

Time-Varying Copula Modelling Between Malaysia and Major Stock Markets

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Abstract

Studies on dependence between stock markets are important because of their implications on the process of decision-making in investment. Many previous studies measure the dependence between markets using static copula. However, in recent years, time-varying copula has been used as an alternative for measuring dependence due to its capability of capturing time-varying dependence between markets. This study uses both static and time-varying copulas to measure the dependence structure between Malaysia and major stock markets (US, UK and Japan) based on the sample data from year 2007 Q1 until year 2017 Q3. The results reveal that the best model for all pairs of indices is the time-varying SJC copula, which also reveals that the Malaysia-US pair has the weakest dependence structure compared to other pairs. In terms of lower and upper tails, the Malaysia-UK and the Malaysia-Japan pairs have the strongest dependence structure respectively. Evidence from this research suggests that diversifications involving Malaysia and US stock markets are not effective.

Keywords: Time-varying copula; Dependence; Malaysia stock market; Diversifications.



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1. Introduction

Copula theory has garnered a lot of interest in the recent few decades. Copula is known as a powerful tool to measure dependency due to several main advantages; copulas are invariant to transformation of data, copulas able to deal with non-linear asymmetric and symmetric distributions, copulas are more flexible to measure co-movements between bivariate data, and copulas allow both models (marginal distributions and dependence structure) to be modelled separately.

There are many previous studies that has used static copula to measure the overall dependence and tail dependence between markets (Messoud and Aloui, 2015; Razak *et al.*, 2016; Salma, 2015; Shamiri *et al.*, 2011). In recent years, however, several researchers initiated the use of dynamic copula to measure time-varying dependency. Time-varying copula is an approach which allows variability in the dependence structure and assumes that the parameter of dependency evolves linearly (Boubaker and Sghaier, 2016). There are several studies on dependence structure between international markets that has been carried out based on time-varying copula. Such studies can be found in Aussenegg and Cech (2008) who investigated the co-movements between Eurostoxx Index and Dow Jones Industrial Index using dynamic Gaussian and student-*t* copula, Kara and Kemaloglu (2016) who found that the USD-EUR pair is best fitted with dynamic tDCC copula while the best model for static copula is Gaussian copula, and Hussain and Li (2017) who revealed that the Chinese stock markets and developed markets (US, Canada, UK, Germany, Japan, and Australia) have low dependencies.

In terms of Malaysia markets, several studies which focused on dependencies between Malaysia and other stock markets can also be found. As examples, Shamiri *et al.* (2011) investigated the dependence structure between Malaysia market (KLCI) and foreign indices (STI and S&P500), Razak and Ismail (2014) explored the interdependence between Malaysia stock markets and found that the KLCI-EMAS pair has strong dependence at upper and lower tails, and Thongkamhong *et al.* (2017) used the Markov switching copula model to measure co-movements between commodities, stocks, and bonds in Malaysia. Generally, we found that most financial studies on Malaysia stock market measure the dependence structure using static copula. To our knowledge, previous studies on dynamic dependence structure based on time-varying copula for the Malaysia stock markets are hardly available.

This paper measures the dependencies between FBMKLCI Index (Malaysia) and other major indices, which are Dow Jones Industrial Average Index (US), FTSE100 (UK) and Nikkei225 (Japan), using both static and time-varying copulas, covering the period of 2 January 2007 until 30 Jun 2017. Our results offer several insights regarding international diversification prospects for the Malaysia stock markets, especially for interested researchers and investors.

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2. Methodology

The estimation of dependence measure (copula) in our study is based on the inference functions of margins (IFM) approach which is a fully parametric method proposed in [Joe \(1997\)](#). The two main steps involved in this approach are the estimation of marginal distribution and the estimation of copula parameter. Firstly, the best volatility model for each series is chosen after considering several ARMA-EGARCH models with four error distributions (normal, skewed normal, student- t and skewed student- t). Secondly, the step involves extracting the standardized residuals from the best marginal model of each paired indices, converting the standardized residuals into pseudo observations using $(u_i, v_i) = [(\text{Rank } O_{1i} / n_{1i} + 1), (\text{Rank } O_{2i} / n_{2i} + 1)]$. Finally, the pseudo observations are used to estimate the copula parameters.

2.1. Marginal Distribution Estimation

In this study, we consider EGARCH models to measure the volatility of univariate data (indices). The main reason for using EGARCH model is that the asymmetric response of variance to shocks can be captured, and the conditional variance is always positive since it uses log of variances. The general EGARCH (p, q) model can be expressed as [\(Nelson, 1991\)](#):

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^p \beta_j \log(\sigma_{t-j}^2) + \sum_{j=1}^q \alpha_j \left| \frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right| + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \quad (1)$$

where σ_t is the conditional variance, ω is a constant, β measures the persistence in conditional volatility, and α indicates the symmetric effect. It should be noted that γ_k represents the leverage effect and commonly has negative value, which indicates that bad news generates higher volatility than positive innovations, and vice versa. The model is asymmetric if $\gamma_k \neq 0$ [\(Ugurlu et al., 2014\)](#).

2.2. Dependence Estimation

In this paper, the dependency between the Malaysia index (FBMKLCI) and other major indices (DJI, FTSE100 and Nikkei225) are modelled using both static and time-varying copulas.

2.2.1. Gaussian Copula

The Gaussian copula has zero tail dependence and can be categorized in the elliptical copula families. The Gaussian copula can be written as:

$$C(u, v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\delta^2}} \exp\left\{-\frac{x^2 - 2\delta xy + y^2}{2(1-\delta^2)}\right\} dx dy \quad (2)$$

$$C = \Phi_{\delta}(\Phi^{-1}(u), \Phi^{-1}(v)), -1 \leq \delta \leq 1 \quad (3)$$

where δ is the linear correlation coefficient, and $\Phi^{-1}(\cdot)$ is the inverse of standard normal distribution.

Under the time-varying copula, the dependence parameter for Gaussian copula can be defined as [Patton \(2006\)](#):

$$\delta_t = \lambda \left(\omega + \beta \delta_{t-1} + \alpha \frac{1}{10} \sum_{j=1}^{10} [\Phi^{-1}(u_{t-j}) \Phi^{-1}(v_{t-j})] \right) \quad (4)$$

where $\lambda = (1 - e^{-x})(1 + e^{-x})^{-1}$ is the modified logistic transformation that keeps the estimates of δ_t in $(-1, 1)$ interval. From equation (4), the time-varying copula parameter, δ_t , follows an ARMA (1,10) type process, the autoregressive term, $\beta \delta_{t-1}$, captures the persistence effect, and the mean of product of the last 10 observations of transformed variables ($\Phi^{-1}(u_{t-j})$ and $\Phi^{-1}(v_{t-j})$) captures the variation effects in the dependence.

2.2.2. Symmetrised Joe-Clayton Copula (SJC Copula)

The SJC copula is a modification of the BB7 copula. It allows simultaneous exploration of upper (τ^U) and lower (τ^L) tail dependences, and its function is given by [Patton \(2006\)](#):

$$C_{SJC}(u, v | \tau^U, \tau^L) = 0.5[C_{JC}(u, v | \tau^U, \tau^L) + C_{JC}(1-u, 1-v | \tau^U, \tau^L) + u + v - 1] \quad (5)$$

To allow for dynamic dependency, both lower and upper tails of the SJC copula can be expressed as:

$$\tau^{U/L} = \tilde{\lambda} \left(\omega^{U/L} + \beta^{U/L} \tau_{t-1} + \alpha^{U/L} \frac{1}{10} \sum_{i=1}^{10} |u_{1,t-i} - u_{2,t-i}| \right) \quad (6)$$

where $\tilde{\lambda}$ is the logistic transformation defined as $\tilde{\lambda} = \left((1 + e^{-x})^{-1} \right)$, which ensures that the estimates of upper/lower dependence parameters are within interval (0,1) at all times.

3. Data and Preliminary Analysis

3.1. Data Description

The Malaysia stock market is represented by the FTSE Bursa Malaysia Kuala Lumpur Composite Index (KLCI). The other three stock indices considered in this study are the DJI (US), the FTSE100 (UK) and the Nikkei225 (Japan) which represent several market locations around the world, which are North America (US), Europe (UK) and Asia Pacific (Japan). The USD currency is used for all indices to avoid issues related to exchange rate risks, and to have homogenous datasets (Shahzad *et al.*, 2017). The data sample consists of 2725 observations of daily closing price index, covering from 2 January 2007 until 30 June 2017. It should be highlighted that the study period covers several recent financial events such as Global Financial crisis (2007-2009), 2008's Lehman Brothers collapse, European Sovereign Debt Crisis in 2009 until 2012, and 2014's Russian Ruble crisis. Due to non-stationary

issues, the price index series are transformed into return series using $R_t = \log P_t - \log P_{t-1}$. All data sets are sourced from *Bloomberg Terminal*.

3.2. Preliminary Analysis

Table 1 provides several statistical measures (mean, median, standard deviation (SD), minimum value, maximum value, skewness and kurtosis) and statistical tests (Jarque-Bera (JB) test and Augmented Dickey-Fuller (ADF) test) for all series.

Table-1. Summary statistics for return series

Stock	Malaysia	US	UK	Japan
Mean	-0.0001	-0.0002	9.90E-05	-7.68E-05
Median	0.0000	-0.0003	-3.58E-04	0.0000
SD	0.0101	0.0117	0.0150	0.0148
Minimum	-0.0537	-0.1051	-1.30E-01	-1.07E-01
Maximum	0.1098	0.0820	1.06E-01	1.13E-01
Skewness	0.5163	0.0984	0.1546	0.4334
Kurtosis	10.6217	13.7267	12.8874	9.3157
JB Test	6714.2000**	13064.0000*	11107.0000*	4612.6000**
ADF Test (p-value)	0.0100	0.0100	0.0100	0.0100

**The values are significant at 1% level

The UK series has the highest mean and the lowest median, while the Malaysia, US and Japan series have average negative returns. In terms of volatility, the UK index is more volatile than other indices as it has the largest standard deviation which is 0.0150. This result is also supported by the largest difference between the minimum and maximum values of the UK return series.

The distribution of return series can be explained by referring to the JB test, skewness and kurtosis. All series have positive skewness, which indicates that all indices have more losses and a few extreme gains (or positive returns). The kurtosis for all series are greater than three, which imply that the distributions are leptokurtic. For JB test statistics, all series show significant p-value at 1% level, which clearly prove that the distributions of all series are not normal. By referring to the ADF test, we find that all indices are stationary since the findings show that the p-value are 0.0100 for all series.

4. Results

4.1. Marginal Models

The volatility of stock markets can be measured by estimating both the conditional mean (ARMA(p, q)) and the conditional variance (EGARCH(P, Q)). We choose lag P and lag q by referring to the autocorrelation function (ACF) and the partial autocorrelation function (PACF), and select the appropriate combination of ARMA(p, q) by referring to the Akaike Information Criterion (AIC). The best ARMA-EGARCH model for each series is then chosen based on the significance of parameter estimates, the lowest information criterion which are AIC, BIC (Bayesian Information Criterion), and the diagnostic tests which are Ljung-Box test on residuals (Q(10)), Ljung-Box

test on squared residuals ($Q^2(10)$) and Langrange Multiplier (LM) test. Summary results for marginal models are not shown here but can be obtained from the authors upon request. Our findings found that the AR(1)-EGARCH(1,1)-skewed-student- t , the EGARCH(1,1)-skewed-student- t , the MA(1)-EGARCH(1,1)-skewed-student- t , and AR(1)-EGARCH(1,1)-student- t provide the best fit for the US, UK, Japan and Malaysia returns series respectively.

4.2. Copula Models

We consider two types of copula families (Gaussian and SJC copula). Apart from estimating the parameters of static copulas, we also estimate the dynamic versions of both copulas. The static copula captures the overall dependence and extreme (or tail) dependence of paired indices, while the dynamic copula captures the time-varying dependence of paired indices.

Table 2 summarizes the parameter estimates under static copula, together with AIC and BIC. The Gaussian copula describes only the overall dependence of bivariate returns, and fails to capture the tail dependence. On the other hand, the SJC copula is able to capture both upper and lower tail dependences of the paired returns. The estimates for Gaussian copula are positive but small, implying that the Malaysia's index correlates positively, but weakly, with the US, UK and Japan indices. The Malaysia-US pair has the weakest overall dependence, which indicates that the Malaysia market is less dependent on the US market compared to the UK and Japan markets.

Table-2. Parameter estimates for static copulas

	Malaysia-US	Malaysia-UK	Malaysia-Japan
Gaussian Copula			
δ	0.1522	0.3733	0.3732
AIC	-63.8562	-408.6839	-408.4944
BIC	-63.8541	-408.6818	-408.4922
SJC Copula			
τ^U	0.0342	0.2244	0.2546
τ^L	0.0175	0.1509	0.0988
AIC	-68.8177	-420.5480	-412.0027
BIC	-68.8134	-420.5437	-411.9983

The estimates of upper tail parameter for SJC copula are slightly higher than the lower tail for all pairs, suggesting the existence of dependency among extreme gains during market blooming. Specifically, the Malaysia-Japan pair has the greatest value of upper tail dependence, which suggest that both indices sometimes reacts similarly during bull market. The Malaysia-UK pair has the highest lower tail dependence compared to other pairs, which indicates that there is some association between extreme losses of Malaysia and UK indices occurring at both periods of crises and bear markets. By referring to the AIC and BIC, the SJC copula has smaller information criterion values and is more suitable to represent the static dependence between Malaysia and other stock markets for years 2007-2017.

The parameter estimates under time-varying copula are shown in Table 3. The results show that dynamic copulas have smaller information criterion compared to static copulas, and the SJC time-varying copula is a better model for modelling the dependence of the bivariate returns. More importantly, this result proves that the dependencies between Malaysia and major stock markets varies across time.

Table-3. Parameter estimates for time-varying copulas

	Malaysia-US	Malaysia-UK	Malaysia-Japan
Time-varying Gaussian Copula			
ω	0.0021	-0.0223	-0.0193
α	0.0264	0.0351	0.0337
β	1.9858	2.1326	2.1265
AIC	-77.9720	-416.7249	-418.2261
BIC	-77.9655	-416.7184	-418.2195
Time-varying SJC Copula			
ω^U	-0.7947	-2.0076	0.9785
α^U	-8.7358	-0.8558	-3.5045
β^U	-1.5448	4.3436	-4.7023
ω^L	-0.5487	-2.0132	-1.0078
α^L	-13.2940	-1.8519	-5.5517
β^L	5.3816	4.8065	1.8738
AIC	-78.2684	-428.8412	-418.4507
BIC	-78.2554	-428.8282	-418.4377

The persistence coefficients, β , are greater than the variation coefficients, α , for all pairs. The values of ω^U are higher than ω^L for all pairs, except for the Malaysia-US pair, which indicates that the dependence between Malaysia and both series (UK and Japan) are stronger during booming period than crisis period. This finding also signifies that diversification between Malaysia and US stock markets is not an effective strategy.

In terms of dynamic dependence, Figures 1-3 illustrate the time path for the three copula parameters: δ_t (Gaussian), τ^u (SJC upper tail), and τ^L (SJC lower tail). For the Malaysia-US pair, the estimates of dependence parameters (δ_t) are between -0.09 and 0.36, the estimates of upper tail dependence (τ^u) range between 0.01 and 0.14, and the estimates of lower tail dependence (τ^L) range between 0.01 and 0.76. The time-varying Gaussian copula in Figure 1 shows that the dependency of the Malaysia-US return series is weak. However, both indices suffered from extreme losses during the financial global crisis, and this situation is reflected by an obvious spike observed in the same figure.

Based on Figure 2, the Malaysia-UK pair has the highest estimates of δ_t , ranging between 0.25 and 0.53. We can also see that the estimates for upper tail dependence, τ^u , range between 0.15 and 0.37, whereas the estimates for lower tail dependence, τ^L , range between 0.08 and 0.38.

Figure 3 revealed that estimates of δ_t for the Malaysia-Japan pair are between 0.16 and 0.49, and the estimates for lower tail dependence, τ^L , move between 0.02 and 0.29. Surprisingly, the estimates for the upper tail dependence, τ^u , is similar to the Malaysia-UK pair which range between 0.15 and 0.36.

Fig-1. Graph path for time-varying copula dependency (Malaysia-US)

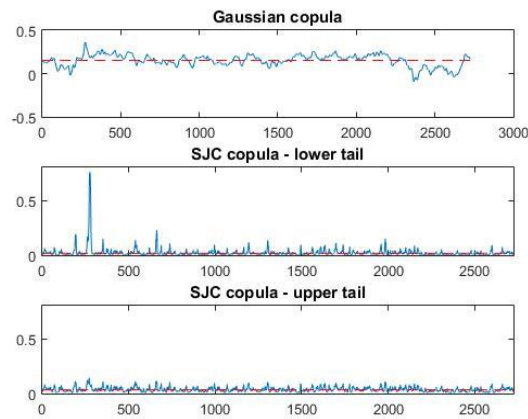


Fig-2. Graph path for time-varying copula dependency (Malaysia-UK)

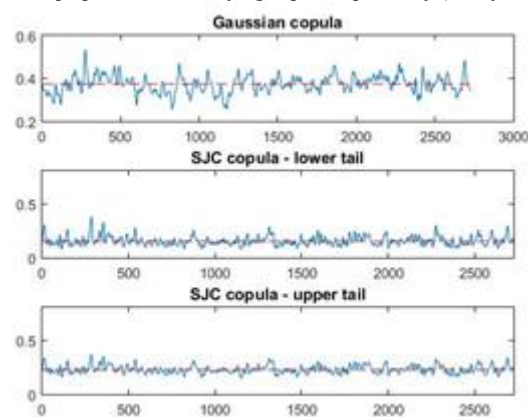
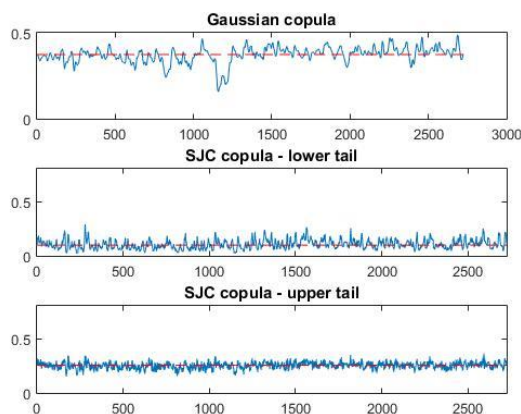


Fig-3. Graph path for time-varying copula dependency (Malaysia-Japan)

5. Conclusion

This paper has examined the dependence structures between Malaysia index (KLCI) and other world indices (DJI, FTSE100 and Nikkei255) using the ARMA-EGARCH model for the marginals, and both copulas (static and time-varying) model for the dependence. Our findings show evidence that the returns series in 2007-2017 have non-normal distribution; they are positively skewed and leptokurtic. We found that the AR(1)-EGARCH(1,1)-skewed-student- t , the EGARCH(1,1)-skewed-student- t , the MA(1)-EGARCH(1,1)-skewed-student- t , and the AR(1)-EGARCH(1,1)-student- t provide the best fit for the US, UK, Japan, and Malaysia returns series, respectively, and thus, these models are used for marginal distributions. We also found that the Malaysia-US pair has the weakest dependence, and this finding is consistent with other related studies from [Shamiri et al. \(2011\)](#) and [Razak and Ismail \(2016\)](#). On the other hand, the Malaysia-UK and the Malaysia-Japan pairs have strong relationship. Based on time-varying copula, we discovered that the estimates of ω^U are higher than ω^L for the Malaysia-UK and the Malaysia-Japan pairs, which suggests that the dependence between both pairs are stronger during booming period compared to crisis period. Finally, our study reveals that the SJC time-varying copula best described the time-varying dependence of the paired indices. Since our scope of study covers international markets and the recent 10-year-period (2007-2017), the time-varying dependence is more appropriate than the static dependence due to several reasons; frequent changes in business cycles, occurrences of major economic events, and incidents of financial disasters ([Hammoudeh et al., 2014](#)).

For future studies, other copula families can be considered to provide better information concerning the co-movements between Malaysia and other stock market indices. We also plan to use tick-by-tick data and high-dimensional copula models to obtain more evidence regarding the dependence structures between Malaysia and other international markets.

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