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## Critical Overview of Some Pumping Test Analysis Equations

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**Abstract:** Possible methods of providing further (and perhaps better) alternative solutions for the exponential integral of aquifer parameter evaluation are investigated. Three known mathematical methods of approach (comprising self-similar, separable variable and travelling wave) are applied, providing three relevant solutions. Further analysis of the self-similar solution reveals that this provides an alternative solution involving normal graph of drawdown versus the measurement intervals. The geomathematical relevance of this method is assessed using data from aquifers from two chronologically different hydrogeological units – the Ajalli Sandstone and Ogwashi-Asaba Formation. The results indicate good functional relationship with satisfactory transmissivity values.

**Keywords:** Pumping test; Groundwater flow; Aquifer; Hydrogeology; Anambra Basin; Nigeria.

### 1. Introduction

The available techniques commonly employed in data acquisition for mathematical and statistical evaluation of aquifer hydrologic parameters fall into two broad groups. These are the pumping test and the grain size methods. Acquisition of pumping test data with normal field layout is usually very costly (in terms of labour, money and equipment) and therefore not commonly done in a depressed economy for mere data acquisition. Most of the available pumping test data in Nigeria, for example, have been recorded by water drilling companies and government establishments during groundwater resources development operations.

In most parts of Nigeria, step drawdown test consisting of three steps of about 1440 minutes each is often done. Problems commonly result during analysis (especially where conventional equations are employed) because of the very few steps of relatively long duration and some potential inherent errors in the analytical technique. To avert some of these problems, constant rate analytical procedure is often employed. Results from this procedure have been considerably satisfactory, more so where each pumping rate is maintained for not less than 24 hours. Recovery test results have also been useful, especially in relation to the validity of one-way test.

Fundamental input into pumping test analysis could be traced to [Darcy \[1\]](#) classical laboratory hydrologic studies. Subsequent contributions by [Thiem \[2\]](#), [Theis \[3\]](#), [Jacob \[4\]](#), [Cooper and Jacob \[5\]](#), [Kruseman and de Ridder \[6\]](#), [Walton \[7\]](#) and [Kehinde and Loenhert \[8\]](#) have, in addition to increasing the number of available analytical procedures, also progressively improved their geomathematical relevance. Development principles and functional properties of some of the available equations have been reviewed in relation to pre-existing boundary conditions and underlying assumptions. We make further attempts at alleviating the potential analytical problems and increasing the validity of associated results here. [Theis \[3\]](#) and its follow-up [\[5\]](#) equations are considered here.

The pioneer solution provided for the exponential integral [\[3\]](#) is the curve matching. Though the exponential integral is geomathematically functional, obtaining working data by mere matching of the field and theoretical curves may be risky, since the inherent error would depend on the geohydrologic differences between the considered environment and that under which the theoretical data were obtained. The result so obtained could be potentially prone to errors (often large but difficult to detect), and thus risky in design operations. The errors involved are operationally fundamental, and therefore not limited to manual evaluation. Thus, the need for alternative or improved analytical procedures and solutions cannot be neglected.

### 2. Previous Investigations

Geoenvironmental variation of relevance in the basic radial flow assumption [\[3\]](#) has been considered in the major escarpment and structurally deformed regions of south-eastern Nigeria [\[9\]](#). Aquifer horizontality constitutes a major fundamental assumption in pumping test method of aquifer property evaluation, providing radial flow in the

test well. Thus, for an aquifer dipping at an angle, both the flow direction and hydraulic conductivity ( $K$ ) are influenced by the value of *dip amount*. The horizontal component ( $K_x = K\cos\theta$ ) is thus required to satisfy the radial flow principle. The geometrical significance of the correction factor ( $\cos\theta$ ) in a dipping confined sandstone aquifer is shown in figure 1.

### 3. Present Study

Three different methods of approach (comprising self-similar, separable variable and travelling wave methods) are applied in providing further solutions for the Theis (1935) exponential integral. Further analysis is limited to the self-similar solution, though similar analysis could be extended to others. Application of the analytical results as a geomathematical function is assessed by analyzing pumping test data from geo-chronologically and hydrogeologically different aquifer units. Three-step tests of twenty-four hour in duration each was obtained for each location, comprising Obinofia Ndiuno and Akpugo (Nkanu) in Ajalli Sandstone, and Nnewi in Ogwashi-Asaba Formation. Each step in the tress-step test is treated as a partially completed constant rate test.

#### 3.1. The Exponential Equation [3]

Theis's method is based on heat flow analogy of transient two-dimensional flow and its radial flow transformation under specified conditions:

$$h(r,0) = h_0, \text{ for all } r \tag{1}$$

$$h(\infty,t) = h_0, \text{ for all } t \tag{2}$$

$$\lim_{r \rightarrow 0} r \frac{\partial h}{\partial r} = \frac{Q}{2\pi T}, \text{ for } t > 0 \tag{3}$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t} \tag{4}$$

$$r = \sqrt{x^2 + y^2} \tag{5}$$

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \tag{6}$$

Where  $dh$  is head loss,  $x$  and  $y$  are coordinate axes (L),  $S$  is storativity,  $T$  is transmissivity ( $L^2T^{-1}$ ),  $t$  is time interval (T) and  $h_0$  is the original head before pumping (L).

Theis (1935) presented his equation in terms of drawdown in exponential integral as:

$$h_0 - h(r,t) = \frac{Q}{4\pi T} W(u) \tag{7}$$

where

$$u = \frac{r^2 S}{4Tt} \tag{8}$$

and

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du \tag{9}$$

is known as the well function. Rearranging (8) yields

$$t = \left(\frac{r^2 S}{4T}\right) \frac{1}{u} \tag{10}$$

This solution is based on the assumed relationship between  $h_0 - h$  and  $t$ ,  $W(u)$  and  $\frac{1}{u}$  due to constant term relationships (see equations (7) and (10)). He thus proposed curve-matching by superposition between log-log plot of  $W(u)$  versus  $\frac{1}{u}$  (type curve); and  $h_0 - h$  (change in drawdown) versus  $t$  (time) on a similar log-log scale.

Corresponding values of  $W(u)$  and  $\frac{1}{u}$ , and  $h_0 - h$  and  $t$  are obtained at match positions. If the type curve is matched with the curve

$$\frac{r^2}{t} = \left(\frac{4T}{S}\right)u,$$

the values  $W(u)$  and  $u$ , and  $h_0 - h$  and  $\frac{r^2}{t}$  are obtained at match points. Aquifer transmissivity ( $T$ ) and storativity ( $S$ ) are then evaluated using these values (along with hydraulic conductivity  $K$  if the aquifer thickness ( $b$ ) is known).

**Remark:** This solution assumes that:-

- A homogenous isotropic aquifer of uniform thickness and infinite areal extent is being considered
- Aquifer is confined and the well penetrates the entire aquifer
- Piezometric surface was initially horizontal or nearly so before pumping began, and
- Flow into the well is radial and steady or transient.

#### 3.2. Analytical relevance of Theis Equation

From equation (1) to (6) and the transformation

$$r^2 = x^2 + y^2 \tag{11}$$

$$h(r, t) \equiv h(x, y, t) \tag{12}$$

we have the following

$$\frac{\partial r}{\partial x} = \frac{x}{r}; \frac{\partial r}{\partial y} = \frac{y}{r} \tag{13}$$

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial r} \frac{\partial r}{\partial x} = \frac{x}{r} \frac{\partial h}{\partial r} \tag{14}$$

$$\begin{aligned} \frac{\partial^2 h}{\partial x^2} &= \frac{\partial}{\partial x} \left( \frac{x}{r} \frac{\partial h}{\partial r} \right) \\ &= \frac{\partial}{\partial x} \left( \frac{1}{r} \right) \frac{\partial h}{\partial r} + \frac{x}{r} \frac{\partial^2 h}{\partial r^2} \frac{\partial r}{\partial x} \\ &= \left( \frac{1}{r} - \frac{x^2}{r^3} \right) \frac{\partial h}{\partial r} + \frac{x^2}{r^2} \frac{\partial^2 h}{\partial r^2} \end{aligned} \tag{15}$$

$$\frac{\partial^2 h}{\partial y^2} = \left( \frac{1}{r} - \frac{y^2}{r^3} \right) \frac{\partial h}{\partial r} + \frac{y^2}{r^2} \frac{\partial^2 h}{\partial r^2} \tag{16}$$

Hence, equation (4) now yields

$$\left( \frac{x^2+y^2}{r^2} \right) \frac{\partial^2 h}{\partial r^2} + \left( \frac{2}{r} - \frac{x^2+y^2}{r^3} \right) \frac{\partial h}{\partial r} - \frac{s}{T} \frac{\partial h}{\partial t} = 0$$

So that applying equation (5) we have

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} - \frac{s}{T} \frac{\partial h}{\partial t} = 0 \tag{17}$$

Thus, the following boundary value problem results:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} - \frac{s}{T} \frac{\partial h}{\partial t} = 0 \tag{18}$$

$$h(r, 0) = h_0; \text{ for all } r \geq 0 \tag{19}$$

$$h(\infty, t) = h_0; \text{ for all } t \geq 0 \tag{20}$$

$$\lim_{r \rightarrow 0} \left( r \frac{\partial h}{\partial r} \right) = \frac{Q}{2\pi T}; \text{ for all } t > 0 \tag{21}$$

Several methods of approach exist for solving this problem. The following three are considered here for comparative analysis.

### 3.3. Self-Similar (Similarity) Solution

1. Let

$$h(r,t) = t^m f(z) \text{ with } z = \frac{r^2 S}{4Tt}$$

Then

$$\frac{\partial z}{\partial r} = \frac{rS}{2Tt}, \text{ and } \frac{\partial z}{\partial t} = -\frac{r^2 S}{4Tt^2}$$

So that we have

$$\frac{\partial h}{\partial r} = \frac{rSt^{m-1}}{2T} \frac{df}{dz} \tag{22}$$

$$\frac{\partial^2 h}{\partial r^2} = \frac{s}{2T} \left[ t^{m-1} \frac{d^2 f}{dz^2} + \left( \frac{r^2 S^2 t^{m-2}}{4T^2} \right) \frac{d^2 f}{dz^2} \right] \tag{23}$$

$$\frac{\partial h}{\partial t} = mt^{m-1} f - \left( \frac{r^2 St^{m-2}}{4T} \right) \frac{df}{dz} \tag{24}$$

Thus, equation (18) now becomes

$$\begin{aligned} \left( \frac{St^{m-1}}{2T} \right) \frac{df}{dz} + \left( \frac{r^2 St^{m-2}}{4T} \right) \frac{d^2 f}{dz^2} + \left( \frac{St^{m-1}}{2T} \right) \frac{df}{dz} - \left( \frac{mSt^{m-1}}{T} \right) f + \left( \frac{r^2 St^{m-2}}{4T} \right) \frac{df}{dz} = 0 \\ \frac{St^{m-1}}{T} \left[ \frac{1}{2} \frac{df}{dz} + z \frac{d^2 f}{dz^2} + \frac{1}{2} \frac{df}{dz} - mf + z \frac{df}{dz} \right] = 0 \end{aligned}$$

Which yields

$$z \frac{d^2 f}{dz^2} + (1+z) \frac{df}{dz} - mf = 0 \tag{25}$$

Using the method of Frobenius, we seek the solution in the form

$$f(z) = \sum_{n \geq 0} \alpha_n z^{n+r}$$

Then the indicial equation is

$$r(r-1) + r = 0 \Rightarrow r^2 = 0 \Rightarrow r_1 = 0 - r_2$$

And the recurrence relation is

$$(n+r+1)^2 \alpha_{n+1} + (n+r-m) \alpha_n = 0 \Rightarrow \alpha_{n+1} = \frac{m-(n+r)}{(n+r+1)^2} \alpha_n, n = 0, 1, 2 \dots$$

So that we then have

$$\alpha_n = \prod_{k=0}^{n-1} \left( \frac{m-k}{(k+1)^2} \right) \alpha_0, n = 1, 2 \dots \tag{26}$$

Thus the solutions are

$$f_1(z) = \sum_{n \geq 0} \alpha_n z^n \text{ and } f_2(z) = \sum_{n \geq 0} \alpha_n z^n \ln z + \sum_{n \geq 0} \beta_n z^n \tag{27}$$

2. Now, let  $m = 0$  so that  $f(z) = h(r, t)$  and  $z$  is as defined. Then, equation (18) now becomes

$$z \frac{d^2 f}{dz^2} + (1+z) \frac{df}{dz} = 0 \tag{28}$$

Thus, we have

$$\frac{d^2 f}{dz^2} \frac{df}{dz} = -\frac{1+z}{z} = -\left( 1 + \frac{1}{z} \right) \tag{29}$$

$$\frac{df}{dz} = A \frac{e^{-z}}{z} \tag{30}$$

$$z \frac{df}{dz} = A e^{-z} \tag{31}$$

Now,

$$r \frac{\partial h}{\partial r} = \frac{r^2 S}{2Tt} \frac{df}{dz} = 2z \frac{df}{dz}$$

But

$$\frac{Q}{2\pi T} = \lim_{r \rightarrow 0} r \frac{\partial h}{\partial r} = 2 \lim_{r \rightarrow 0} z \frac{df}{dz} = 2A \lim_{r \rightarrow 0} e^{-z} = 2A$$

So that

$$A = \frac{Q}{2\pi T} \tag{32}$$

Thus,

$$\frac{df}{dz} = \frac{Q}{4\pi T} \frac{e^{-z}}{z}$$

Hence,

$$f(z) = f_0 - \frac{Q}{4\pi T} W(z) \tag{33}$$

### 3.4. Travelling Wave Solution

Let  $h(r, t) \equiv h(r \pm ct + d) = h(y)$  so that we now have

$$\frac{d^2 h}{dy^2} \pm g(y) \frac{dh}{dy} = 0 \tag{34}$$

Where

$$g(y) = \mp \left(1 + \frac{1}{y}\right)$$

Hence, we obtain as before

$$h(y) = h_0 - \frac{Q}{4\pi T} W(y) \tag{35}$$

### 3.5. Separable Variable (Fourier) Solution

Let  $h(r, t) \equiv R(r)G(t)$ . Then, we have

$$\frac{\partial h}{\partial t} = R(r) \frac{dG}{dt}; \quad \frac{\partial^2 h}{\partial r^2} = \frac{d^2 R}{dr^2} G(t) \tag{36}$$

So that equation (18) now becomes, after dividing through by  $R(r)G(t)$ ;

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{1}{rR} \frac{dR}{dr} = \frac{S}{G} \frac{dG}{dt} \tag{37}$$

For Periodic solutions, we now have

$$\frac{1}{R} \left( \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) = -\omega^2 = \frac{S}{G} \frac{dG}{dt} \tag{38}$$

Hence, we have the following two ordinary differential equations

$$\frac{dG}{dt} + \frac{\omega^2 T}{S} G = 0$$

Which has the solution

$$G(t) = C \exp \left\{ -\frac{\omega^2 T}{S} t \right\}$$

and

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \omega^2 R = 0$$

or equivalently

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \left( \omega^2 - \frac{0}{r^2} \right) R = 0 \tag{40}$$

Which is a *Bessel equation* and admits as a solution the *Bessel function* of the first kind  $J_0(\omega r)$  given by

$$R(r) = J_0(\omega r) = \sum_{n \geq 0} \frac{(-1)^n}{n!n!} \left( \frac{\omega r}{2} \right)^{2n} \tag{41}$$

Thus,

$$h(r, t) \equiv R(r)G(t) = \sum_{n \geq 0} \frac{(-1)^n}{n!n!} \left( \frac{\omega r}{2} \right)^{2n} \exp \left\{ -\frac{\omega^2 T}{S} t \right\} \tag{42}$$

### 3.6. Alternative Analysis

Further analysis of the solutions obtained from the self-similar, travelling wave and Fourier methods of approach would provide viable options in evaluating aquifer parameters from pumping test data. As the first in such possible series of analytical procedures, further analysis on the self-similar solution is hence discussed. We note that the basic equation now is

$$f_0 - f = \frac{Q}{4\pi T} W(z), \quad \text{where } W(z) = \int_z^\infty \frac{e^{-u}}{u} du \tag{43}$$

and observe that

$$e^{-u} = \sum_{r \geq 0} \frac{(-1)^r u^r}{r!}$$

Then

$$\frac{e^{-u}}{u} = \sum_{r \geq 0} \frac{(-1)^r u^{r-1}}{r!}$$

$$= \frac{1}{u} - 1 + \frac{u}{2!} - \frac{u^2}{3!} + \frac{u^3}{4!} - \dots$$

$$\int_z^\infty \frac{e^{-u}}{u} du = \left[ \ln u - u + \frac{u^2}{2.2!} + \frac{u^3}{3.3!} + \frac{u^4}{4.4!} - \dots \right]_z^\infty$$

Hence,

$$W(z) = -0.5772 - \ln z + z - \frac{z^2}{2.2!} + \frac{z^3}{3.3!} + \frac{z^4}{4.4!} - \dots \quad (44)$$

Equation (44) provides the infinite series solution by Cooper and Jacob (1946). Thus, for very small values of  $z$  (small values of  $r$  or large values of  $t$ ) we have

$$W(z) \sim -0.5772 - \ln z$$

Thus,

$$h_0 - h = \frac{Q}{4\pi T} (-0.5772 - \ln z)$$

$$= \frac{Q}{4\pi T} (-\ln 1.78 - \ln z)$$

$$= \frac{Q}{4\pi T} \ln \left( \frac{4T}{1.78r^2 S} t \right)$$

$$= \frac{2.3Q}{4\pi T} \log_{10} \left( \frac{2.25T}{r^2 S} t \right)$$

$$= \frac{0.575Q}{\pi T} \log_{10} \left( \frac{2.25T}{r^2 S} t \right)$$

$$= \frac{0.183Q}{\pi T} \log_{10} \left( \frac{2.25T}{r^2 S} t \right)$$

Here  $h_0 - h$  represents the change in drawdown after a time  $t$ , thus the corresponding changes after times  $t_1, t_2, t_3, \dots, t_k$  would be  $h_0 - h_1, h_0 - h_2, h_0 - h_3, \dots, h_0 - h_k$  respectively. Hence, the difference between the changes in drawdown after times  $t_1$  and  $t_k$  is

$$h_0 - h_k - (h_0 - h_1) = \frac{0.575Q}{\pi T} \left[ \log_{10} \left( \frac{2.25T}{r^2 S} t_k \right) - \log_{10} \left( \frac{2.25T}{r^2 S} t_1 \right) \right]$$

Which then yields

$$h_1 - h_k = \frac{0.575Q}{\pi T} \log_{10} \left( \frac{t_k}{t_1} \right) \quad (46)$$

Choosing time intervals to represent *log cycles* would provide the choice

$$t_n = 10^{n-1} t_1 \quad (47)$$

Thus, there are  $(n - 1)$  log cycles between  $t_1$  and  $t_n$ , leading to the equation

$$h_0 - h_n = \frac{0.575Q}{\pi T} (n - 1), \quad n = 1, 2, 3, \dots \quad (48)$$

Defining  $\Delta_n = h_1 - h_n$ , an arithmetic plot of  $\Delta_n$  versus  $n$  gives a straight line graph through (0,1) with slope

$$m = \frac{0.575Q}{\pi T} \quad (49)$$

So that aquifer transmissivity is then given by

$$T = \frac{0.575Q}{m\pi} \quad (50)$$

Observe that

$$\log_{10} x = \log_{10} a \cdot \log_a x$$

We may, therefore, modify (46) to become

$$h_1 - h_k = \frac{0.575Q \log_{10} r}{\pi T} \log_r \left( \frac{t_k}{t_1} \right) \quad (51)$$

Where  $1 < r \leq 10$ . Thus, choosing time intervals to represent log cycles (in base  $r$ ) yields

$$t_n = r^{n-1} t_1 \quad (52)$$

We therefore, have

$$h_1 - h_n = \frac{0.575Q \log_{10} r}{\pi T} (n - 1), \quad n = 1, 2, 3 \dots \quad (53)$$

An arithmetic plot of  $\Delta_n$  versus  $n$  again gives a straight line graph passing through (1,0) with slope

$$m = \frac{0.575Q \log_{10} r}{\pi T} \quad (54)$$

Aquifer transmissivity  $T$  is still given by

$$T = \frac{0.575Q}{m\pi} \quad (55)$$

As in (50). An obvious advantage of (52) over (47) is that with  $t_1 = 1$  minute, using (47)  $t_4 = 1000$  minutes whereas using (52) then  $t_4 = 125$  minutes when  $r = 5$  and  $t_4 = 8$  minutes when  $r = 2$ .

## 4. Discussion

The functional relationship (in terms of head loss  $h$ ) of the  $f(r, t)$  originally adopted in Thiem [2] for cylindrical inland flow model is also incorporated in the boundary values establishing the Theis [3] semi-log approach. The resultant assumptions and radial flow transformation equation (15) are often difficult to satisfy in most natural conditions. Some empirical corrections would thus be required when applying such analytical methods in some

hydrogeological conditions. For instance, cases of partial saturation, leaky and perched aquifer conditions, geostratigraphic discontinuity and geometric configuration, among other factors, have the potential to affect significantly, the expected results. Deviation from horizontal orientation, for instance, is likely to reduce the relevance of radial flow assumption (5), thus increasing the need for dip correction (fig. 1).

Mathematical solutions of the exponential integral have been provided using self-similar (similarity), separable variables (Fourier) and travelling wave methods of approach. Three analytical solutions with potentials to produce relevant results on further analysis have been obtained (equations (33), (35) and (42)). Further analysis of the self-similar solution has shown a good functional association in a *normal (arithmetic)* plot of drawdown versus the reading time intervals taken in log cycles (equations (47) and (53)). This method has been applied in analyzing some pumping test data from parts of Anambra Basin (fig. 2). The results give the transmissivity ( $T$ ) values of Ajalli sandstone at Obinofia Ndiuno and Akpugo-Nkanu as  $4.88m^2hr^{-1}$  and  $0.622m^2hr^{-1}$ . The relatively high values of  $T$  for Ajalli Sandstone seem typical of the averagely medium grained sand with minimal cement and clay content.

Among the peculiar problems associated with the application of normal plot method in Nigeria is the difficulty in obtaining step drawdown tests of duration long enough to cover three to four cycles (time equivalent of 1000 to 10,000 minutes). This has, however, been adequately addressed by the further consideration (equation 52) since the base of the logarithm function to be used could be determined from the perceived duration of the experiment and the number of observations/ readings desired. Adequate provision for any desired duration and/ or number of readings would be possible if considered at the design stage, or a constant rate would be adopted. Appreciably good results could be achieved within a short time and with a more readily affordable technique.

## 5. Conclusion

Wide differences often observed in results from the same pumping test data analysed with different methods indicate pre-existing errors in the analytical techniques. Improvement on the validity of aquifer parameters obtained from these methods is thus necessary. This could be achieved through constant review of the available methods of analysis and the underlying assumptions.

Analysis of pumping test data using normal graph plot constitutes a good alternative or, at least, a supplementary procedure to curve- matching and semi-log methods. Further geomathematical analysis is necessary to increase the available options, and increase the accuracy in pumping test analysis.

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## Appendix

Tables-1. A, B, C. Pumping test data from parts of Anambra State

A: Otolo Nnewi ( $Q = 6.8m^3hr^{-1}$ )

time (minutes)	drawdown (meters)	time (minutes)	drawdown (meters)
10.00	2.00	60.00	4.60
120.00	5.56	180.00	5.90
240.00	6.00	360.00	6.20
540.00	6.30	720.00	6.40
940.00	6.50	1440.00	6.80

B: Obinofia Ndiuno ( $Q = 40m^3hr^{-1}$ )

time (minutes)	drawdown (meters)	time (minutes)	drawdown (meters)
1.00	0.15	2.00	0.58
3.00	0.80	4.00	1.12
5.00	1.32	7.00	1.49
10.00	1.90	15.00	2.20
20.00	2.42	25.00	2.65
30.00	3.00	45.00	3.34
60.00	3.56	75.00	3.95
90.00	4.34	105.00	4.59
120.00	5.05	180.00	5.27
360.00	5.58	540.00	5.62
720.00	5.64	900.00	5.64
1440.00	23.94		

C: Akpugo (Nkanu L.G.A.) ( $Q = 50m^3hr^{-1}$ )

time (minutes)	drawdown (meters)	time (minutes)	drawdown (meters)
1.00	0.23	1.50	0.70
2.00	1.13	2.50	1.50
3.00	1.83	4.00	2.20
5.00	2.68	10.00	3.07
30.00	3.71	45.00	4.20
60.00	4.51	75.00	4.93
105.00	5.55	120.00	6.03
135.00	6.45	165.00	6.60
210.00	6.65	220.00	6.70
280.00	6.70	460.00	6.70
820.00	6.70	1000.00	6.70
1440.00	6.70		

Fig-1. Geometric model of a Dipping, Confined Aquifer

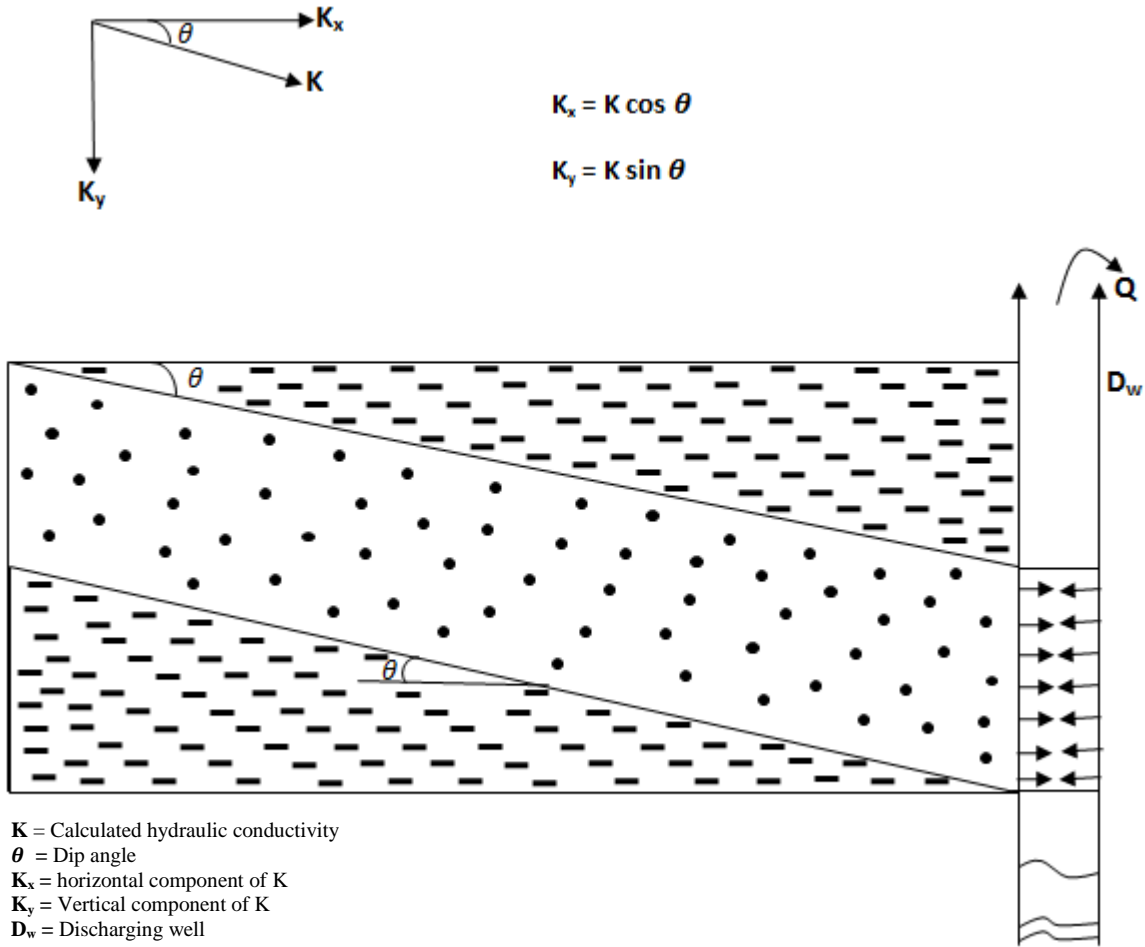
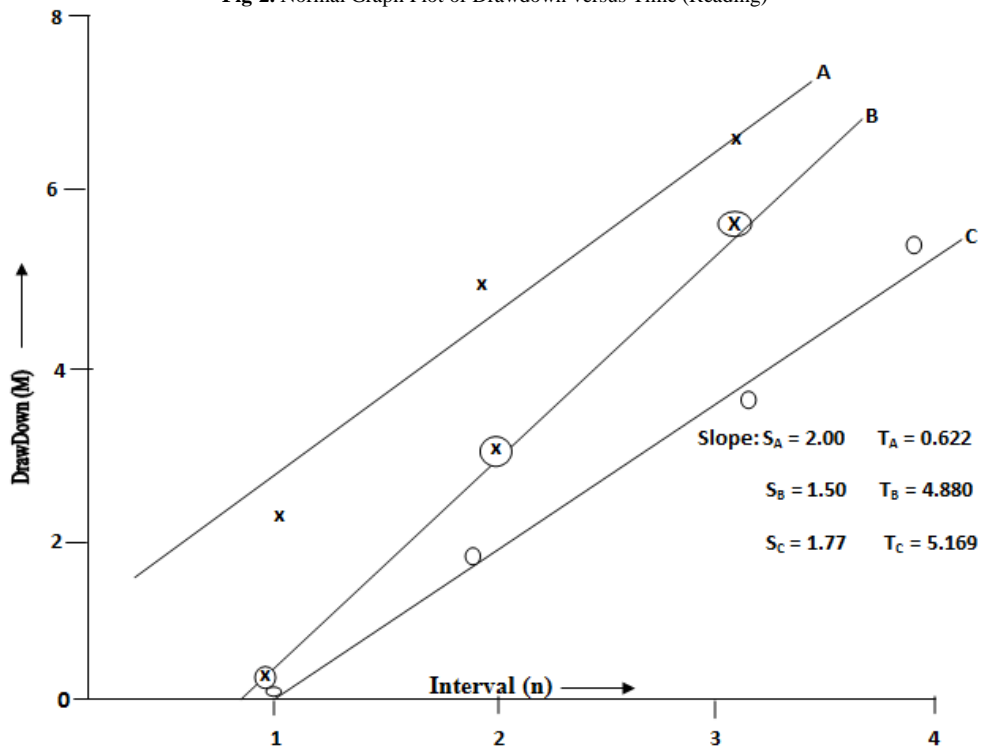


Fig-2. Normal Graph Plot of Drawdown versus Time (Reading)



Interval (n). A = Nnewi; B = Obinofia Ndiuno; C = Akpugo Nkanu L.G.A.