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Model of Optimizing the Delivery Moment Taking Into Account the Uncertainty of Demand

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Abstract

The majority of trade enterprises in Russia are guided by the average demand and delivery time indicators in stock management, and only a few large companies use simulation of logistics processes, which improves the efficiency and performance by reducing costs of storage and shortages. The article presents a model of stock management, or a model of determining the optimal delivery moment with due account for the uncertainty of demand. The criterion of minimizing the integral costs, with due account for the costs of excess stocks and the costs of the lack of goods in stock, is an efficiency criterion. The triangular distribution is considered as a law of random demand distribution, being one of the most appropriate in the context of insufficient statistical data. The economic mathematical model under consideration allows to optimize the delivery moment if risks are minimized, based on statistical data on the demand for goods in the previous period, or to use expert estimates if such data are not available. These data are sufficient for building a probability distribution for a random variable of demand. The model allows to determine the day of delivery for a certain amount of goods in the case of random demand if risks are minimized. In the case of a triangular distribution, this optimization problem has an analytical solution, which is reduced to formula evaluation.

Keywords: Stock management; Cost minimization; Delivery moment; Uncertainty of demand; Triangular distribution.

1. Introduction

The majority of trade enterprises in Russia are guided by the average demand and delivery time indicators in stock management, and only a few large companies use simulation of logistics processes, which improves the efficiency and performance by reducing costs of storage and shortages.

In reality, the uncertainty often arises, associated with inaccuracy or incompleteness of information about demand, temporary delays in deliveries, damage to products, etc. Accounting for uncertainty factors in models allows to find the most efficient stock management strategy in the context of such uncertainties. Various models that take into account such uncertainty were considered in (Brodezkiy, 2007); (Kosorukov and Sviridova, 2009a); (Kosorukov and Sviridova, 2009b); (Kosorukov and Sviridova, 2012); (Kosorukov and Sviridova, 2015).

As such, any trading company needs an optimal stock management strategy for success – otherwise, this will result in an increase in costs and product prices and hence the loss of competitiveness of the entire enterprise. Besides, financial resources that are unnecessarily invested in stocks could generate additional profit. This raises the issue of finding the optimal business model of stock management in the context of uncertainty, the objective function of which is to minimize additional costs, with due account for the constraints imposed by the economic environment and the specifics of the enterprise operation.

The economic mathematical model under consideration allows to optimize the delivery moment if risks are minimized, based on statistical data on the demand for goods in the previous period, or to use expert estimates if such data are not available. These data are sufficient for building a probability distribution for a random variable of demand.

Risk in this model is understood as a deviation of real values of demand for goods from the expected. Besides, it is assumed that delay or premature delivery of ordered goods is impossible, i.e. if an order is made for moment t^*, then the goods arrive at this very moment.

2. Problem Formalization

Uncertainty about the moment when the goods are out of stock α is expressed by formula (1):

$$\alpha = \alpha_0 + \Delta\alpha, \qquad (1)$$

where α_0 is the expected time when the goods are out of stock;

$\Delta\alpha$ is a random variable that describes a deviation of the real time when the goods are out of stock from the expected.

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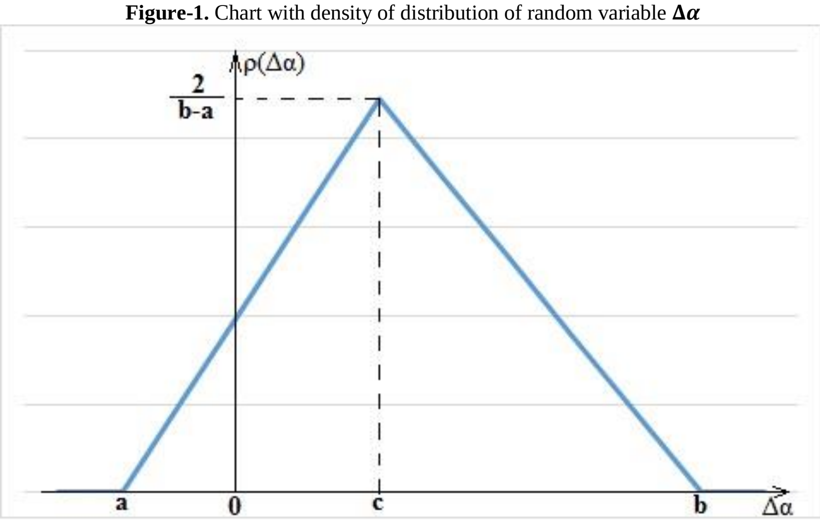
It is assumed that random variable $\Delta\alpha$ is distributed according to the triangular distribution law on interval $[a, b]$. Consideration of the triangular distribution as the least demanding to the volume of the initial statistical data distinguishes this model from the models considered in (Brodezkiy, 2007); (Kosorukov and Sviridova, 2009a); (Kosorukov and Sviridova, 2009b); (Kosorukov and Sviridova, 2012); (Kosorukov and Sviridova, 2015). Parameters $a, b,$ and c are determined from statistical data or using expert estimates if the following condition is met: $a \leq c \leq b, a < b$, where a is the lower limit, b is the upper limit, and c is the mode (most common value in the distribution). In the special case of $a = c$ or $c = b$, the triangular distribution is built with two points. In this case, the real time when the goods are out of stock α also has a triangular distribution of a random variable on interval $[\alpha_0 + \text{a}; \alpha_0 + \text{b}]$. Figure 1 presents a chart with density of distribution of random variable $\Delta\alpha$. This assumption is required for the possibility to conduct analytical studies of the model under consideration. On the other hand, the triangular distribution adequately reflects practical situations where statistical data are not available, since expert estimates can be used for it.

Figure-1. Chart with density of distribution of random variable $\boldsymbol{\Delta\alpha}$



Cumulative average expected costs are an optimization criterion in this model. The aggregate costs reflecting the risks of the chosen stock management strategy include, first of all, the costs of storing goods at stock, and, secondly, the costs associated with untimely delivery of goods, and hence with incomplete satisfaction of demand.

On the one hand, costs for storing surplus at stock may arise due to the early delivery of goods, as well as losses in the liquidation of unsold goods. Assume the lot volume is fixed and equal to Q. Then the costs of storing volume Q from the delivery moment t^* till the actual running out of stock α, in the case if the delivery occurred earlier than time t^* $(t^* < \alpha)$, according to formula (2), are as follows:

$$I = pQ(\alpha - t^*), \tag{2}$$

where $p = const$ is daily storage cost per unit.

On the other hand, a shortage of goods may arise due to the late delivery of goods, which will lead to lost profits and the risk of losing customers. Then the costs of the goods shortage from the moment of actual running out of stock α till the moment t^* of delivery in the volume Q, in the case if the delivery occurred later than time t^* $(t^* < \alpha)$, according to formula (3), are as follows:

$$D = \frac{Q}{\alpha_0} z(t^* - \alpha), \tag{3}$$

where $z = const$ is profit from a unit sale, and

Q/α_0 is the average daily volume of goods sold.

Total costs are calculated according to formula (4):

$$I + D = \begin{cases} pQ(\alpha - t^*), t^* < \alpha; \\ \dfrac{Q}{\alpha_0} z(t^* - \alpha), t^* > \alpha. \end{cases} \tag{4}$$

The expected value is considered as a function of total costs, which is a random variable in stochastic models.

The uncertainty of demand in this model is described by a continuous random variable $\Delta\alpha$, which has a law of triangular distribution with density represented in formula (5):

$$\rho(\Delta\alpha) = \begin{cases} 0, \text{for } \Delta\alpha < a; \\ \dfrac{2(\Delta\alpha - a)}{(b-a)(c-a)}, \text{for } a \le \Delta\alpha < c; \\ \dfrac{2}{(b-a)}, \text{for } \Delta\alpha = c; \\ \dfrac{2(\text{b} - \Delta\alpha)}{(b-a)(b-c)}, \text{for c} < \Delta\alpha \le b; \\ 0, \text{for } b < \Delta\alpha. \end{cases} \qquad (5)$$

The expected value of total costs is calculated according to formula (6):

$$F(t^*) = \int_a^b \frac{Q}{\alpha_0} z(t^* - \alpha_0 - \Delta\alpha)\rho(\Delta\alpha)d\Delta\alpha + \\ + \int_a^b pQ(\alpha_0 + \Delta\alpha - t^*)\rho(\Delta\alpha)d\Delta\alpha. \qquad (6)$$

The problem of minimizing the risks of stock management described by expression (7) is to find the time of scheduling the delivery t^*, at which the expected value of total costs will be the least.

$$F(t^*) \to \min_{t^*}. \qquad (7)$$

3. Analytical Solution of the Problem

Let us consider expression (6) in parts, according to formula (8):

$$F(t^*) = F_D(t^*) + F_I(t^*). \qquad (8)$$

The first term of expression (8) is represented by formula (9):

$$F_D(t^*) = \int_a^b \frac{Q}{\alpha_0} z(t^* - \alpha_0 - \Delta\alpha)\rho(\Delta\alpha)d\Delta\alpha. \qquad (9)$$

The integral in formula (9) exists for $t^* > \alpha$, which is equivalent to the fact that $t^* > \alpha_0 + \Delta\alpha$, and hence integral (9) also exists for $t^* - \alpha_0 > \Delta\alpha$.

Let us consider four possible cases.

In the first case, for $t^* - \alpha_0 < a$, the inequality $t^* - \alpha_0 > \Delta\alpha$ fails, which means that integral $F_D(t^*)$ in $(-\infty; a)$ does not exist and takes the look according to formula (10):

$$F_{D_1}(t^*) = 0. \qquad (10)$$

Let us consider the second case for $a \le t^* - \alpha_0 < c$. The inequality $t^* - \alpha_0 > \Delta\alpha$ is true on interval $[a; t^* - \alpha_0]$ and fails on interval $(t^* - \alpha_0, b)$, therefore:

$$F_{D_2}(t^*) = \int_a^{t^*-\alpha_0} \underbrace{\frac{Q}{\alpha_0}}_{K_1} z\,(t^* - \alpha_0 - \Delta\alpha)\frac{2(\Delta\alpha - a)}{(b-a)(c-a)}d\Delta\alpha$$

If transformations and calculations are omitted, formula (9) results in formula (11) in the second case:

$$F_{D_2}(t^*) = \frac{K_1}{3(b-a)(c-a)}(t^* - a + \alpha_0)^3. \qquad (11)$$

Let us consider the third case, when $a < c \le t^* - \alpha_0 < b$. Inequality $t^* - \alpha_0 > \Delta\alpha$ is true on intervals $[a; c]$ and $[c; t^* - \alpha_0]$ and fails on interval $(t^* - \alpha_0; b)$, therefore:

$$F_{D_3}(t^*) = \int_a^c \underbrace{\frac{Q}{\alpha_0}}_{K_1} z\,(t^* - \alpha_0 - \Delta\alpha)\frac{2(\Delta\alpha - a)}{(b-a)(c-a)}d\Delta\alpha + \int_c^{t^*-\alpha_0} \underbrace{\frac{Q}{\alpha_0}}_{K_1} z\,(t^* -$$

$$-\alpha_0 - \Delta\alpha)\frac{2(\mathrm{b} - \Delta\alpha)}{(b-a)(b-c)}d\Delta$$

If transformations and calculations are omitted, formula (9) results in formula (12) in the third case:

$$F_{D_3}(t^*) = \frac{K_1}{3(b-a)}\Big[(a-c)(3\alpha_0 + a + 2c - 3t^*) + \\ + \frac{1}{(b-c)}(\alpha_0 + c - t^*)^2(\alpha_0 + 3b - 2c - t^*)\Big]. \qquad (12)$$

In the fourth case, for $a < b \le t^* - \alpha_0$, inequality $t^* - \alpha_0 > \Delta\alpha$ is true on the entire interval $[a, b]$, therefore:

$$F_{D_4}(t^*) = \int_a^c \underbrace{\frac{Q}{\alpha_0}}_{K_1} z\,(t^* - \alpha_0 - \Delta\alpha)\frac{2(\Delta\alpha - a)}{(b-a)(c-a)}d\Delta\alpha + \int_c^b \underbrace{\frac{Q}{\alpha_0}}_{K_1} z\,(t^* - \alpha_0 - \\ -\Delta\alpha) + \int_c^b \underbrace{\frac{Q}{\alpha_0}}_{K_1} z\frac{2(\mathrm{b} - \Delta\alpha)}{(b-a)(b-c)}d\Delta\alpha$$

If transformations and calculations are omitted, formula (9) results in formula (13) in the fourth case:

$$F_{D_4}(t^*) = \frac{K_1}{3}(3t^* - a - b - c - 3\alpha_0). \qquad (13)$$

Now let us consider the second term of expression (8), which is expressed by the following formula (14):

$$F_I(t^*) = \int_a^b pQ(\alpha_0 + \Delta\alpha - t^*)\rho(\Delta\alpha)d\Delta\alpha. \qquad (14)$$

The integral in formula (14) exists for $t^* < \alpha$, which is equivalent to the fact that $t^* < \alpha_0 + \Delta\alpha$, which means that integral (14) also exists for $t^* - \alpha_0 < \Delta\alpha$.

Let us consider four possible cases.

In the first case, for $t^* - \alpha_0 < a < b$, inequality $t^* - \alpha_0 < \Delta\alpha$ is true on the entire interval $[a, b]$, therefore:

$$F_{I_1}(t^*) = \int_a^c \underbrace{pQ}_{K_2}(\alpha_0 + \Delta\alpha - t^*)\frac{2(\Delta\alpha - a)}{(b-a)(c-a)}d\Delta\alpha + \int_c^b \underbrace{pQ}_{K_2}(\alpha_0 + \Delta\alpha - t^*)\cdot \\ \cdot\frac{2(\mathrm{b} - \Delta\alpha)}{(b-a)(b-c)}d\Delta\alpha$$

If transformations and calculations are omitted, formula (14) results in formula (15) in the first case:

$$F_{I_1}(t^*) = \frac{K_2}{3}(3\alpha_0 + a + b + c - 3t^*). \qquad (15)$$

Let us consider the second case for $a \le t^* - \alpha_0 < c < b$. The inequality $t^* - \alpha_0 < \Delta\alpha$ is true on intervals $[t^* - \alpha_0; c]$ and $[c; b]$ and fails on interval $[a; t^* - \alpha_0]$, therefore:

$$F_{I_2}(t^*) = \int_{t^*-\alpha_0}^c \underbrace{pQ}_{K_2}(\alpha_0 + \Delta\alpha - t^*)\frac{2(\Delta\alpha - a)}{(b-a)(c-a)}d\Delta\alpha + \int_c^b \underbrace{pQ}_{K_2}(\alpha_0 + \Delta\alpha - \\ -t^*)\frac{2(\mathrm{b} - \Delta\alpha)}{(b-a)(b-c)}d\Delta\alpha$$

If transformations and calculations are omitted, formula (14) results in formula (16) in the second case:

$$F_{I_2}(t^*) = \frac{K_2}{3(b-a)}\Big[\frac{1}{(c-a)}(t^* - \alpha_0 - c)^2(t^* - \alpha_0 - 3a + 2c) + \\ +(b-c)(3\alpha_0 + b + 2c - 3t^*)\Big]. \qquad (16)$$

Let us consider the third case for $a < c \le t^* - \alpha_0 < b$. The inequality $t^* - \alpha_0 < \Delta\alpha$ is true on interval $[t^* - \alpha_0; b]$ and fails on interval $[a; t^* - \alpha_0]$, therefore:

$$F_{I_3}(t^*) = \int_{t^*-\alpha_0}^{b} \underbrace{pQ}_{K_2} (\alpha_0 + \Delta\alpha - t^*) \frac{2(\mathrm{b} - \Delta\alpha)}{(b-a)(b-c)} d\Delta\alpha = \frac{2K_2}{(b-a)(b-c)} \cdot$$
$$\cdot \int_{t^*-\alpha_0}^{b} (\alpha_0 + \Delta\alpha - t^*)(\mathrm{b} - \Delta\alpha) d\Delta\alpha$$

As such, formula (14) takes the following look (17) in the third case:

$$F_{I_3}(t^*) = \frac{-K_2}{3(b-a)(b-c)}(t^* - \alpha_0 - b)^3. \qquad (17)$$

In the fourth case, for $b \le t^* - \alpha_0$, the inequality $t^* - \alpha_0 < \Delta\alpha$ fails, therefore, integral $F_I(t^*)$ in $(b; +\infty)$ does not exist and takes the look according to formula (18):

$$F_{I_4}(t^*) = 0. \qquad (18)$$

Let us find the expected value of total costs in each of the considered intervals by adding the obtained formulas (10) and (15), (11) and (16), (12) and (17), (13) and (18).

Adding formulas (10) and (15), expression (19) is obtained:

$$F_1(t^*) = F_{D_1}(t^*) + F_{I_1}(t^*) = \frac{K_2}{3}(3\alpha_0 + a + b + c - 3t^*). \qquad (19)$$

Adding formulas (11) and (16), expression (20) is obtained:

$$F_2(t^*) = F_{D_2}(t^*) + F_{I_2}(t^*) = \frac{K_1}{3(b-a)(c-a)}(t^* - a + \alpha_0)^3 +$$
$$+ \frac{K_2}{3(b-a)}\left[\frac{1}{(c-a)}(t^* - \alpha_0 - c)^2(t^* - \alpha_0 - 3a + 2c) + \right. \qquad (20)$$
$$\left. +(b-c)(3\alpha_0 + b + 2c - 3t^*)\right].$$

Adding formulas (12) and (17), expression (21) is obtained:

$$F_3(t^*) = F_{D_3}(t^*) + F_{I_3}(t^*) = \frac{K_1}{3(b-a)}\left[(a-c)(3\alpha_0 + a + 2c - 3t^*) + \right.$$
$$\left. + \frac{1}{(b-c)}(\alpha_0 + c - t^*)^2(\alpha_0 + 3b - 2c - t^*)\right] - \qquad (21)$$
$$- \frac{K_2}{3(b-a)(b-c)}(t^* - \alpha_0 - b)^3.$$

Adding formulas (13) and (18), expression (22) is obtained:

$$F_4(t^*) = F_{D_4}(t^*) + F_{I_4}(t^*) = \frac{K_1}{3}(3t^* - a - b - c - 3\alpha_0). \qquad (22)$$

Let us find the minimum expected costs in each of the intervals.

$F_1(t^*) = \frac{K_2}{3}(3\alpha_0 + a + b + c - 3t^*)$ is a linear decreasing function, which means that the minimum value is reached at the right end of interval $(-\infty; a + \alpha_0]$, and therefore, at point $t^* = a + \alpha_0$. The least value of function $F_1(t^*)$ is represented by formula (23):

$$\min_{t^*=a+\alpha_0} F_1(t^*) = \frac{K_2}{3}(3\alpha_0 + a + b + c - 3a - 3\alpha_0) = \frac{K_2}{3}(b + c - 2a). \quad (23)$$

To find the least value for $F_2(t^*)$, let us take the derivative of function (20) and equate it to zero.

$$\frac{dF_2(t^*)}{dt^*} = \frac{K_1}{3(b-a)(c-a)} 3(t^* - a + \alpha_0)^2 + \frac{K_2}{3(b-a)}\left[\frac{1}{(c-a)} 2(t^* - \alpha_0 - \right.$$
$$\left. -c)(t^* - \alpha_0 - 3a + 2c) + \frac{1}{(c-a)}(t^* - \alpha_0 - c)^2 + (b-c)(-3)\right]$$

If transformations and calculations are omitted, formula (24) is obtained:

$$\frac{dF_2(t^*)}{dt^*} = \frac{(K_1 + K_2)}{(b-a)(c-a)}(t^* - a - \alpha_0)^2 - K_2. \qquad (24)$$

Let us equate expression (24) to zero:

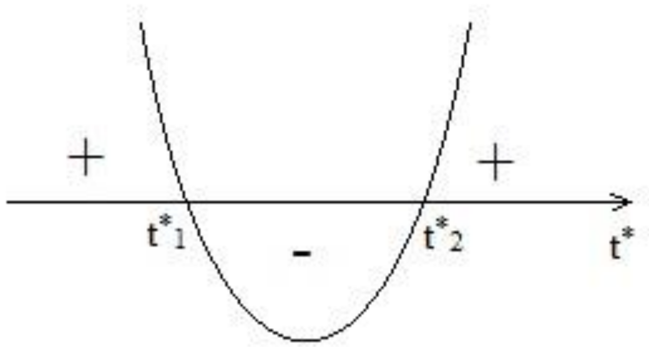
$$\frac{dF_2(t^*)}{dt^*} = 0.$$

The roots of the resulting equations are found:

$$t_{1,2}^* = a + \alpha_0 \pm \sqrt{\frac{K_2(b-a)(c-a)}{(K_1 + K_2)}}.$$

The graph of function $\frac{(K_1+K_2)}{(b-a)(c-a)}(t^* - a - \alpha_0)^2 - K_2 = 0$ is a parabola (Figure 2), the branches are directed upwards, since $\frac{(K_1+K_2)}{(b-a)(c-a)} > 0$.

Figure-2. Schematic representation of the graph of function $F_2{}'(t^*)$



It can be seen from Figure 2 that derivative $F_2{}'(t^*)$ changes its sign from minus to plus at point t_2^*, which is as follows (25):

$$t_2^* = a + \alpha_0 + \sqrt{\frac{K_2(b-a)(c-a)}{(K_1 + K_2)}}. \qquad (25)$$

This means that t_2^* is the least value of function $F_2(t^*)$.

Let us find the value of function $F_2(t^*)$ in the minimum point, for which (25) is substituted into (20). If transformations are omitted, the following result is obtained:

$$\min_t F_2(t) = F_2(t^*) = \frac{K_2}{3}\left(b + c - 2a - -2\sqrt{\frac{K_2(b-a)(c-a)}{(K_1 + K_2)}}\right). \qquad (26)$$

To find the minimum of $F_3(t^*)$, the derivative of function (21) is taken and equated to zero. The results of transformations are provided.

$$\frac{dF_3(t^*)}{dt^*} = K_1 - \frac{(K_1 + K_2)}{(b-a)(b-c)}(t^* - b - \alpha_0)^2. \qquad (27)$$

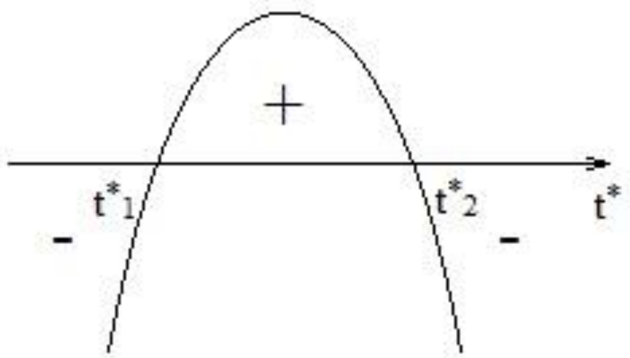
Let us equate expression (27) to zero and find the roots of the equation:

$$\frac{dF_3(t^*)}{dt^*} = 0;$$

$$t^*_{1,2} = b + \alpha_0 \pm \sqrt{\frac{K_1(b-a)(b-c)}{(K_1+K_2)}}.$$

The graph of function $K_1 - \frac{(K_1+K_2)}{(b-a)(b-c)}(t^* - b - \alpha_0)^2 = 0$ is a parabola (Figure 2), the branches are directed downwards, since $-\frac{(K_1+K_2)}{(b-a)(b-c)} < 0$.

Figure-3. Schematic representation of the graph of function $\boldsymbol{F_3}'(\boldsymbol{t}^*)$



It can be seen from Figure 3 that derivative $F_3{}'(t^*)$ changes its sign from minus to plus at point t_1^*, which is as follows (28):

$$t_1^* = b + \alpha_0 - \sqrt{\frac{K_1(b-a)(b-c)}{(K_1+K_2)}}. \qquad (28)$$

This means that t_1^* is the least value of function $F_3(t^*)$.

Let us find the value of function $F_3(t^*)$ in the minimum point, for which (28) is substituted into (21) and the necessary algebraic transformations are performed to obtain the following result:

$$\min_t F_3(t) = F_3(t^*) = \frac{K_1}{3}\left(2b - a - c - 2\sqrt{\frac{K_1(b-a)(b-c)}{(K_1+K_2)}}\right). \qquad (29)$$

$F_4(t^*) = \frac{K_1}{3}(3t^* - a - b - c - 3\alpha_0)$ is a linear increasing function, which means that the minimum value is reached at the left end of interval $[b + \alpha_0; +\infty)$ or in point $t^* = b + \alpha_0$. The minimum value of function $F_4(t^*)$, is represented by formula (30):

$$\min_{t^* = b + \alpha_0} F_4(t^*) = \frac{K_1}{3}(3b + 3\alpha_0 - a - b - c - 3\alpha_0) = \frac{K_1}{3}(2b - a - c). \quad (30)$$

To find the least value of function $F(t^*)$, formulas (23), (26), (29) and (30) are compared and the least value among them is found.

Let us compare (23) with (26) by subtracting (26) from (23):

$$\frac{K_2}{3}(b + c - 2a) - \frac{K_2}{3}\left(b + c - 2a - 2\sqrt{\frac{K_2(b-a)(c-a)}{(K_1+K_2)}}\right) =$$

$$= \frac{2K_2}{3}\sqrt{\frac{K_2(b-a)(c-a)}{(K_1+K_2)}} > 0.$$

The resulting expression is greater than zero, which means that (23) is greater than (26).

Now let us compare (29) with (30) by subtracting (30) from (29):

$$\frac{K_1}{3}\left(2b-a-c-2\sqrt{\frac{K_1(b-a)(b-c)}{(K_1+K_2)}}\right)-\frac{K_1}{3}(2b-a-c)=$$
$$=-\frac{2K_1}{3}\sqrt{\frac{K_1(b-a)(b-c)}{(K_1+K_2)}}<0.$$

The resulting expression is less than zero, which means that (29) is less than (30).
Finally, let us compare (26) with (29) by subtracting (29) from (26):

$$\frac{K_2}{3}\left(b+c-2a-2\sqrt{\frac{K_2(b-a)(c-a)}{(K_1+K_2)}}\right)-\frac{K_1}{3}\Bigg(2b-a-c-$$
$$-2\sqrt{\frac{K_1(b-a)(b-c)}{(K_1+K_2)}}\Bigg)=\frac{1}{3}\Bigg((K_2-2K_1)b+(K_1-2K_2)a+(K_1+K_2)c-$$
$$-2\sqrt{\frac{(b-a)}{(K_1+K_2)}}\left(K_2\sqrt{K_2(c-a)}-K_1\sqrt{K_1(b-c)}\right)\Bigg).$$

The sign in the resulting expression depends on the following parameters: a, b, c, $K_1=\frac{Qz}{\alpha_0}$, $K_2=Qp$.

The solution to this problem, based on the theory of mathematical analysis optimization, is not provided. Only the final version of the solution to problem (7), described by relation (31), is presented.

$$\text{If } p\left(b+c-2a-2\sqrt{\frac{p(b-a)(c-a)}{\left(\frac{z}{\alpha_0}+p\right)}}\right)<\frac{z}{\alpha_0}\Bigg(2b-a-c-$$
$$-2\sqrt{\frac{\frac{z}{\alpha_0}(b-a)(b-c)}{\left(\frac{z}{\alpha_0}+p\right)}}\Bigg)\text{, then } t^*=a+\alpha_0+\sqrt{\frac{p(b-a)(c-a)}{\left(\frac{z}{\alpha_0}+p\right)}};$$
$$\text{If } p\left(b+c-2a-2\sqrt{\frac{p(b-a)(c-a)}{\left(\frac{z}{\alpha_0}+p\right)}}\right)>\frac{z}{\alpha_0}\Bigg(2b-a-c-$$
$$-2\sqrt{\frac{\frac{z}{\alpha_0}(b-a)(b-c)}{\left(\frac{z}{\alpha_0}+p\right)}}\Bigg)\text{, then } t^*=b+\alpha_0-\sqrt{\frac{\frac{z}{\alpha_0}(b-a)(b-c)}{\left(\frac{z}{\alpha_0}+p\right)}}. \quad (31)$$

4. Conclusion

As such, the model allows to determine the day of delivery for a certain amount of goods in the case of random demand if risks are minimized. In the case of triangular distribution, this optimization problem has an analytical solution, which is reduced to formula evaluation (31).

The result, or the type of analytical expression and its content, depend on the input parameters of the model, namely α_0 – the expected time when the goods are out of stock, p – the daily cost of storing the unit of production, z – the profit from the sale of a unit of production, as well as parameters of the triangular distribution a, b, c of random variable $\Delta\alpha$, which describes a deviation of the real time when the goods are out of stock from the expected.

The problem in a similar formulation was considered in papers (Kosorukov and Sviridova, 2009a); (Kosorukov and Sviridova, 2009b); (Kosorukov and Sviridova, 2012), but it was for the case when a random variable describing the deviation of the real time when the goods were out of stock from the expected one could be considered normally distributed. First of all, this is not always true, and secondly, the company often lacks statistical data to test a sample of realizations of a random variable for its compliance with the normal distribution law. In this regard, the authors note the practical feasibility of the results, since the parameters of the triangular distribution can be evaluated by experts if a sufficient volume of statistical data is missing.

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