



Mover-Stayer Analysis of Students' Academic Progress in Modibbo Adama University of Technology, Yola

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Abstract

This paper studies the pattern of students' movement within and around the various classes of degrees in Modibbo Adama University of Technology, Yola, Nigeria. In this paper, a transition matrix was developed for the five classes of degrees using movement patterns in ten consecutive semesters (2011 – 2016). The probabilities of moving across the five different classes were obtained. Furthermore, a fundamental matrix was obtained to determine the expected number of students who stay within each particular class (stayers).

Keywords: Mover-stayer; Polya-aeppli; Negative binomial; Classes of degree.



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1. Introduction

The importance of education in Nigeria cannot be overemphasized. This realization has led to changes from traditional form of formal education to be able to compete in the world market. Higher education is giving priority to prepare students for the challenges of the generality of the populace. It is important in every system of education, to evaluate the progress of students. Every higher education institution is a place where students stay in a given stage for one academic year and then move to the next stage or leaves the system as a graduate or drop out (Brezavscek *et al.*, 2017). In order to assess student's progress through the educational system, and to help managers of the educational institutions formulate workable education policy, since academic decision-making is a necessary part of university administration, the pattern of movement across various classes of degrees by students becomes relevant. Modibbo Adama University of Technology, Yola (MAUTECH), consists of seven Schools aside School of Post graduate studies (SPGS), School of General Studies and the Centre for Distance learning (CDL), which follow a regular semester system. A typical student takes 5 years to complete the required credit hours; students of School of Management and management Technology are exceptions because they take 4 years to complete the credit hours. The seven Schools are:

1. Physical Sciences
2. Life Sciences
3. Agric. and Agric. Technology
4. Engineering & Engineering Technology
5. Technology and Science Education
6. Environmental Sciences
7. Management & Information Technology

In the literature, there are studies, which modelled the student's progress and performance in higher education using Markov chain. However, the Mover-Stayer model is an extension of the Markov Chain model for dealing with a very specific type of unobserved heterogeneity in the population. For instance, Alawadhi and Konsowa (2010) presented an application of Markov chain analysis of students flow at Kuwait University. The analysis was based on a random sample of 1100 students from 1996/97 to 2004/05 academic years. Stratified random sample was used for the sample collection, based on the colleges' total number of students. The data from each college was studied and the corresponding Markov chain analysis was conducted. A frequency matrix was constructed for the university, from which the transition probabilities were estimated. The matrix which represents the transition probabilities of remaining in or progressing to another state is presented. The students' mean lifetimes in different levels of study in the colleges as well as the percentage of dropping out of the system are estimated. Adeleke *et al.* (2014), studied the pattern of students' enrolment and their academic performance in the Department of Mathematical Sciences (Mathematics option), Ekiti State University, Ado-Ekiti, Nigeria. They developed a transition matrix for ten (10) consecutive academic sessions. The probabilities of absorptions (Graduating and Withdrawal) were obtained. They equally obtained a fundamental matrix to determine the expected length of students' stay before graduating. Prediction was made on the enrolment and academic performance of students. Brezavscek *et al.* (2017), applied the Markov chain with five (5) transient and two (2) absorbing states to investigate the pattern of students' enrolment

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and their academic performance in a Slovenian higher education institution. Based on the students' intake records, the transition matrix was developed considering eight (8) consecutive academic seasons from 2008/09 to 2016/17. The students' progression towards the next stage of the study programme was estimated. The expected time that a student spends at a particular stage as well as the expected duration of study is determined. The graduation and withdrawal probabilities were equally obtained. Consequently, a prediction of the students' enrolment for the next three (3) academic years was made. Some useful studies also attempts to apply the Markov chain to analyse the processes of higher education study (Hlavaty and Domeova, 2014 ; Moody and Duclouy, 2014; Symeonaki and Kalamatianou, 2011).

The aim of the paper is to apply the alternative approach to the extension of the Mover-Stayer model developed by Adams and Abdulkadir (2018), to analyse the movement pattern of students of Modibbo Adama University of Technology, Yola, across various classes of degrees and to estimate the stayer population within the period 2011 – 2016.

2. Methodology

The Mover-Stayer model was first introduced by Blumen *et al.* (1955), in their study of the movement of workers among various industrial aggregates in the US. This model assumes that, there are two types of individuals in the population under consideration: a) the 'Stayer' who with probability one remains in the same category during the entire period of study; b) the 'Mover' whose changes in category overtime can be described by a Markov chain with constant transition probability matrix. The transition probability matrix for Movers, and the proportion of Stayers among the individuals in each category at, say the initial point in time, are unknown parameters. As a result, BKM suggested decomposing the population into movers and stayers,

$$P(1) = S + (I - S)M, \tag{1}$$

where S is a diagonal matrix containing as entries the proportion of persons in each origin state who remain there permanently, I – S is a diagonal matrix with entries which indicate the proportion in a state who are potentially mobile, I is an identity matrix and M is the transition matrix for mobile individuals

$$P(1) = \begin{bmatrix} P_{11} & \cdot & \cdot & \cdot & P_{1m} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ P_{m1} & \cdot & \cdot & \cdot & P_{mm} \end{bmatrix} \tag{2}$$

$$M = \begin{bmatrix} M_{11} & \cdot & \cdot & \cdot & M_{1m} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ M_{m1} & \cdot & \cdot & \cdot & M_{mm} \end{bmatrix} \tag{3}$$

where n is the number of states of the process and P(1) is a one-step population transition matrix.

The assumptions of the mover-stayer models can be summarized as follows; i) there exists a proportion of the population in each state that never moves, ii) the other proportion of the population which is mobile, is homogenous in its pattern of movement, that is, it follows a Markov process, and iii) the process is stationary.

To address the heterogeneity problem, Spilerman (1972) worked on the extension of the existing mover-stayer model proposed by BKM. He assumed a unimodal distribution to model rate of mobility within the movers (using gamma distribution). The negative binomial was obtained because of the combination of Poisson and Gamma distributions.

Motivated by the work of Spilerman (1972); Adams and Abdulkadir (2018) assumed negative binomial for the rate of movement (π_i). The choice of the negative binomial as a mixing distribution is informed by considering the number of transitions required to achieved desire events.

$$\begin{aligned}
 \pi_i = f(\lambda) &= \binom{k-1}{v-1} (1-\rho)^v \rho^{k-v} \\
 & \quad K=1, 2, \dots \\
 r_v(t) &= \sum_{v=1}^{\infty} \frac{(\lambda t)^v e^{-\lambda t}}{v!} \binom{k-1}{v-1} (1-\rho)^v \rho^{k-v} \\
 &= e^{-\lambda t} \sum_{v=1}^{\infty} \frac{\{\lambda t(1-\rho)\}^v}{v!} \binom{k-1}{v-1} \rho^{k-v} \\
 r_v(t) &= e^{-\lambda t} \sum_{v=1}^{\infty} \frac{\{\lambda t(1-\rho)\}^v}{v!} \binom{k-1}{v-1} \rho^{k-v} ; \quad k=1, 2, 3, \dots
 \end{aligned}
 \tag{4}$$

Equation (4) coincides with the Pólya-Aeppli distribution, where v takes values from 1 to k . If the parameter $\rho = 0$, the distribution in (4) will coincide with the classical homogenous Poisson distribution (Chukova and Minkova, 2012; Minkova, 2002).

The mean and variance of the Pólya-Aeppli distribution are given as;

$$E(N) = \frac{\lambda}{1-\rho} \quad \text{and} \quad \text{Var}(N) = \frac{\lambda(1+\rho)}{(1-\rho)^2}
 \tag{5}$$

2.1. Estimation of the Stayer Population

The transition matrix for the entire population, P , is defined as

$$P(1) = S + (I - S)M$$

According to Goodman (1961) and Morgan *et al.* (1983), S has s_i (the proportion initially in the i^{th} state who are stayers) down the diagonal, and I is the identity matrix. Consequently,

$$P_{ii} = S_i + (1 - S_i)M_{ii} \quad \dots \tag{6}$$

$$\Rightarrow M_{ii} = \frac{P_{ii} - S_i}{1 - S_i} ; \quad i = 1, 2, 3, \dots, n \tag{7}$$

Using the estimator of m_{ii} presented above, we obtain the following estimator of s_i ;

$$\therefore S_i = \frac{P_{ii} - M_{ii}}{1 - M_{ii}} , \text{ for } m_{ii} < 1 \quad \dots \tag{8}$$

2.2. Data

The model was applied to data collected on academic performance, captured by the current Grade Point Average (GPA) of students who enrolled and graduated within the period 2011 – 2016, from seven (7) Departments, one each from seven Schools of the Modibbo Adama University of Technology, Yola. The Department are; Statistics and Operations Research, Management Technology, Agric. Economics, Mathematics Education, Biochemistry, Architecture and Civil Engineering

The data collected on GPA was coded as follows;

Code	Class of Degree
1	First Class
2	Second Class Upper
3	Second Class Lower
4	Third Class
5	Pass

This gives five (5) classes of degree defined as states of the process. Results were collected per semester, and a maximum of nine (9) semesters were recorded, given rise to a maximum of eight (8) trials per student. Sequel to the definition of Polya-Aeppli distribution, the number of moves (v) represents the clusters while the distribution of the rate of movement is a repeated independent trials which can result in a success with probability ρ and failure with probability $1 - \rho$, then the distribution of the number of moves, with the k^{th} success on a specified number of trials follow the negative binomial distribution. In other words, the distribution of the number of successes in a sequence of trials before a specified number of failures occurs.

3. Results

The distribution of the number of moves within the five states (classes of degree) is given in Table 1.

Table-1. Distribution of the number of moves from Academic Performance Data

No. of moves (v)	$f_v(1)$	1000* $f_v(1)$ Population	Polya-Aeppli $\lambda = 1.2087$	Estimates $\rho = 0.3911$
0	0.3791	379	299	
1	0.1360	136	220	
2	0.1279	128	167	
3	0.1170	117	117	
4	0.0922	92	82	
5	0.0649	65	49	
6	0.0449	45	34	
7	0.0251	25	18	
8	0.0129	13	11	
Total	1.0000	1000	977	

NB: $\bar{v} = 1.985$ and $\sigma^2 = 4.5348$

Table 1 shows the result obtained from the baseline distribution as compared to the distribution of number of moves estimated by the Polya-Aeppli distribution. We noticed that at $v = 3$, the baseline coincides with the estimated value. The observed transition matrix for mover individuals as well as the population transition matrix is given below;

$$M = \begin{bmatrix} 0.489 & 0.398 & 0.114 & 0.000 & 0.000 \\ 0.067 & 0.367 & 0.478 & 0.085 & 0.003 \\ 0.004 & 0.309 & 0.415 & 0.260 & 0.012 \\ 0.000 & 0.086 & 0.510 & 0.330 & 0.074 \\ 0.000 & 0.014 & 0.143 & 0.557 & 0.286 \end{bmatrix}$$

$$P(1) = \begin{bmatrix} 0.516 & 0.212 & 0.195 & 0.070 & 0.007 \\ 0.033 & 0.569 & 0.273 & 0.113 & 0.012 \\ 0.016 & 0.172 & 0.649 & 0.147 & 0.017 \\ 0.011 & 0.131 & 0.281 & 0.548 & 0.030 \\ 0.008 & 0.097 & 0.223 & 0.218 & 0.452 \end{bmatrix}$$

3.1. Individual-Level Transition Matrix

Using the P(1) matrix obtained above, with λ and ρ as computed under Table 1, M^* the estimate of the individual-level transition matrix was constructed using;

$$M^* = \begin{bmatrix} 0.506 & 0.208 & 0.191 & 0.079 & 0.007 \\ 0.032 & 0.557 & 0.268 & 0.120 & 0.012 \\ 0.016 & 0.168 & 0.636 & 0.154 & 0.016 \\ 0.011 & 0.128 & 0.285 & 0.537 & 0.029 \\ 0.007 & 0.095 & 0.219 & 0.224 & 0.443 \end{bmatrix}$$

Matrix M^* , therefore, indicates how individuals (students) move across various classes of degrees each semester. It shows in particular, that, second class lower division is the most successful class in retaining students when they move, followed by second class upper, third class, first class degree respectively. Pass degree is the least successful.

In a similar manner, 20.8%, 19.1%, 7.9% and 0.7% of the students, moved from first class degree grade to second class upper, second class lower, third class and pass degree grades respectively during the study period. 3.2%, 26.8%, 12.0% and 1.2% moved from second class upper to first class, second class lower, third class and pass degree grades respectively. 1.6%, 16.8%, 15.4% and 1.6% moved from second class lower to first class, second class upper, third class and pass degree grades respectively. 1.1%, 12.8%, 28.5% and 2.9% moved from third class to first class, second class upper, second class lower and pass degree grades respectively, while 0.7%, 9.5%, 21.9% and

22.4% moved from pass degree to first class, second class upper, second class lower and third class grades respectively during the period.

The results also show that, 20.8% students moved from first class to second class upper, 19.1% from first class to second class lower, 28.5% from third class to second class lower, 21.9% from pass degree to second class lower as well as 22.4% from pass degree to third class grade which is more when compared to other grades, with third class to second class lower recording the highest movements, while pass degree to first class and first class to pass degree recording the lowest movements.

3.2. Estimation of the Stayer Population on Academic Performance

The total proportion of stayer population based on the sample collected, shows that, 24 out of 374 of the students, representing 0.0686 (6.9%) are stayers. However, the proportions per class of degree computed from individual-level transition matrix as proposed by Goodman (1961) and Morgan *et al.* (1983) follow;

$$\therefore S = \begin{bmatrix} 0.020 & 0 & 0 & 0 & 0 \\ 0 & 0.027 & 0 & 0 & 0 \\ 0 & 0 & 0.036 & 0 & 0 \\ 0 & 0 & 0 & 0.024 & 0 \\ 0 & 0 & 0 & 0 & 0.016 \end{bmatrix}$$

This shows that, 2.0% of the students who started with first class as their baseline grade retained that grade throughout the period of study; 2.7% started with second class upper and remained there through the entire study period. Similarly, 3.6%, 2.4% and 1.6% of students who started as second class lower, third class and pass degree respectively retained the grades throughout the period of study.

4. Discussion and Conclusion

In the context of academic performance of student's, data was collected on academic performance, captured by the current Grade Point Average (GPA) of students who enrolled and graduated within the period 2011 – 2016, from seven (7) Departments, one each from seven Schools of the Modibbo Adama University of Technology, Yola. The five (5) classes of degree were coded and analyzed accordingly. The proportions of stayers to various classes of degree were equally estimated. Furthermore, the individual-level transition matrix was estimated, where it was observed that, second class lower degree is most successful in retaining students when they move across all the other classes of degrees within the period under study. This was followed closely by second class upper, third class, first class degree respectively. Pass degree is however the least successful. Majority of the students, moved from third class grade to second class lower grade, while only few students break-even, by moving from either pass degree to first class or first class to pass degree during the entire study period. The results obtained were similar to that of Alawadhi and Konsowa (2010) who presented an application of Markov chain analysis of students flow at Kuwait University. They constructed a frequency matrix for the university, from which the transition probabilities were estimated. The matrix which represents the transition probabilities of remaining in or progressing to another state was also presented. The paper however, did not estimate the stayer population.

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