The Journal of Social Sciences Research
ISSN(e): 2411-9458, ISSN(p): 2413-6670
Special Issue. 5, pp: 80-84, 2018
URL: https://arpgweb.com/journal/journal/7/special_issue
DOI: https://doi.org/10.32861/jssr.spi5.80.84

Academic Research Publishing Group

Original Research **Open Access**

Construction of Trajectories for System Stable Functioning by Their Hierarchical Models

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Abstract

The paper proposes the language for system model description in the form of a hierarchy of so-called control units (CUs). Each CU is characterized by a set of state parameters Q combining a set of X inputs and a set of internal parameters P; a set of secondary control units B1, ..., Bn; the control function F(i, X(t), Q(t-1)), which, by the number of the secondary unit i, the input parameters of the unit at time t and the state parameters at time t-1, determines the values of the input parameters of the secondary block Bi in moment of time t; the recalculation function of H(X(t), Q(t-1), Q1(t),..., Qn(t)), which determines P(t) values of the unit internal parameters according to the state parameter values of the secondary units at the moment of time t. An important advantage of this method of the model description for a particular system S is the possibility to construct the Boolean function FRI(QI(t-1), QI(t)) in the form of a BDD within a limited time, true t., etc. when QI(t) are the values of the unit I state parameters for the model of the system S at time t, and QI(t-1) are the values of the state parameters of the unit I at time t-1. The control functions in various CU from the model of the system S are, as a rule, nondeterministic. The task consists in finding the variants of their calculation at each moment of time t so that the state parameters of certain model CU satisfy certain requirements. If these requirements are written in the form of Boolean formulas or CTL logic formulas, the problem of finding the stable operation paths of the system S is reduced to the search for all executing sets of some Boolean function given in the form of BDD. This provides an effective solution to the problem of finding the trajectories of system S stable functioning. The obtained results were used in the development of complex socio-economic system models and "correct" programs for their development.

Keywords: System models; Model checking; BDD of Boolean functions; Verification algorithms; University development programs; Promela language.

1. Introduction

At present, the methods of program-targeted management by the development of various socio-economic systems of a complex structure, such as territories, regions, regional systems (education, health, tourism), as well as various separate socio-economic objects, and in particular, education objects, such as universities, which can also be seen as systems. Therefore, the actual task is the development of "right" programs for the development of systems, similar with the development of correct computer programs - a developed discipline in theoretical programming. In this paper, we will consider the problem of "correct" programs for the development of various systems using the example of university development programs. The importance of the programs for the development of modern universities was emphasized by many authoritative organizations and authors (Altbach *et al.*, 2009). The development programs have been published for a 10-year or 15-year period in almost all major Russian universities. But it is quite difficult to be convinced of their "correctness", i.e. of the fact that their implementation will lead to the declared goals. To solve the problem of "correct" development programs, we apply the modification of the method Model Checking (Giunchiglia and Traverso, 1999), used to verify computer programs. In Model Checking, the model M of a particular system is represented by the constructions of finite machine type or Kripke models, and the requirements for the correct functioning of the system are described in the form of the formula φ of some language of L. specifications. Often, the formula of the temporal logic LTL or CTL are used as L language (Gurfinkel *et al.*, 2003). The verification task is reduced to verification of the formula φ truth on the model M. But during the study of a particular system operation, the constructing of its model in the form of an adequate finite machine or Kripke model is very laborious and cumbersome.

In the first part of this paper, they proposed the language for system model description in the form of a hierarchy of so-called control units. Expressive capabilities of this language are demonstrated on the example of "University" system development model. The task of the "right" program development to develop the system for which the hierarchical model is constructed is reduced to the problems of finding trajectories for the stable functioning of the system and the trajectories of the system sustainable development, the exact formulations and solutions of which are given in the second part of this paper (Shatilova *et al.*, 2018).

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2. Methods

The concept of the system hierarchical model

Type hierarchy. The elements of the model are the control units of different types. Each type □ is defined by the set of components <T,X,P,F,G>. Let's describe each of them.

T=<□□1,…,□n> is a tuple of not necessarily different types, each of which we will call the child type for □. This means that each CU A of type □ must contain the types B1,…,Bn of secondary units □1,…,□n respectively. The type □ will be the ancestor for all types □i, as well as for all types for which at least one □i is an ancestor.

X =<x1,…,xm> is the list of input parameters. We assume that the values of each parameter xi are an integer in the finite interval [0, Ni], where Ni is the natural number. At each instant t, the input parameters are transferred to the CU of □ type from main CU.

P=<p1,…, pk> is the list of internal parameters whose values at any time t serve to determine the current state of each BU A of type □. Let's assume that the values of each parameter pi are some integer in the interval [0, Ki], where Ki is a natural number.

The combined tuple of input and internal parameters Q = <X;P> will be called the parameters of CU A of the type □. We will denote by Q (t) the list of values of Q state parameters at time t.

F(i,X(t),Q(t-1)) is a control function. Its value is the set of control parameter values Xi(t), which the CU A of type □ at the time t transmits to its secondary CU Bi of type □i as a set of input parameters.

H(X(t),Q(t-1),Q1(t),…,Qn(t)) is the recalculation function. Its value is the set of values P(t) of CU A internal parameters of the type □□at the moment t. Here Qi(t)- is the set of state parameters of the secondary CU Bi at time t. CU types that do not have secondary types, i.e. which have an empty T set, will be called basic types. For base-type CU P(t)=H(X(t),Q(t-1)), i.e. the state of the basic CU at the time t is determined by the input parameters and its state at the previous time moment.

The set of CU types in the model must satisfy the following conditions:

1). Each □ type is not an ancestor of itself.

2). There is a unique type □ without ancestors, which we will call the main type of the model.

3). The main type has no input parameters.

Example. Let us briefly describe the construction of the "University" system model. At the first stage we will establish the main type of university, in which we will define the daughter types of the institute, the faculty, the research center, the admission committee, the service of academic exchanges, etc.

A lot of parameters that characterize the state of the university-type CU at each time point will be divided into groups: the parameters describing financial stability; the parameters that describe the foundation of development; the parameters describing competitiveness, etc. As a control function, a nondeterministic function of the reception plan and budget financing redistribution between institutes, faculties, scientific centers, etc. can be used here. The non-determinism of the control function is explained by the fact that at different times this redistribution of resources can be carried out according to different principles: "in proportion to internal ratings", "in proportion to the number of students", "on the principle of leader support" , "by the results of foresight research", etc.

As a recalculation function, it is logical to use the natural functions of university parameter value recalculation by the known values of its subdivision parameters.

Further, the types of institute, faculty, research center, admission committee, the service of academic exchanges, etc. are described similarly. Thus, we will build a hierarchyof CU types included in the model of the "University" system.

Specific CU task. In order to set a particular CU of the type □, it is necessary to specify the specific secondary BUs B1, ..., Bn of □1,…,□n types respectively and to set the parameters of its state Q(0) at the initial moment of time. In this case, each particular CU can be a secondary one only for one CU. The model of a specific system is the CU task of the basic type of the model, which we will denote by I and which will represent a certain hierarchical structure of nested ones CU per se.

Model functioning. We imagine time as a sequence of consecutive moments, which we denote by 0,1,2, ..., j, ... 0 is the initial instant of time. The simulated system functions in time, passing from one state to another.

Let the states of all CUs entering the model at the initial time moment t = 0 are known. Let us show how to calculate the states of all CUs from the model at time moment t = 1.

The rank R1(□) of the type □ will be the number of types that are the ancestors of □. The induction by R1(□) proves easily that at the time t = 1 they calculate the input parameters of each CU A of the type □, entering the model. At that they use the control functions included in the descriptions of the corresponding types.

The rank R2 (□) of the type □ will be the number of types that are the descendants of □. After the calculation of the input parameters of all the CUs entering the model at the time t = 1, the induction on R2 (□) proves easily that at the time t = 1, they calculate the internal parameters of each CU A of the type □□entering the model. In this case, the conversion functions are used from the descriptions of the corresponding types.

Continuing this process, we will first calculate the values of the input parameters of all CUs entering the model at the instant of time t = 2, and then the values of the internal parameters of all CUs from the model. Similarly, by reasoning, we calculate the values of the input and internal parameters of all CUs from the model at an arbitrary time instant t.

Trajectories of system functioning. Let QA = <q1, ..., qs> be the set of state parameters defined for the control unit A. Let the values of all the parameters qi be known at the initial time moment t = 0.

Let's denote the state of the unit A at the moment t = 0 by QA(0) = <q1 (0),..., qs(0)>, where qi(0) is the value of the parameter qi at the moment t = 0. Similarly, we denote by QA(t) = <q1 (t), ..., qs(t)> the state of the unit A at the moment t. Here qi(t) is the value of the parameter qi at time t.

The trajectory of CU A functioning of type □□□is the sequence QA(0),QA(1),…,QA(j),QA(j+1),… , such that QA(0) is the initial state of CU A, and for each j, the state QA(j + 1) is calculated by the state QA(j) according to the rules of model functioning described above.

The states of the whole system will be identified with the states of the unit I of the constructed model. I.e. the trajectory of the system functioning is the sequence of states of the system QI(0), QI(1), ..., QI(j), QI(j + 1), ..., such that QI(0) is the initial state of the system, and for each j the state of QI(j + 1) is calculated by QI(j) state according to the rules of model operation described above.

The transition ratio and its characteristic function. Let k be the number of parameters of the main type of the model. The transition ratio for CU I is ratio RI (v1, ..., vk, w1, ..., wk) such that for arbitrary sets of the parameters <a1, ..., ak> and <b1, ..., bk> of the unit I RI (a1, ..., ak, b1, ..., bk) is true if and only if there is a trajectory of CU I functioning and the instant t = j, such that QI (j) = <a1, ..., ak>, QI (j + 1) = <b1, ..., bk>.

Obviously, it is difficult to specify the RI relation by an explicit enumeration of the sets on which it is true. Let's show how under certain assumptions we can develop a Boolean characteristic function for this ratio.

Suppose that for all CUs entering the model:

- all state parameters are boolean ones;

- for the control function F(i,X(t),Q(t-1)) from the description of CU type the functions Fi(X(t), Q(t-1))=F(i,X(t),Q(t-1)) are Boolean ones in the sense that each component of the set Xi(t), the input parameters of the i-th secondary unit is computed using the Boolean function.

- all the projections of the function H(X(t),Q(t-1),Q1 (t), ..., Qn(t)) recalculation that compute the components of the set of internal parameters Q(t) of the unit are also Boolean ones.

Then for each CU A of the type □□ by induction on the rank R2(□) we define the following lists of pairwise distinct variables

VXA(t),VPA(t),V*PA(t),V*PA(t-1) and the Boolean function

ΦA(VXA(t),V*PA(t-1),V*PA(t)).

At R2(□)=0 VXA(t) is the list of pairwise distinct Boolean variables of the length m, VPA(t) is the list of pairwise distinct Boolean variables of the length k,

V*PA(t-1) is the list of pairwise distinct Boolean variables of length k, V*PA(t) = VPA(t), and the lists VXA (t), VPA(t) and V*PA(t-1) should not have common variables. The numbers m and k are from the description of the type □□

ΦA(VXA(t),V*PA(t-1),V*PA(t)) = (HA(VXA(t),V*PA(t-1)) ≡ V*PA(t)), where HA is the recalculation function from the description of the type □□

Let R2(□)>0, B1,…, Bn are the secondary units of the unit A for which the corresponding lists and formulas have been developed.

Then VXA (t) is the list of pairwise distinct Boolean variables of the length m, VPA(t) is the list of pairwise distinct Boolean variables of the length k,

VPA (t-1) is the list of pairwise distinct Boolean variables of the length k,

V*PA(t) = VPA(t)#V*PB1(t)#...#V*PBn(t),

V*PA(t-1) = VPA(t-1)#V*PB1(t-1)#...#V*PBn(t-1),

where # is the operation of list concatenation, at that the lists VXA(t), VPA(t),

VPA(t-1),V*PB1(t),...,V*PBn(t),V*PB1(t-1),...,V*PBn(t-1), should not have common elements.

ΦA(VXA(t),V*PA(t-1),V*PA(t)) = ΦB1(FA1(VXA(t),VPA(t-1)),V*PB1(t-1),V*PB1(t))

&…& ΦBn(FAn(VXA(t),VPA(t-1)),V*PBn(t-1),V*PBn(t)) &

(HA(VXA(t),V*PA(t-1),V*PB1(t),…,V*PBn(t)) ≡ V*PA(t)),

where FAi is the control function from the description of the type □□□□ calculating the input parameters of the secondary unit Bi, i=1,…,n.

Let's consider the formula ΦI (V*PI(t-1), V*PI(t)). Let W1 is the list of all variables from V*PI(t-1) that are not in the list of VPI(t-1), W2 is the list of all variables from V*PI(t) that are not included in VPI(t) list. Let's develop the Boolean function ΦRI(VPI(t-1),VPI(t)) = ∃W1 ∃W2 ΦI(V*PI(t-1),V*PI(t)), which is the desired characteristic of Boolean function for the transition ratio RI.

Let's note that if all control and recalculation functions in the model are specified in the form of BDD (Wegener, 2000), then the formula ΦRI(VPI(t-1),VPI(t)) can also be effectively developed as BDD.

3. Results and Discussions

3.1. Problems Solved Using the Hierarchical Models of Systems

The problem of finding stable operation trajectories. Suppose that we need to develop a program for the development of some system S. Let's a set of parameters P = <p1, ..., pk> will be given for the system S, whose values determine the system state. The initial state of the system is described by the set P(0) = <p1(0), ..., pk(0)>. We want to develop a program for S system development, such that the result of its implementation is the system transition into the state that satisfies some target condition G. Besides, during the transition from the initial state to the target state, the system should only be only in the states that satisfy certain admissibility requirements C.

In order to solve this problem, let's develop the hierarchical model M of the system S, the main type of which will correspond to the set of parameters $P = \langle p1, ..., pk \rangle$. Let's note that for the main CU I of the model M, the sets of internal parameters and state parameters coincide, i.e. Q = P. Now our problem is reduced to finding the trajectories of QI(0), QI (1), ..., QI(N) model operation, such that QI(0) is the initial state of the CU I, the state QI(N) satisfies the condition G and for each j, $0<j<N$ the state QI(j) satisfies the requirements of C.

Suppose that the target condition G and the admissibility requirements C can be specified by the boolean functions ΦG and ΦC, respectively. I.e. the boolean function ΦG (p1(t), ..., pk(t)) takes on the value of the truth m, etc., when the state $P(t) = \langle p1 (t), ..., pk (t) \rangle$ satisfies the condition G, and the Boolean function ΦC(p1 (t), ..., pk (t)) takes on the value of the truth m, etc., when the state $P(t) = \langle p1 (t), ..., pk(t) \rangle$ satisfies the requirements C.

The solution of the problem consists in the development of the following Boolean functions.

1) FRI (VPI (t-1), VPI (t)) is the characteristic Boolean function for the transition ratio RI.

2) FRCI (VPI (t-1), VPI (t)) = FFI (VPI (t-1), VPI (t)) & FC (VPI (t-1)) & FC (VPI (t)) is the characteristic Boolean function for the transition ratio RI between the states that satisfy the admissibility requirements.

3) FRC*I(VPI (t-1), VPI (r)) characterizes the transitive closure of the transition ratio RI between states that satisfy the admissibility requirements. The function FRC*I (VPI (t-1), VPI (r)) satisfies the following conditions:

3.1) If FRCI (VPI (t-1), VPI (r)) = 1, then FRC * I (VPI (t-1), VPI (r)) = 1

3.2) If there is a set of values of the variables W, such that

ΦRCI(VPI(t-1),W)=1 и ΦRC*I(W,VPI(r)), then ΦRC*I(VPI(t-1),VPI(r))=1, i.e. ΦRC*I(VPI(t-1),VPI(r)) = ΦRCI(VPI(t-1),VPI(r)) v ∃W(ΦRCI(VPI(t-1), W) &ΦRC*I(W, VPI(r))).

Let's note that if ΦRCI(VPI(t-1),VPI(t)) function is set as BDD, then BDD for the function FRC*I(VPI (t-1), VPI (r)) is also developed effectively [3].

ΦRC^I(VPI(t)) = ΦRC*I(PI(0),VPI(t)) is the characteristic Boolean function for the ratio "PI(t) is the state attainable from the initial state along the trajectories, all of whose intermediate states are admissible."

2) ΦRCGI(VPI(t)) = ΦRC^I(VPI(t)) & ΦG(VPI(t)) is the characteristic Boolean function for the relation "PI(t) - the target state achievable from the initial state along the trajectories, all the intermediate states of which are admissible ones." If this function takes a true value on a certain set of parameters it is meaningful to develop a development program, i.e. from the initial state, the system can go to the target state along a path whose intermediate states are all admissible.

3) ΦRCN*I(VPI(1),…,VPI(N))= ΦRCI(PI(0),VPI(1))&ΦRCI(VPI(1),VPI(2))&…& ΦRCI(VPI(N-1),VPI(N))&ΦG(VPI(N)) - characterizing Boolean function for the set of admissible trajectories of the N-length model functioning ending by the target state.

Any development program is developed for a certain period of time (a certain number of years, months, weeks). Therefore, any executing set for this function corresponds to the "correct" of the system development, in the sense that when it is implemented, the system will move from the initial state to the target in N stages, while the system will be in acceptable states at all intermediate stages.

Target indicators in the "University" system, characterizing the quality of educational programs, are considered in Mikhailov (2015). The target conditions given there are simple in their structure and at a certain coding of indicator values can be written in the form of Boolean formulas.

Let's note that there are often the problems in which the admissibility requirements and target conditions have a more complex structure that can be expressed by the formulas of the temporal logics CTL or LTL.

Let us consider, for example, the problem of finding trajectories for a system stable development. On the sets $Q = \langle q1, ..., qk \rangle$ of the state parameter values of the main CU model, in addition to C admissibility requirements and the target conditions G, the binary preference relation is introduced $<$. If the relation $Q1<Q2$ is satisfied, then we say that the state Q2 is preferable to the state Q1.

The trajectory of QI(0),QI(1), ..., QI(N) operation is

a stable development trajectory of the system if there is an infinite sequence of time instants $t1<t2<... <tj<...$ such that for all j less than some number N, C(Q(tj)) and $Q(tj)<Q(tj + 1)$ conditions are satisfied, and for all j greater than or equal to N, C(Q(tj)) and G(Q(tj)) is fulfilled.

In this task, the admissibility requirements and target conditions are written in the form of LTL logic formulas. This problem is solved by the translation of the constructed hierarchical model of the system S into the program written by Promela language and by the use of software verifier Spin (Checker, 2003).

4. Conclusions

The above-described approach to the development of proper system development programs was successfully applied in the development of the Competitiveness Enhancement Program of the Kazan Federal University and the development programs for the territories of several municipalities in Cuba (Vega, 2018). This makes it possible to draw the conclusion about the effectiveness of the Model Checking approach with the proposed means of system model description during the study of the prospects for the development of complex socio-economic systems.

5. Summary

The problem of finding trajectories for the stable functioning of specific socio-economic systems requires the development of their adequate models, since the carrying out of experiments on such systems is not possible in most cases. The proposed class of system models, for which the problem of stable functioning is solved quite effectively, is a convenient tool to study the behavior of various types of socio-economic systems.

Acknowledgements

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

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