**Abstract:** This research paper aims at investigating disturbance rejection associated with a highly oscillating second-order process. The PD-PI controller having three parameters are tuned to provide efficient rejection of a step input disturbance input. Controller tuning based on using MATLAB control and optimization toolboxes. Using the suggested tuning technique, it is possible to reduce the maximum time response of the closed loop control system to as low as 0.0095 and obtain time response to the disturbance input having zero settling time. The effect of the proportional gain of the PD-PI controller on the control system dynamics is investigated for a gain ≤ 100. The performance of the control system during disturbance rejection using the PD-PI controller is compared with that using a second-order compensator. The PD-PI controller is superior in dealing with the disturbance rejection associated with the highly oscillating second-order process.

**Keywords:** Disturbance rejection; Highly oscillating second-order process; PD-PI controller; Controller tuning; Control system performance.

1. Introduction

This is the second paper in a series of research papers investigating disturbance rejection associated with a highly oscillating second-order-like process. The first paper was about using a feedforward second-order compensator for this purpose. Here, is the second paper investigating using the PD-PI controller for the same purpose.

Skogestad [1] used modified integral term of the PID controller to improve disturbance rejection associated with integrating processes. Skogestad [2] used a single tuning rule for first order and second order time delay models. Sorensen, et al. [3] developed a combined feedback and input shaping controller to address sources of oscillation by motion of the bridge or crane or from environmental disturbances. They applied the developed controller on a 10 ton bridge crane of the Georgia Institute of Technology.

Jain and Nigram [4] explored the idea of model generation and optimization for a PD-PI controller. They obtained promising results when using the PD-PI controller with the highly nonlinear inverted pendulum. Matsu, Matusu, et al. [5] compared three different control designs for SISO system with harmonically time-varying delay. They compared the three methods through simulation example for both set point tracking and disturbance inputs. Jujuly [6] developed a unified framework for the internal model control (IMC) based on PI/PID controller design and analysis. He developed a generalized 2DOF IMC-PI/PID controller design methods for first order, second order and other processes without time delay and compared with other existing methods. Asad, et al. [7] proposed a fuzzy PD-based control strategy to transfer loads using overhead cranes. They presented a comparative analysis of fuzzy PD and classical PI controllers.

Rajnikanth and Lathe [8] proposed a bacterial foraging optimization algorithm based approach to tune an IMC-PID controller for a class of first order plus time delayed unstable systems. They confirmed the efficiency of their tuning procedure through a comparison with other algorithms such as particle swarm optimization and ant colony optimization. They obtained robust performance in reference tracking with perturbed model parameters. Herbst [9] carried out a simulative study using generic first and second order plants for quick virtual assessment of the abilities of disturbance rejection control. He concluded that active disturbance rejection control can be considered as a strong alternative for solving practical control problems. Agarwal [10] proposed tuning rules for PI and PID controllers for unstable first order plus dead time processes. His tuning method is based on the satisfaction of gain and phase margin specifications.

Hassaan [11] examined using a PD-PI controller in controlling first order delayed processes. The results showed better performance when compared with PID controller with two different tuning techniques for set-point tracking. Hassaan [12] investigated using a PD-PI controller for disturbance rejection associated with delayed double integrating processes. He showed that the PD-PI controller is superior compared with other disturbance rejection technique based on PID controller for the same process. Hassaan [13] investigated the possibility of using a 2DOF controller.
controller in disturbance rejection associated with delayed double integrating processes. He showed that the 2DOF controller was able to compete with a PID plus first order lag controller, but it could not compete with I-PD and PD-PI controllers. Shin [14] presented PID controller with disturbance rejection, low sensitivity and notch filter against bending frequency by the disturbances. They certified the performance of the designed system by simulation and experimentally indicating the improvement of system performance in cases of existence of external disturbances.

2. Process
The controlled process is second-order-like process having the transfer function, \( G_p(s) \):
\[
G_p(s) = \frac{(\omega_n^2)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}
\]
(1)
Where:
\( \omega_n \) = process natural frequency = 10 rad/s
\( \zeta \) = process damping ratio = 0.05
This level of damping ratio generates a dynamic system having a highly oscillating characteristics when subjected to a step input. This highly oscillating nature is characterized by an 85.4 % damping ratio.

3. Controller
The controller used is a PD-PI controller having the structure shown in Fig.1 [4].

The block diagram of the control system of Fig.1 has two inputs: reference input \( R(s) \) and disturbance input \( D(s) \). The PD-PI controller has two feedforward control loops. The first loop incorporates the derivative gain \( K_d \), while the second loop incorporates the proportional gain \( K_{pc} \) and the integral gain \( K_i \).

The transfer function of the PD-PI controller using the 2 loops in Fig.1, \( G_c(s) \) is given by Hassaan [11]:
\[
G_c(s) = \frac{K_{pc}K_d s^2 + (K_{pc} + K_i K_d) s + K_i}{s}
\]
(2)

4. Control System Transfer Function
For the purpose of disturbance rejection, the reference input, \( R(s) \) is omitted from Fig.1 and the disturbance input \( D(s) \) is considered the control system input and \( C(s) \) is its output. The closed-loop transfer function of the closed-loop control systems with disturbance input is obtained from the system block diagram of Fig.1 with \( R(s) =0 \) as:
\[
\frac{C(s)}{D(s)} = \frac{b_0 s}{[s^3 + a_0 s^2 + a_1 s + a_2]}
\]
(3)
Where:
\[
\begin{align*}
b_0 &= \omega_n^2 \\
a_0 &= 2\zeta\omega_n + K_{pc}K_d\omega_n^2 \\
a_1 &= \omega_n^2 + \zeta\omega_n(K_{pc} + K_i K_d) \\
a_2 &= K_i \omega_n^2
\end{align*}
\]

5. Controller Tuning and System Time Response
The controller has three parameters: \( K_{pc} \), \( K_i \), and \( K_d \). The controller parameters are tuned as follows:
- The control and optimization toolboxes of MATLAB is used to assign the three parameters of the controller \( (K_{pc} \), \( K_i \), and \( K_d \) [15], [16].
- The MATLAB command ‘fminunc’ is used [16].
- A number of objective functions based on the error between the step time response of the control system for a unit disturbance input and its zero desired value are selected to tune the controller. They are ITAE, ISE, IAE, ITSE and ISTSE [17-19].
- The step response of the closed-loop control system is plotted using the command ‘step’ of MATLAB [15].
- The time-based specifications of the control system are extracted using the MATLAB command ‘stepinfo’ [15].
A sample of the tuning results is shown in Table 1 for the highly oscillating second-order-like process.

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>ITAE</th>
<th>ISE</th>
<th>IAE</th>
<th>ITSE</th>
<th>ISTSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_{pc}</td>
<td>9.9369</td>
<td>8.9705</td>
<td>8.9750</td>
<td>9.9361</td>
<td>9.9377</td>
</tr>
<tr>
<td>K_{i}</td>
<td>1.6175</td>
<td>0.4974</td>
<td>0.4949</td>
<td>1.6190</td>
<td>1.6176</td>
</tr>
<tr>
<td>K_{d}</td>
<td>0.0457</td>
<td>0.0164</td>
<td>0.0020</td>
<td>0.0458</td>
<td>0.0457</td>
</tr>
<tr>
<td>c_{max}</td>
<td>0.095</td>
<td>0.145</td>
<td>0.188</td>
<td>0.095</td>
<td>0.095</td>
</tr>
<tr>
<td>T_{cmax} (s)</td>
<td>0.12</td>
<td>0.10</td>
<td>0.09</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>T_s (s)</td>
<td>4.2</td>
<td>14.0</td>
<td>14.0</td>
<td>4.2</td>
<td>4.2</td>
</tr>
</tbody>
</table>

The effect of the five objective functions on the tuning process is investigated by their effect of the time response of the control system during the disturbance rejection process. The time response of the control system for a unit step disturbance input is shown in Fig. 2.

The IAE objective function is not suitable at all for this application, while the other four objective functions generate almost the same time response to the unit step disturbance input.
Because of the nonlinearity of the optimization problem, each guessed proportional parameter K_{pc} generates a local minimum for the objective function (ITAE) and hence a set of tuned PD-PI controller parameters. The effect of the proportional gain on the dynamics of the control system during the disturbance rejection process is shown in Fig. 3 for a proportional gain in the range 5.005 ≤ K_{pc} ≤ 29.697.
The effect of higher levels of $K_{pc}$ on the disturbance time response during the process of disturbance rejection is shown in Fig. 4 for response time up 2 s.

The effect of the proportional gain of the PD-PI controller on some of the time-based specifications of the control system is shown in Figs. 4 and 5 using the ITAE objective function.
Both the maximum time response and the settling time of the disturbance time response decreases as the proportional gain of the PD-PI controller increases.

6. Comparison with other Research Work

The first compensator investigated for disturbance rejection associated with this highly oscillating second-order-like process was the second-order compensator proposed by the author and used successfully in set point input tracking associated with the highly oscillating second-order-like process [20]. The same compensator was tried for disturbance rejection by the author, but it did not give satisfactory results [21]. The comparison of the time response of the control system for a unit disturbance input using the second-order compensator and the present PD-PI controller is shown graphically in Fig. 7.

The time based specifications of the two time responses in Fig. 7 are compared in Table 2.

### Table 2: Comparison of control system specifications

<table>
<thead>
<tr>
<th>Controller / compensator</th>
<th>$e_{\text{max}}$</th>
<th>$T_{\text{cmax}}$ (s)</th>
<th>$T_s$ (s)</th>
<th>$e_{\text{ss}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second-order compensator</td>
<td>1.739</td>
<td>1.20</td>
<td>7.3</td>
<td>0.047</td>
</tr>
<tr>
<td>Present PD-PI controller</td>
<td>0.095</td>
<td>0.120</td>
<td>4.2</td>
<td>0</td>
</tr>
</tbody>
</table>

7. Conclusion

- The dynamic problem of disturbance rejection associated with highly oscillating second-order-like process was investigated using a PD-PI controller.
- The PD-PI controller was tuned using control and optimization toolboxes with five different error-based objective functions.
- The IAE objective function did not give satisfactory performance.
- The ITAE, ISE, ITSE and IISTSE objective functions have given almost the same time response to a unit disturbance input.
- The effect of using different levels of guessed proportional gain of the PD-PI controller was investigated for the range $5 \leq K_{pc} \leq 100$.
- The proposed PD-PI controller was completely successful in rejecting the disturbance associated with the highly oscillating second-order-like process.
- The PD-PI controller could generate a time response without any oscillation around the steady-state value and succeeded to reduce the steady-state error to zero.
- It was possible to reduce the maximum time response of the control system in response to a unit step disturbance input to less than 0.01.
- It was possible with the PD-PI controller to reduce the settling time of the time response to zero.
- In comparison with the previously investigated second-order compensator, the PD-PI controller was superior in rejecting the load disturbance associated with the highly oscillating second-order process.
References


