



Analysis Method of Enterprise Network Relationship Based on Graph Theory

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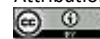
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Abstract

Network relationship is positively correlated with enterprise development and performance, which is an important basis for studying enterprise development and judging enterprise development situation. In this paper, complex network theory and methods are used to study enterprise network relationships. Based on graph theory, a graph model of enterprise network relationships is constructed, and the Laplacian matrix of the graph model is used to analyze important indicators of network relationships, relationship strength, stability, reciprocity, centrality and other indicators of numerical analysis methods.

Keywords: Graph theory; Network relationship; Corporate development; Graph model; Complex systems.

1. Introduction

An enterprise does not exist in isolation in its operation and development process. The enterprise and other market entities form a network of relationships. All market activities of the enterprise occur in the enterprise network. Network relationships can show that the enterprise is in production and operation activities, and its relationship with the outside of company between capital, technology, and business is an important factor that affects business operations, and it is also a key research content in management and economics.

At present, there are many researches on network relationship including the relationship between network relationship and enterprise performance, competitiveness and innovation ability. Conventional enterprise network relations network analysis method mainly through the questionnaire survey offer the decision basis to improve their own performance to obtain relevant information and data for data analysis, surveying computation, research network relationship of strength, reciprocity, stability and centrality index and enterprise performance, such as the link between the growth and weight of the corresponding for the enterprise cluster and the industrial development. Most of the enterprise network relations research mainly concentrated in the link between the network relationship and enterprise development. The study of quantitative analysis of enterprise network relations, the same type enterprises, enterprise network relations between the different stages of development has less involved, and no network related numerical analysis method is put forward.

The overall network relationship of an enterprise is constituted by the sub-network relationship among the subjects. The network relationship of an enterprise includes the relationship between itself and other enterprises as well as the network relationship between other enterprises. Therefore, the network of enterprise relations is a complex system, which needs to be studied from the perspective and method of complex system.

This paper introduces the theory of complex systems, transforms the study of enterprise network relationships into the study of complex network relationships, and proposes a graph theory-based enterprise network modeling method. According to the Laplacian matrix of the enterprise network relationship graph model, proposed an analysis methods based on graph model for network scale, relationship strength, stability, reciprocity, centrality and other indicators.

2. The Relationship Between Network Relations and Corporate Development

In the process of enterprise development, the enterprise is not an isolated individual, and its development is closely related to its customers, suppliers, partners, service providers, competitors, etc., forming a complex network of relations among all parties, that is, enterprise network relations. Enterprise development is closely related to the complex network in which it is located. Many scholars at home and abroad have conducted many researches on the relationship between network relationship and enterprise performance, competitiveness and innovation ability, and obtained many valuable results.

2.1. Correlation Between Network Relationship and Enterprise Performance

The relationship between network relationship and enterprise performance is a key research area of enterprise network relationship. The "weak relationship" school represented by Granovetter believes that weak relationship can provide diversified and unique information and knowledge, which is conducive to enterprise innovation and performance improvement (Granovetter, 1973). The "strong relationship" school represented by Coleman believes that strong relationship can bring emotional resources such as commitment and trust to enterprises, which is conducive to accelerating the flow and sharing of information and knowledge among enterprises, thus promoting enterprise cooperation and performance improvement (Coleman and James, 1988).

In addition, many scholars also focus on the research on the deep relationship between the strength of network relationship and enterprise performance. For example, Yang et al. studied the influence of resource acquisition behavior on enterprise performance in the strength of network relationship (Yang *et al.*, 2009). Peng and Fu study the influence of resource integration on enterprise performance in the strength of network relationship (Peng and Fu, 2013). Wang wu et al. studied the relationship between relationship strength and enterprise performance from the perspective of competition and cooperation among enterprises (Wang and Wu, 2019).

Relationship strength on behalf of the network between enterprises of the interactive relationship between frequency and duration, and trust, so most of the research results show that the entrenched in our country under the background of human society and the credit system, as a developing country under the condition of contract system is not perfect. The cooperation and interaction between enterprises is more need to promote and indication of strong relationship. Strong network can bring rich emotional resources for enterprises, promote mutual trust and commitment to one another between the enterprises, which is beneficial to complex knowledge and tacit knowledge transfer and sharing, and good for business and technical related cooperation between enterprises to keep the long and stable business cooperation relationship and improve enterprise performance.

2.2. Research on the Relationship Between Network Relationship and Enterprise Competitiveness

Enterprise competitiveness refers to the ability of an enterprise to design, produce and sell products or export labor services, that is, the price and non-price quality and performance of its products and labor services are more attractive than other competitors, which is the basis for the survival of the enterprise and reflects the vitality of the enterprise.

The research on the relationship between network relations and enterprise competitiveness is also one of the important contents of the research on enterprise network relations. For example, XIE et al. focused on studying the competitive strategy of enterprises and how enterprise clusters affect enterprise relations so as to promote the improvement of enterprise competitiveness (Xie and Lan, 2004; Xie and Liu, 2005; Xie and Liu, 2007).

Various studies have shown that the relationship strength, density, reciprocity and other three factors between enterprises have a positive impact on the development of enterprises. A good enterprise relationship network can consolidate and enhance the exchange and cooperation between enterprises and other enterprises, partners, intermediary agencies, financial institutions, research and development institutions, and ultimately promote the development of enterprises and technology and enhance the competitiveness of enterprises.

2.3. The Relationship Between Network Relationship and Enterprise Innovation Ability and Efficiency

With the increasing complexity of innovation activities, corporate innovation has become a multi-party interactive corporate behavior. Many enterprises' innovative behaviors are generated through the interaction between customers, suppliers, partners and other entities. Corporate innovation depends on the network environment, the type and number of contacts between the enterprise and other organizations. the good network relationship to form a combination of product chain and industry chain can improve innovation performance and environment.

There are many researches on the relationship between the characteristics of innovation networks and the innovation ability of enterprises. For example, He et al. studied the influence of network characteristics and attributes on the innovation capabilities of SMEs and put forward suggestions for SMEs to improve their innovation capabilities through network relationship construction (He *et al.*, 2005). Helper's research considers that corporate innovation networks are positively related to corporate innovation capabilities as a whole (Helper, 1995). Hsu proposed that the strong connection of the network has a positive impact on technological innovation capabilities and studied the relationship between network relationships and technological innovation capabilities in terms of relationship strength (Hsu, 1997). Oliver has studied the impact of network reciprocity on innovation ability and performance, and believes that on the basis of reciprocity, members of network organizations are willing to work for common interests and goals, and then jointly improve innovation performance (Oliver, 1990). Ronald studied the relationship between the diversity of network members, information, and resources to enhance innovation capabilities (Ronald, 1992). Smmie also believes that geographic clusters will increase the ability to innovate (Simmie). The research by Li et al, shows that centrality, intensity, and level of reciprocity have a positive impact on some variables of corporate innovation capabilities, which are the fundamental factors affecting innovation performance (Li *et al.*, 2007).

From the above research, it can be concluded that there are significant positive correlations between the five network structure variables such as network density, reciprocity, centrality, stability, and resource abundance in the enterprise network relationship and the innovation performance of the enterprise. Centrality, as a measure of the

position of an enterprise in the embedded network, has the largest correlation coefficient with innovation performance.

2.4. The Relationship Between Network Relations and Corporate Growth

The growth of an enterprise is related to performance, innovation ability, competitive relationship and correlation, and also related to enterprise technology and market direction, and decision-making ability. It is a synthesis of various enterprise development factors and the key to judging whether an enterprise is worth investing. Enterprise growth has become the focus of attention in the field of venture capital.

In the study of corporate network relationships, the relationship between network relationships and corporate growth is also an important focus for many researchers on the study of complex network relationships. For example: Christiana, Plummer, Leyden, Xiao, Yin and other scholars have studied the factors and correlations of network relationships and corporate growth (Leyden *et al.*, 2014; Plummer *et al.*, 2016; Weber and Kratzer, 2013; Xiao *et al.*, 2018; Yin *et al.*, 2019). Most of the research results believe that a good network is conducive to knowledge transfer, information and resource exchange between the two partners, and it has a positive impact on the growth of the company.

To sum up, network scale, network heterogeneity, relationship strength, and relationship quality all have a positive impact on corporate growth, promote corporate growth and performance, and promote technological innovation and competitiveness.

3. Graph Theory and Network Relationships

3.1. The Concept of Graph Theory

Graph Theory is a branch of mathematics. It takes pictures as research objects. A graph in graph theory is a figure composed of a number of given points and lines connecting two points. This kind of figure is usually used to describe a specific relationship between certain things. Points are used to represent things and two points are connected. The line indicates that there is such a relationship between the corresponding two things.

Graph theory itself is part of applied mathematics. The written record of graph theory first appeared in Euler's treatise in 1736. The original question he considered had a strong practical background. The widespread application of graph theory has promoted its own development. In the 1940s and 1960s, matroid theory, hypergraph theory, pole graph theory, as well as algebraic graph theory, topological graph theory, etc., have made great progress.

This paper uses $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ to represent a simple undirected graph (that is, an undirected graph without multiple edges and self-loops). Where $\mathcal{V} = (\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_n)$, which represents the set of all nodes in graph G , $J = \{1, 2, \dots, n\}$ is the label set of each point; v_i represents the i -th node in the graph; $\mathbb{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of all edges between nodes in the graph, and the connected edges can be expressed as $e_{ij} = (\mathcal{E}_i, \mathcal{E}_j)$; $\mathcal{A} = [a_{ij}]$ is a weighted adjacency matrix, and the elements in the matrix are defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} > 0$. When point \mathcal{E}_i has a continuous edge to point \mathcal{E}_j , and both point i and point j can communicate with each other in the agent system, then $a_{ij} > 0$. At this point, point j is called the neighbor of point i , which is expressed as $N_i, N_i = \{j \in \mathcal{V} : (i, j) \in \mathbb{E}, j \neq i\}$. The out degree matrix of graph \mathcal{G} is defined as $D = \text{diag}\{d_1, d_2, \dots, d_n\} \in \mathbb{R}^{n \times n}$, where $d_i = \sum_{j \in N_i} a_{ij}$ and $i = \{1, 2, \dots, n\}$ is the diagonal element of the matrix. Then the Laplacian matrix of graph \mathcal{G} is defined as $L = D - A$.

3.2. The Relationship Between Graph Theory and Complex Systems

A complex system is a system with a moderate number of intelligent, adaptive subjects that make actions based on local information. In the 1980s abroad, the concept of complex science was proposed. Cheng, a well-known scholar in China, made an in-depth analysis of the basic methods and tools of complex systems and their application prospects (Cheng, 1999). At this stage, complex system theory and statistical physics, applied mathematics, nonlinear science, and theoretical physics have become emerging research hotspots to study the development of physical laws in the financial market. It is called financial physics, and research on complex systems has spread in many aspects of finance. The feasibility and effectiveness of this method have been recognized by many researchers. Applications of complex system theory in management, economics, finance and other fields include: the study of financial market based on complex system; the study of financial crisis transmission theory based on complex system theory; the study of stock price volatility and risk contagion based on complex system theory; and the application of complex system theory in the analysis of enterprise network relations.

The complex network theory developed rapidly in the past decades has provided an important support point for the study of complexity and the science of complex systems. It highly summarizes the important characteristics of complex system. It has strong vitality both in theory and in applications. Modeling complex systems into complex networks is a common method for complex system research (Wang *et al.*, 2008), and complex network modeling problems can be attributed to the establishment of various graphs (Duan, 2008) based on graph theory to study complex network problems. Graph theory is an important research and modeling method in the study of the relationship between complex systems and complex networks (Li Jinhua, 2009). It can simplify various complex systems and complex networks into nodes and sets of edges connecting nodes, which can help understand basic mechanisms for the evolution and function formation of complex systems and complex networks. For example, Han abstracted complex systems into a directed graph described by triples to study the fault diagnosis of complex systems (Han *et al.*, 2005).

Using graph theory and complex network relationships can effectively carry out analysis and research on enterprise network relationships.

4. The Graph Model of Network Relationships

The establishment of graph models is the basis of network relationship analysis. According to the relevant knowledge of graph theory and the actual situation of enterprise network relationships, the establishment of enterprise network relationship graph models includes identifying the correct network relationships and the establishment of network relationship graph models.

4.1. Identify Valuable Network Relationships

Network relationship risk identification and network relationship analysis conclusions can be used as an important input for optional risk control. At the same time that venture capital institutions conduct due diligence or conduct value assessments and risk index evaluations, they also increase the dimension of network relationship risks to study the development of enterprises. To assist venture capital institutions to better carry out risk control work.

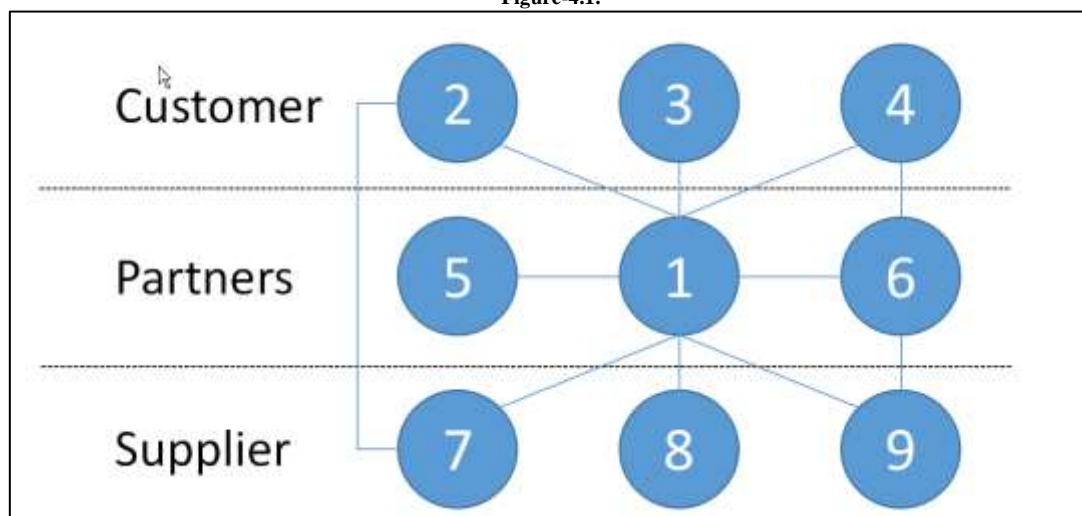
The confirmation of network relation is the first step and the key step of constructing Laplace matrix of enterprise network relation. In enterprise network relations, the majority of enterprises in the network scale present phenomenon of excessive, particularly with some partners in many invalid relationships, a large number of invalid relationships is difficult to bring direct or indirect economic benefits for the enterprise or technology and the improvement of efficiency. If such invalid network included in the scope of network analysis, which affect the network analysis. At the same time, if the focus on the relationship between the revenue and expenditure situation of both sides to analyze effective or ineffective relationships, it cause deviation. Such as some relationships between the industrial internet platform enterprises and some old customers, there is zero revenue situation. The relationship about the equipment data docking of two sides, It does not have the revenue or expenditure contacts, but still should be considered the effective relationship. The cooperation about technology research and development and marketing of enterprises, also be deemed to be valid relationship. Therefore, the emphasis should be placed on the confirmation of effective network relations in the analysis of network relations. Only by realizing the effective identification of network relations, we can construct the Laplace matrix that truly reflect the network relations of target enterprises, and carry out the research on network scale, strength, centrality and other aspects.

4.2. Graph Model of Network Relations

The concept of graph theory is used to describe the network relationship structure between enterprises, and the enterprise is regarded as the point in the graph, and the relationship between enterprises is regarded as the edge between the points.

Assume that the objective company has nine effective network relationships: customers, suppliers, and partners. At the same time, there are relationships between its customers and suppliers or partners. Three customers (Enterprise 2, Enterprise 3, Enterprise 4), two horizontally related enterprises, such as cooperative enterprises, affiliated enterprises (Enterprise 5, Enterprise 6), and three suppliers (Enterprise 7, Enterprise 8, Enterprise 9). At the same time, the customer enterprise 4, the supplier enterprise 9, and the partner enterprise 6 are also connected, and the customer enterprise 2 and the supplier enterprise 7 are connected. At this time, we can construct the network relationship of enterprise 1 into a topology diagram as shown in FIG 3-1

Figure-4.1.



In Figure 3-1, each dot represents an enterprise, and the line between dots and dots represents the relationship between the enterprises. The value of the relationship between the enterprises is defined as, where i is the serial num of the objective enterprise, and j is The serial num of its associated company. a_{ij} represents the value of the relationship between the i -th enterprise and the j -th enterprise. And we define the set of enterprises that can have a

network relationship with enterprise i as the adjacent point set of enterprise i , defined as N . At this time, if there is no relationship between the two enterprises, their relationship value is $a_{ij} = 0$, and we also define $a_{ii} = 0$.

According to graph theory knowledge, the enterprise network relationship in Figure 3-1 can be represented by the following matrix:

$$\begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} \\ a_{21} & 0 & 0 & 0 & 0 & 0 & a_{27} & 0 & 0 \\ a_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & 0 & 0 & 0 & 0 & a_{46} & 0 & 0 & 0 \\ a_{51} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{61} & 0 & 0 & a_{64} & 0 & 0 & 0 & 0 & a_{69} \\ a_{71} & a_{72} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{81} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{91} & 0 & 0 & 0 & 0 & a_{96} & 0 & 0 & 0 \end{bmatrix} \quad (3-1)$$

It can be seen that the matrix (3-1) is the adjacency matrix of Fig. 3-1 and is defined as A . At the same time, D is defined as the in-and-out degree matrix in Figure 3-1, which is a diagonal matrix, where is the number of in-and-out degrees of point i , and the Laplacian matrix $L = D-A$, that is:

$$\begin{bmatrix} \sum_{j \in N} a_{1j} & -a_{12} & -a_{13} & -a_{14} & -a_{15} & -a_{16} & -a_{17} & -a_{18} & -a_{19} \\ -a_{21} & \sum_{j \in N} a_{2j} & 0 & 0 & 0 & 0 & -a_{27} & 0 & 0 \\ -a_{31} & 0 & \sum_{j \in N} a_{3j} & 0 & 0 & 0 & 0 & 0 & 0 \\ -a_{41} & 0 & 0 & \sum_{j \in N} a_{4j} & 0 & -a_{46} & 0 & 0 & 0 \\ -a_{51} & 0 & 0 & 0 & \sum_{j \in N} a_{5j} & 0 & 0 & 0 & 0 \\ -a_{61} & 0 & 0 & -a_{64} & 0 & \sum_{j \in N} a_{6j} & 0 & 0 & -a_{69} \\ -a_{71} & -a_{72} & 0 & 0 & 0 & 0 & \sum_{j \in N} a_{7j} & 0 & 0 \\ -a_{81} & 0 & 0 & 0 & 0 & 0 & 0 & \sum_{j \in N} a_{8j} & 0 \\ a_{91} & 0 & 0 & 0 & 0 & -a_{96} & 0 & 0 & \sum_{j \in N} a_{9j} \end{bmatrix} \quad (3-2)$$

At this time, we have transformed the enterprise relationship described in Figure 3-1 into a Laplacian matrix and can use the graph theory to describe it, laying the foundation for the quantitative research of enterprise relationship networks in the next stage.

5. Analysis of Network Relationships Based on Graph Models

5.1. Network Scale

The analysis dimensions of network scale include the scale and changes in network relationships.

1. Scale. Represents the number of partners, customers, suppliers, etc. that the company contacts. This indicator can be intuitively represented by the number of points on the relationship graph, and is finally expressed as the dimension of the Laplacian matrix of the relational network. The higher the matrix dimension, the larger the enterprise's network scale.
2. Changes in the participants in the corporate relationship network. The change of participants cannot be directly analyzed according to the Laplacian matrix of the relationship network, so before the relationship modeling process, the participants of the latest graph model and the participants of the previous modeling need to be compared to determine the network relationship Changes in each subject.

5.2. Network Relationship Value

5.2.1. Network Relationship Value Analysis

The network relationship value (a_{ij}) consists of three index parameters, including: relationship strength, reciprocity, and stability. The sum of the three index values is both the network relationship value in the network relationship matrix. The strength of the relationship indicates the closeness of the relationship between the companies in the network, including the closeness of the relationship and the degree of trust between the two parties.

Reciprocity means that in the network, the activities of both parties can bring benefits to each party or the improvement of technical and management capabilities. Stability, which means that companies can maintain stable relationships, including business relationships, technology and business cooperation. In the analysis of specific relationship values, we define $a_{ij} = c_1k_1 + c_2k_2 + c_3k_3$, which k_1 , k_2 , k_3 respectively represent three parameter values of relationship strength, reciprocity, and stability, and the specific values of k_1 , k_2 , k_3 are analyzed and evaluated by the assessor according to the enterprise situation and related methods. c_1 , c_2 , c_3 are the corresponding weight parameters, and $c_1 + c_2 + c_3 = 1$. In the actual analysis process, the value of c_1 , c_2 , c_3 can be adjusted according to the industry of the analyzed enterprise and the position of the industrial chain.

5.2.2. Relationship Strength Analysis

The strength of the network relationship (k_1) is mainly analyzed and measured based on two inputs. One is the number of projects in which the two parties in the network cooperate or have business relationships. The second is the frequency of interaction between the two sides of the network. In a network relationship, a unified scoring standard can be set according to the number of interaction items and the frequency of interaction between the two sides of the relationship, which can quantitatively score the relationship strength. The greater the number of related projects and the higher the frequency of interaction, the stronger the relationship.

5.2.3. Stability Analysis

The analysis and measurement inputs for stability (k_3) include two elements, one is the duration of the relationship between the two parties. The second is interdependence, such as whether there is a business, data or technology dependency relationship between the enterprises, and the strength of the dependency relationship. Based on these two data, the stability of the relationship network can be quantitatively analyzed and scored. The longer the duration of the relationship between the two parties, the higher the dependence on business, data or technology, the better the stability of the dual-issue network relationship.

5.2.4. Reciprocity Analysis

Reciprocity (k_2) is mainly the evaluation of the two sides of the network relationship in terms of income, resources, customers, technology and other capabilities, based on the characteristics of reciprocity, construct a scoring method to study the reciprocity of the two parties. Let the reciprocity score include a comprehensive score and a supplementary score on revenue: $k_2 = \hat{k}_2 + \bar{k}_2$, \hat{k}_2 is a comprehensive score and a supplementary score on revenue. Comprehensive scoring based indicators include: data interaction, technology and product development, business and project cooperation, business promotion, revenue and expenditure, and others. The supplementary score of the revenue situation is calculated by the formula: $\bar{k}_2 = \beta Ct^{-1}$, where C is the total revenue obtained by the company through this network relationship in the year under analysis, t is time, β is the weight, and the analyst considers that the revenue relationship accounts for the reciprocity score according to the actual setting. The higher the specific gravity, the β larger. At this time, an analysis and calculation method of the network relationship value between the two parties of the industrial Internet platform is formed. Accumulating grading experience cases based on this method can improve the accuracy of scoring.

5.3. Overall Analysis and Evaluation of the Values of Network Relationship

Based on the numerical analysis of relationship strength, stability, and reciprocity, the network relationship value of a company in a specific network relationship can be calculated. Based on this, it is proposed to use the average value of the characteristic value of the network relationship matrix to analyze and evaluate the company's Relationship strength, stability, reciprocity.

Since the research object is the network relationship value of the target company, so the influence of network relationships between other companies should be ruled out before the study. The general form of the corporate network relationship Laplacian matrix is

$$\begin{bmatrix} \sum_{j \in N} a_{1j} & -a_{12} & -a_{13} & \cdots & -a_{1n} \\ -a_{21} & \sum_{j \in N} a_{2j} & -a_{23} & \cdots & -a_{2n} \\ -a_{31} & -a_{32} & \sum_{j \in N} a_{3j} & \cdots & -a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & -a_{n3} & \cdots & \sum_{j \in N} a_{nj} \end{bmatrix} \quad (4-1)$$

Process the network relation Laplace matrix. Let when $i \neq 1$ and $j \neq 1$, all of $a_{ij} = 0$, then the matrix (4-1) becomes

$$\begin{bmatrix} \sum_{j \in N} a_{1j} & -a_{12} & -a_{13} & \cdots & -a_{1n} \\ -a_{21} & a_{21} & 0 & \cdots & 0 \\ -a_{31} & 0 & a_{31} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & 0 & 0 & \cdots & a_{n1} \end{bmatrix} \quad (4-2)$$

The matrix (4-2) is a relation value judgment matrix constructed for the target company. According to the matrix theory, the matrix (4-2) has n eigenvalues, and all eigenvalues have positive real parts. Therefore, the average of the eigenvalues of the matrix (4-2) is calculated, and the result is used as the entire network relationship matrix network relationship Quantitative description of the value. When the average value of the characteristic value is larger, the value of the network relationship of the enterprise is better, and the strength, stability, and reciprocity of the relationship between the target enterprise and each subject of the network are better.

5.4. Network Centrality

Centrality indicates the position of the enterprise in the entire network relationship. The stronger the centrality, the higher the irreplaceable status in the entire network. The parties in the network need to pass the target enterprise to make business contacts or realize business transactions. In network relationships, centrality is embodied by the irreplaceability of platform companies. How companies provide customers with unique and irreplaceable technical services and play a key role in network relationships is their core competitiveness. Therefore, centrality is an industry An important key indicator of Internet relations for Internet companies.

In the network relationship matrix, the degree of centrality can be reflected by the number and value of $a_{ij}, (i, j \neq 1)$ the network relationship values that are not related to the target enterprise in the network relationship Laplacian matrix. The more $a_{ij}, (i, j \neq 1)$, the more communication between the various parties, The greater $a_{ij}, (i, j \neq 1)$, the greater the closer the relationship between the two companies.

Carry on the central research of corporate network relations through the Laplacian matrix of corporate network relations. Because the centrality of the corporate network relationship is mainly to study the situation of direct relationships between other enterprises in the network relationship except the target enterprise, this article uses the central benchmark curve to analyze the centrality of the enterprise and determine the strength of the centrality.

Centrality mainly studies the network relationships of other enterprises except the target enterprise. At the same time, it can be known that under the ideal state of network relationship centrality, each enterprise in the network relationship only has a relationship with the target enterprise. At this time, there is no network relationship between the enterprises. The value of the network relationship between enterprises is 0, that is, all at the same time. At the same time, in order to reduce the interference of the change in the value of the target enterprise network relationship on the centrality analysis, according to the form of the corporate network relationship Laplacian matrix (matrix (4-1)), a centrality judgment matrix is constructed:

$$\begin{bmatrix} a_{21} & 0 & \cdots & 0 \\ 0 & \ddots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{n1} \end{bmatrix} \quad (4-3)$$

According to the knowledge of matrix theory, the matrix (4-3) is a diagonal matrix, and its eigenvalues are diagonal elements.

Therefore, we can place the eigenvalues of the centrality judgment matrix on the rectangular coordinate system. On the rectangular coordinate system, the ordinate is the value size of eigenvalue; the abscissa is the serial number of eigenvalue. Then connect the points to construct a central reference curve, which can be used as an ideal description of corporate centrality.

At the same time, we construct a matrix of other enterprise relations based on the matrix (4-1), which is used to describe the network relations of other companies in addition to the target enterprise. Then the other enterprises relation matrix can be written as

$$\begin{bmatrix} \sum_{j \in N} a_{2j} - a_{21} & -a_{23} & \cdots & -a_{2n} \\ -a_{32} & \sum_{j \in N} a_{3j} - a_{31} & \cdots & -a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n2} & -a_{n3} & \cdots & \sum_{j \in N} a_{nj} - a_{n1} \end{bmatrix} \quad (4-4)$$

The values of all elements of the matrix (4-4) are not related to the network relationship of the target enterprise, and all matrix characteristics are only related to the network relationship between other enterprises. According to the characteristics of the graph theory Laplacian matrix, matrix (4-4) has an eigenvalue of 0 and other eigenvalues have positive real parts. According to the method of constructing a central reference curve, place the real parts of the eigenvalues of the matrix (4-4) on the same rectangular coordinate system, and rank by the size. Thus tectonic the network relationships curves of other enterprises to describe the network relationship between other enterprises. The stronger the network relationship of other enterprises, the larger the eigenvalue, and the larger the slope of the corresponding network relationship curve.

If there is no network relationship for any company other than the target company, all elements in the matrix (4-4) are 0, and the corresponding curve of the eigenvalues is a straight line overlapping the X axis.

The analysis of the centrality of the network relationship can be performed according to the eigenvalue curves of the matrix (4-3) and the matrix (4-4). Since the dimensions of the matrix (4-3) and the matrix (4-4) are equal, the number of eigenvalues of the two matrices is the same. The eigenvalue curves of the two matrices are superimposed on the same coordinate axis, analysis the centrality, perform the estimation and identification of centrality

First analyze the ideal situation when other companies do not have a network relationship. At this time, the difference between the centrality baseline curve and each characteristic value of the other enterprises network relationship curve is a positive number. thus the difference is equal to the eigenvalues of the centrality judgment matrix (matrix (4-3)), it is the ideal state of the centrality of the network relationship. Then by comparing and analyzing the actual situation and the perfect situation, a neutral judgment conclusion can be drawn: When the centrality baseline curve deviates positively from other enterprise network relationship curves and the larger the deviation, the more central the target company is in its network relationship; Conversely, the smaller the deviation of the two curves, or even negative deviations from other enterprise network relationship curves, the worse the centrality. According to the above analysis and calculation steps, the centrality of the enterprises can be directly analyzed.

6. Conclusion

This paper uses the method of graph theory to model the network relationships of enterprises and construct the Laplacian matrix of network relationships according to the graph model. Use the matrix to analyze important indicators of network relationships, such as network scale, relationship strength, stability, reciprocity, centrality. Use the matrix dimension to determine the network scale; construct a judgment matrix for corporate relationship values, study and judge the strength, stability, and reciprocity of the corporate relationship; use the deviation of the central reference curve from other corporate network relationship curves to analyze centrality is strong or weak.

The conclusion of this paper provides an effective numerical analysis method for the research of enterprises network relationship, it can provide a new method for the analysis and research of enterprise network relationship in risk control, industry management and other fields.

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