of production factors, consumption habits and productivity are constant; short term or temporary shocks are assumed as stemming from the demand side of the economy. On the other hand, in the long run, the quantity of production factors, habits, expectations, efficiency and technology are considered to be dynamic. In this case, permanent or long-term shocks must be accepted as originating from the supply side of the economy.

The starting point of the model is the ordering of the variables in the three-variable VAR model and the inclusion of constraints into the model. In the Cholesky decomposition framework, variables included in the model are unemployment rate, real GDP, and the inflation rate measured by consumer price index. The presentation of the model and the constraints to be applied are as follow:

$$\begin{aligned} \Delta u_t &= \sum_{K=0}^{\infty} A_{11}(K)\varepsilon_{1,t-k} + \sum_{K=0}^{\infty} A_{12}(K)\varepsilon_{2,t-k} + \sum_{K=0}^{\infty} A_{13}(K)\varepsilon_{3,t-k} + \psi_u \\ \Delta y_t &= \sum_{K=0}^{\infty} A_{21}(K)\varepsilon_{1,t-k} + \sum_{K=0}^{\infty} A_{22}(K)\varepsilon_{2,t-k} + \sum_{K=0}^{\infty} A_{23}(K)\varepsilon_{3,t-k} + \psi_y \\ \pi_t &= \sum_{K=0}^{\infty} A_{31}(K)\varepsilon_{1,t-k} + \sum_{K=0}^{\infty} A_{32}(K)\varepsilon_{2,t-k} + \sum_{K=0}^{\infty} A_{33}(K)\varepsilon_{3,t-k} + \psi_\pi \end{aligned}$$

In other words, it can be written as;

$$\begin{bmatrix} \Delta u \\ \Delta y \\ \pi \end{bmatrix}_t = \begin{bmatrix} A_{11}(L) & A_{12}(L) & A_{13}(L) \\ A_{21}(L) & A_{22}(L) & A_{23}(L) \\ A_{31}(L) & A_{32}(L) & A_{33}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} + \begin{bmatrix} \psi_u \\ \psi_y \\ \psi_\pi \end{bmatrix}_t$$

In the above notation  $[\psi_u, \psi_y, \psi_\pi]$  refers to the deterministic trend vector, and  $A_{ij}(L)$  is the lag operator. Lag level is determined by the formula  $E(\varepsilon_i \varepsilon_i') = I$ . Since shocks cannot be observed, the VAR model should be estimated in the following form:

$$\begin{bmatrix} \Delta u \\ \Delta y \\ \pi \end{bmatrix}_t = \begin{bmatrix} H_{11}(L) & H_{12}(L) & H_{13}(L) \\ H_{21}(L) & H_{22}(L) & H_{23}(L) \\ H_{31}(L) & H_{32}(L) & H_{33}(L) \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta y \\ \pi \end{bmatrix}_{t-1} + \begin{bmatrix} \psi_u \\ \psi_y \\ \psi_\pi \end{bmatrix}_t + \begin{bmatrix} \mu_u \\ \mu_y \\ \mu_\pi \end{bmatrix}_t$$

Correspondingly, the residual terms from the VAR model can be written as:

$$\begin{bmatrix} \mu_u \\ \mu_y \\ \mu_\pi \end{bmatrix}_t = \begin{bmatrix} \psi_u \\ \psi_y \\ \psi_\pi \end{bmatrix}_t + \begin{bmatrix} A_{11}(0) & A_{12}(0) & A_{13}(0) \\ A_{21}(0) & A_{22}(0) & A_{23}(0) \\ A_{31}(0) & A_{32}(0) & A_{33}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}_t$$

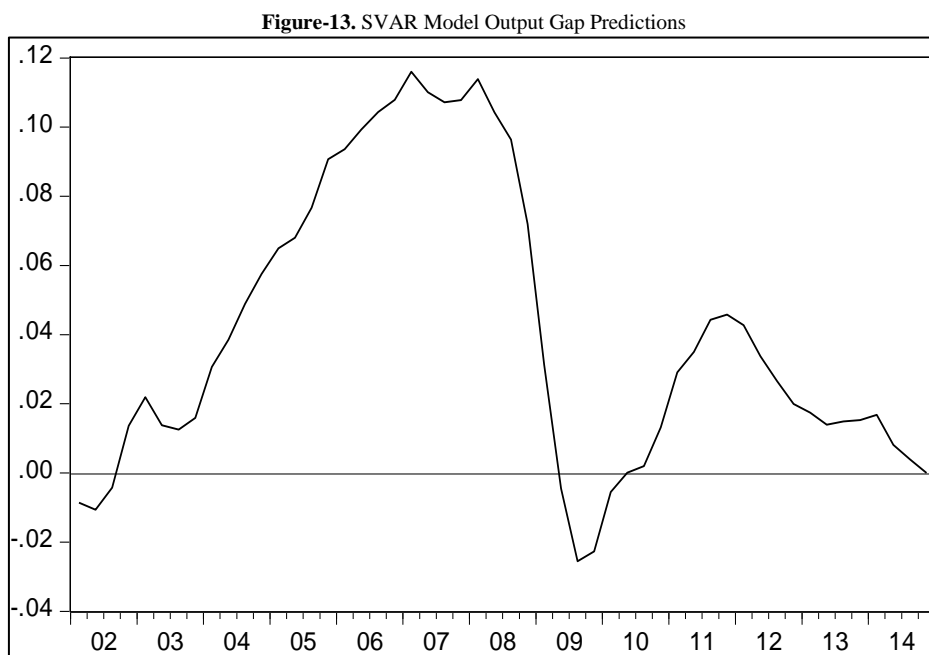
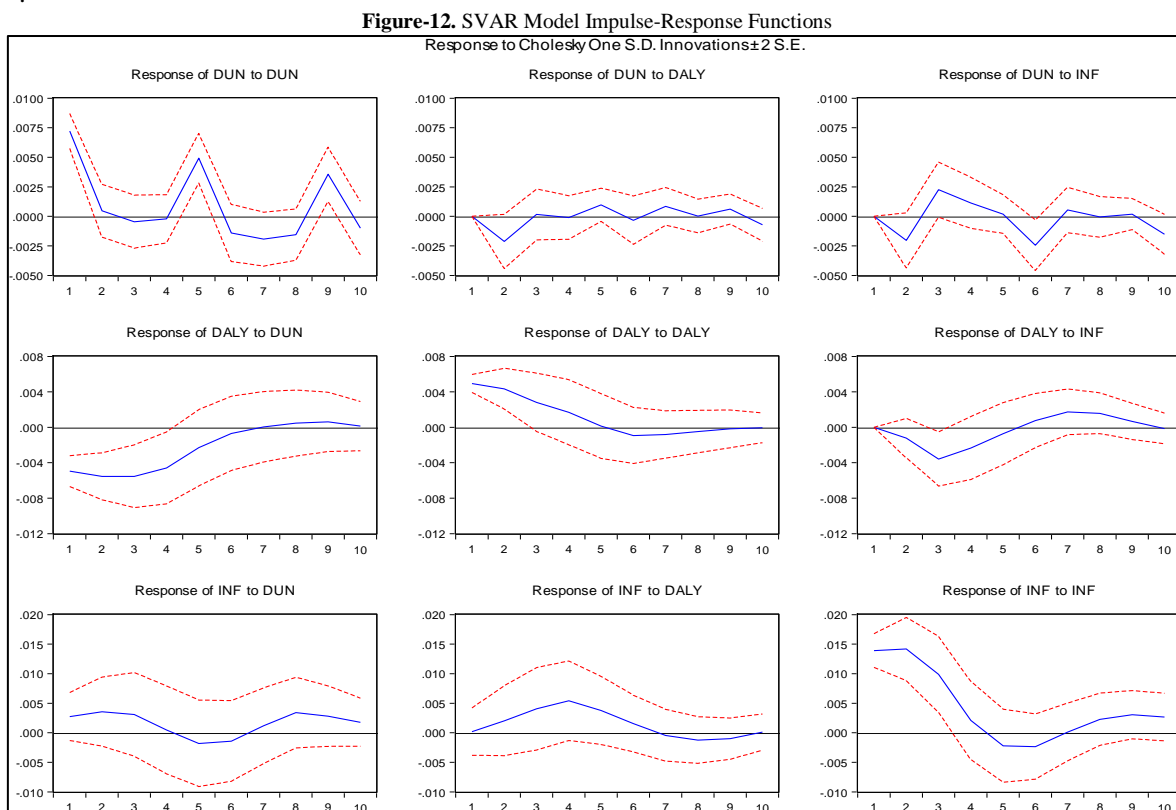
Considering the last obtained model, while  $A(0)$  describes the simultaneous effects of shocks in the system,  $\varepsilon_1$  shows the permanent (aggregate supply) shocks,  $\varepsilon_2$  shows the real demand (aggregate demand) shocks and  $\varepsilon_3$  shows the nominal demand (inflation) shocks in the  $[\varepsilon_1, \varepsilon_2, \varepsilon_3]$  vector. In this system of equations, the SVAR model is estimated by imposing the long-term restriction  $H_{ij}(L)=0$  for  $i = 1,2,3$ .

Table-2. SVAR Model Forecast Results

	$\Delta u_t$	$\Delta y_t$	$\pi_t$
$\Delta u_{t-1}$	-0,181 (0,144)	-0,025 (0,086)	-0,025 (0,227)
$\Delta y_{t-1}$	-0,085 (0,128)	0,806 (0,076)	1,013 (0,201)
$\pi_{t-1}$	-0,001 (0,012)	0,015 (0,007)	0,759 (0,019)
RSS = 0,00805			
Log Likelihood =0,457			
Akiake Criteria = -17,60			

Impulse response functions obtained from the SVAR model produce results that are consistent with the economic theory as seen Figure 12 below. For instance, production gives a serious negative response to a positive shock in unemployment and this effect has a lasting influence in ten quarters. On the other hand, production shows

gradually negative response to a positive shock in inflation from the begging of the second quarter and this effect has a lasting influence in ten quarters. The output gap values calculated according to the potential output estimations obtained from the above model can be seen [Figure 13](#):



Except for negative values for 2009, the output gap values obtained in this model are often positive at varying scales. If we look at the periodic developments in Turkish economy that we shortly discussed in the first part of the study and consider the statistical properties of potential output, we can conclude that the SVAR model produces more realistic output gap estimations than other models.

#### 4. Evaluation of Output Gap Predictions

In this section, we will compare the models which we have used up to now for predicted output gap. We will also make a general and empirical evaluation of the following models: linear trend models, the Hodrick-Prescott filtering, multivariate structural equation and three-variable SVAR models. Figure 14 below shows the potential output estimates obtained from the models, and Figure 16 shows the output gap values defined as the difference between the actual and the potential output values.

Figure-14. Potential Output Estimates

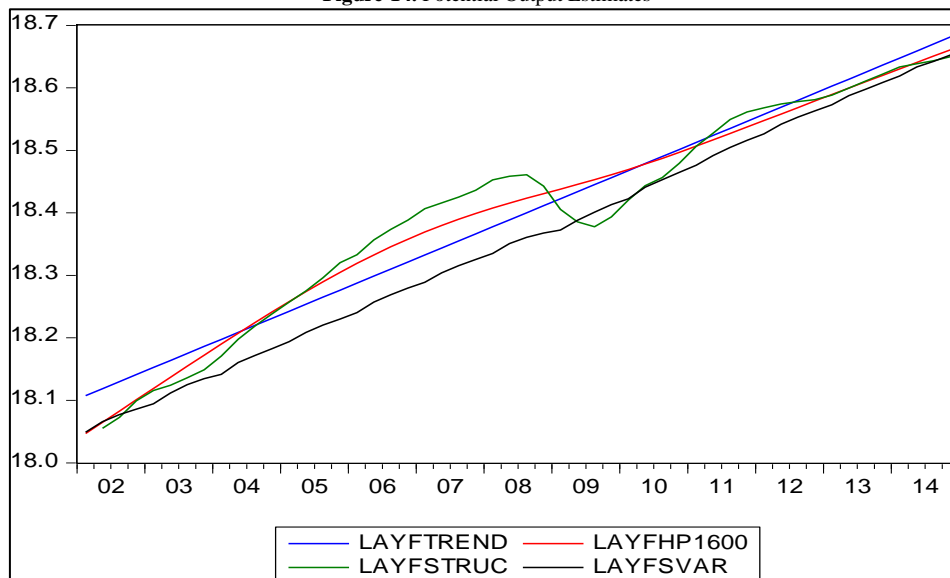
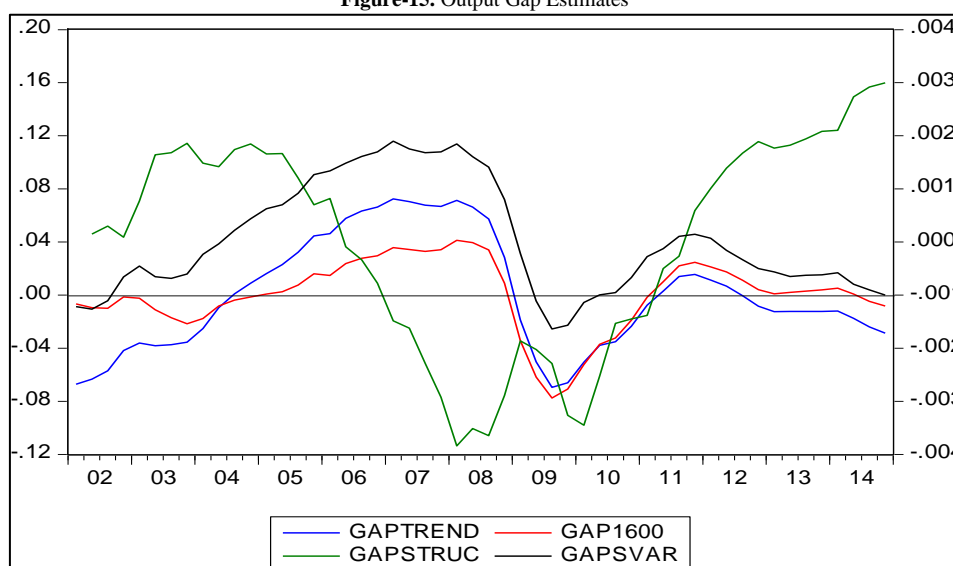


Figure-15. Output Gap Estimates



The first issue in output gap estimations is that the estimates for each quarterly period are estimated within a certain range, which is quite large from time to time. This naturally results in contradictory outcomes on the idle capacity in the economy. These contradictions are more crucial when the estimation result of multivariate structural model is compared with the results of other models. For instance, all the models used in this study produce negative output gap estimates during 2008 global financial crisis. However, the multivariate structural model starts negative values in the last quarter of 2006 while other models predict the starting time of negative gap values as the last quarter of 2008. In other words, during the period of 2007-2008, the multivariate structural model shows negative output gap values, while linear trend, HP filtering and SVAR models show positive output gap values. The same situation applies to the last estimation period. While the three techniques listed above show a negative output gap at the end of 2014, the multivariate structural model now gives a positive output gap value. It is possible to increase the number of such contradictory periods. However, it is also necessary to emphasize a common characteristic of estimated output gap values. Accordingly, the output gap estimates present similar characteristics. This can be observed in Table 3, which shows the correlation coefficients among the respective output gap values.

Table-3. Output Gap Correlation Table

	Gap Trend	Gap HP	Gap Structural	Gap SVAR
Gap Trend	1,000 --			
Gap HP	0,852 (0,000)	1,000 --		
Gap Structural	-0,239 (0,092)	0,056 (0,696)	1,000 --	
Gap SVAR	0,976 (0,000)	0,800 (0,000)	-0,284 (0,043)	1,000 --

According to the table, the other models except the multivariate structural model have valid correlations as statistical values ranging from 0.98 to 0.80. This shows that the output gap values obtained from the linear trend, HP filtering and SVAR models are substantially similar. The correlation between the output gap values obtained from the multivariate structural model and the values obtained with other methods is often inversed and weak. On the other hand, inverse correlation coefficients according to the standard error values in parentheses below the correlation coefficients are not statistically valid.

Besides the drawbacks of various output gap models mentioned above, the estimation results obtained vary according to the estimation method used. This makes it difficult for us to assess the usefulness of estimation methods, since their outcomes sometimes implicate serious contradictions with each other. Hence, an empirical choice criterion is necessary to determine the usefulness of such prediction models. Unfortunately, there is no empirical criterion developed in the context of output gap in the literature. For this reason, we will also prefer to take advantage of the economic theory using the generally preferred method. Theoretically, the output gap has a significant contribution in explaining the inflation in the country. Positive output gap values mark the periods when inflationary pressures increase, and negative output gap values should be regarded as periods when inflationary pressures are alleviated. For this reason, the output gap values that we have estimated using various methods so far can be evaluated according to their strengths or contributions to explain this theoretical situation. The inflation model which we use for this purpose is a basic mark-up inflation model. The following "gap" variant of this model, given in the basic form below, requires the use of output gap values which we have previously estimated through various methods:

$$\Delta p_t = \alpha_0 - \alpha_1 P_{t-1} + \alpha_2 ulc_{t-1} + \alpha_3 ip_{t-1} + \alpha_4 \Delta ulc_t + \alpha_5 gap_{t-i} + \xi_t$$

In this equation,  $p$  shows consumer price index;  $ulc$  is the index value of unit labor cost,  $ip$  is the import price index,  $gap$  is the output gap value and  $\xi$  is the error term. The lower case, notation as before, indicates that the corresponding variable is logarithmically included in the model. Note that, in the mark-up inflation model expressed in the error correction form, the lag structure of  $p$ ,  $ulc$  and  $ip$  variables are definite but there is no information about the lag structure in the gap variable. The lag structure of this variable will be determined separately for each gap variable using the Akaike Information Criteria. In order to see the contribution of gap variable to the model, a model in which output gap is not included and another model in which realized economic growth rates are included instead of output gap growth rates will be estimated. Thus, the performance comparison can be made. The estimation results are summarized in Table 4.

Table-4. Mark-Up Inflation Models and Impacts of the Output Gap

Coefficient	No Output Gap	Realized Growth Model	Linear Trend	HP Filter	Structural Equation	SVAR
$\alpha_0$	4,86 (0,77)	4,85 (0,73)	3,60 (1,20)	3,54 (0,96)	6,23 (0,78)	3,34 (1,16)
$-\alpha_1$	-1,07 (0,19)	-1,04 (0,18)	-0,95 (0,21)	-1,18 (0,18)	-1,11 (0,25)	-0,94 (0,20)
$\alpha_2$	0,16 (0,03)	0,15 (0,04)	0,12 (0,05)	0,22 (0,04)	0,16 (0,04)	0,10 (0,05)
$\alpha_3$	0,16 (0,12)	0,17 (0,12)	0,05 (0,20)	0,50 (0,17)	0,02 (0,12)	0,11 (0,20)
$\alpha_4$	-0,57 (0,22)	-0,69 (0,22)	-0,50 (0,23)	-0,43 (0,21)	-0,10 (0,21)	-0,44 (0,23)
$\alpha_5$	--	0,49 (0,20)	0,56 (0,41)	1,35 (0,51)	1,79 (0,68)	0,76 (0,43)
Adj R <sup>2</sup>	0,34	0,68	0,75	0,78	0,77	0,89
Q(12)	0,01	0,51	0,38	0,43	0,67	0,73

Note: Adj R<sup>2</sup> is the adjusted coefficient of determination, Q (12) shows the level of significance of the Box-Pierce test which proves that the autocorrelation function is zero. The values in parentheses are the standard errors of the corresponding estimations.

In the analysis of the table, the first issue to remark is that the inflation model without output gap variable lacks the explanatory power. This situation is a natural result of an inflation process which is mostly in the direction of decline during the examination period and it indicates the existence of a permanent inflation problem. However, if a measure of output gap is included in the inflation model, the explanatory power of the model rises significantly. Note that by incorporating such a measure to the model, the problem of serial correlation in residuals no longer exists. On the other hand, regardless of the estimation method, all models produce better results than the realized growth rates model. This indicates the fact that the output gap is an important variable that must be included in the inflation equation. In this context, output gap estimates obtained with SVAR model describes inflation most adequately.

Regarding the slope coefficients and the constant terms of equations, it can be said that whichever way the output gap is estimated, the changes observed over time in the output gap can seriously help to determine the inflation. We can explain the reason of this in two different ways. First, the inflationary effect caused by output gap in the empirical inflation models is determined by the average value of the output gap, not by whether the output gap is zero or not. This is similar to what we have done in this study. Estimated output gap values have different average

values. So, the constant terms in each estimated inflation equation has different levels. Since the constant term obtained through which the output gap series obtained by using multivariate structural model is highly different from the other models constant terms, output gap values produced by this method can be regarded as unreliable estimates. Second, in the empirical inflation models we have dealt with in this study, the effect of different business cycle phases on inflation in the estimation of the output gap is balanced by the acquisition of different slope coefficients. For example, the output gap estimate which produces a larger cycle, creates a smaller output gap coefficient in inflation equation. For this reason, the predictive power of the equations are close even if the predicted parameters change considerably.

Given that the aggregate demand is one of the fundamental sources of the changes in inflation, higher value of forecast errors in the no-output gap equation indicates that inclusion of any gap measure to the inflation equation is expected to reduce the forecast errors. Table 5 shows the out of sample forecasting performance of the various methods used in this study for the last two years. The performance evaluation is made by using three basic criterias: RMSE (Root Mean Squared Error), MAE (Mean Absolute Error) and Theil Inequality Coefficient (Theil's U).

Table-5. Out of Sample Estimation Errors

Criteria	No Output Gap	Growth Model	Linear Trend	HP Filter	Structural Equation	SVAR
RMSE	0,066	0,062	0,065	0,041	0,052	0,024
MAE	0,051	0,049	0,049	0,029	0,043	0,028
THEIL U	0,204	0,190	0,199	0,189	0,193	0,097
Biasness	0,000	0,000	0,000	0,000	0,000	0,000
Variance	0,100	0,086	0,095	0,084	0,123	0,002
Covariance	0,900	0,914	0,905	0,916	0,877	0,998

According to the table, SVAR model shows the best performance in estimating the inflation. Two models with lowest RMSE values are SVAR and HP filtering techniques. The MAE value which indicates the deviation of the predictions from the average in absolute terms is again the minimum for these two techniques. When Theil inequality coefficient is taken as the basis, all models except SVAR model give approximate results, whereas SVAR model gives the lowest value. On the other hand, when we look at the distribution of this error, it is understood that the error percentage due to bias and variance is very close to zero, and that 99,8% of the errors are caused by covariance. This represents a distribution very close to the ideal in the distribution of prediction errors. According to the table, the most effective output gap values in describing and forecasting inflation are the output gap values obtained from the SVAR model and HP filtering technique.

## 5. Conclusion

According to the output gap estimates, the estimates for each quarterly period were made within a certain range. However, from time to time this range has expanded considerably. This situation naturally leads to contradictory conclusions about the idle capacity in the economy. These contradictions increase the importance of the estimation of the multivariate structural model that we have realized by theoretical relations and the fact that the output gap values produced by the other three methods are very different from each other. The output gap estimates present similar characteristics. Regarding the slope coefficients of the output gap estimates and the constant terms of the equations, it can be said that whichever output gap estimate is considered, the changes observed over time in output gap can seriously help in determining the inflation. We can explain the reason for this from two perspectives. First, the inflationary effect caused by output gap in the empirical inflation models is determined by the average value of the output gap, not by whether the output gap is zero or not. This is similar to what we have done in this study. Estimated output gap values have different average values. So, the constant terms in each estimated inflation equation has different levels. Since the constant term obtained through which the output gap series obtained by using multivariate structural model is highly different from the other models, output gap values produced by this method can be regarded as unreliable estimates. Second, in the empirical inflation models we have dealt with in this study, the effect of different conjuncture phases on inflation in the estimation of the output gap is balanced by the acquisition of different slope coefficients. For example, the output gap estimate which produces a larger cycle, creates a smaller output gap coefficient in inflation equation. For this reason, the predictive power of the equations are close even if the predicted parameters change considerably. According to the results obtained by diversifying the mark-up inflation model, the SVAR model was chosen as the best model for comparing the forecast results and guiding the inflation of the output gap calculated by different methods. The HP filter is the second best model. These results were also supported by the measurement of out-of-sample forecast errors.

## References

- Bjørnland, H. C., Brubakk, L. and ve Jore, A. S. (2006). Forecasting inflation with an uncertain output gap, Norges bank working paper series, no. 2006/2.
- Blanchard, O. J. and Quah, D. (1989). The dynamic effect of aggregate demand and supply disturbances. *American Economic Review*, 79(4): 655–73.

- Brouwer, G. D. and Ericsson, N. (1995). Modelling inflation in Australia, Reserve bank of australia research discussion paper no. 9510 (in Turkish).
- Çiçek, S. (2009). *Küreselleşme ve enflasyon: Küresel çıktı açığı hipotezi - türkiye örneği*. PhD Dissertation, İstanbul: Yıldız Teknik Üniversitesi.
- Debelle, G. and Vickery, J. (1997). *Is the phillips curve a curve? Some evidence and implications for Australia*. Reserve Bank of Australia Research Paper No. 9706.
- Diebold, F. X. and Senhadji, A. S. (1996). The uncertain unit root on GNP: Comment. *American Economic Review*, 86(5): 1291–98.
- Guay, A. and St. Amant, P. (1996). Do mechanical filters provide a good approximation of business cycles?, bank of canada technical report no. 78.
- Hodrick, R. and Prescott, E. (1997). Post-War US Business Cycles: An Empirical Investigation. *Journal of Money, Credit and Banking*, 29(1): 1–16.
- Horn, G. A., Logeay, C. and Tober, S. (2007). Methodological issues of medium-term macroeconomic projections – the case of potential output, final report research report prepared for the federal ministry of economics and technology, (no. 4).
- Laxton, D. and Tetlow, R. (1992). A simple multivariate filter for the measurement of potential output, bank of canada technical report no. 59.
- Yavan, Z. A. and ve Türker Kaya, Y. (2007). Üretim fonksiyonu yaklaşımına vurguyla potansiyel çıktı açığı tahmin etme yöntemleri ve yapısal İşsizlik ögesi: Literatür değerlendirmesi ve türkiye örneği, tüslad- koç üni. Ekonomik araştırma forumu Konferansı. Available: <http://eaf.ku.edu.tr/sites/eaf.ku.edu.tr/files/zay.pdf>