



The Mystical Corrections on the Fractional Binomial Expansion

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Abstract

For so many years now a lot of scientist have used the series of positive binomial expansion to solve that of positive fractional binomial expansion, Negative binomial expansion, and Negative fractional binomial expansion which was generated/derived using Maclaurin series to derive the series of positive fractional binomial expansion, Negative binomial expansion, and Negative fractional binomial expansion just as it was used to provide answers to positive binomial expansion but fails for All the other expansion due to a deviation made. This Manuscript contains the correct solution/answers to both positive fractional binomial expansions and Negative fractional binomial expansion with proofs through worked examples and a general formula to solve questions involving both positive fractional binomial expansions and Negative fractional binomial expansions.

Keywords: Solve; Equation; Substituting.



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1. Introduction

According to Coolidge [1] the binomial Theorem, familiar at least in its elementary aspects to every students of algebra, has a long and reasonably plain history. Most people associate it vaguely in their minds with the name of Newtons; he either invented it or it was carved on his tomb. In some way or the other it was his theorem. Well, as a matter of fact it wasn't, although his work did mark an important advance in the general theory.

We find the first trace of Binomial Theorem in Euclid II, 4, "if a straight line be cut at random, the square on the whole is equal to the square on the segments and twice the rectangle of the segments." if the segments are a and b this means in algebraic language

$$(a + b)^2 = a^2 + b^2 + 2ab$$

The corresponding formula for the square difference is found in Euclid II, 7, "if a straight line be cut at random, the square on the whole and that on one of the segments both together, are equal to twice the rectangle contained by the whole and said segment, and the square on the remaining segment,".

From the formula above it is seen that from the left hand side that if a& b are summed up together and then squared the results obtained must be the same for the right hand side that is a squared plus b squared plus 2 time the product of a & b.

Therefore, if the result for both the positive fractional binomial expansions and Negative fractional binomial expansions estimated from the left hand side of an equation is not exactly equal to the Right hand of the equation just as in the case of positive binomial then we can either get an approximate results or a wrong result. With relevant worked examples in this Manuscript shows the exact solution to both the positive fractional binomial expansions, Negative fractional binomial expansions and the deviations made using the Maclaurin series to obtain the expansion for both positive fractional binomial expansions and that of the Negative fractional binomial expansions [2-6].

2. Methodology

For so many years, there has been a Mistake made in using the series of positive binomial to finding the expansion of both positive fractional binomial expansions and Negative fractional binomial expansions. This material presents new solutions to the series of both positive fractional binomial expansions and Negative fractional binomial expansions using worked examples.

2.1. Positive Fractional Binomial Expansion

Example 2.1.

Solve the following;

$$\sqrt{1+x}$$

Solution

Given that

$$(1+x)^{\frac{1}{2}} = p \tag{1}$$

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∴ on squaring both sides of the above equation we've

$$p^2 = 1 + x \quad (2)$$

$$p^2 - 1 = x$$

$$(p-1)(p+1) = x$$

$$\Rightarrow (p-1) = \frac{x}{p+1} \quad (3)$$

$$p = 1 + \frac{x}{p+1} \quad (4)$$

Substituting the value of p from equation (1) above into (4) we've

$$\sqrt{1+x} = 1 + \frac{x}{1+\sqrt{1+x}} \quad (5)$$

Example 2.2

Solve the following;

$$\sqrt[3]{1+x}$$

Solution

Given that

$$(1+x)^{\frac{1}{3}} = p \quad (6)$$

$$p^3 = 1 + x \quad (7)$$

$$p^3 - 1 = x \quad (8)$$

$$(p-1)(p^2 + p + 1) = x \quad (9)$$

$$\Rightarrow p = 1 + \frac{x}{p^2 + p + 1} \quad (10)$$

Substituting the value of p from equation (6) above into (10) we've

$$\sqrt[3]{1+x} = 1 + \frac{x}{1 + \sqrt[3]{1+x} + (\sqrt[3]{1+x})^2} \quad (11)$$

Example 2.3.

Solve the following;

$$\sqrt[4]{1+x}$$

Solution

Given that

$$(1+x)^{\frac{1}{4}} = p \quad (12)$$

$$p^4 = 1 + x \quad (13)$$

$$p^4 - 1 = x \quad (14)$$

$$(p^2 - 1)(p^2 + 1) = x \quad (15)$$

$$(p-1)(p+1)(p^2 + 1) = x \quad (16)$$

$$(p-1)(p^3 + p^2 + 1) = x \quad (17)$$

$$\Rightarrow p = 1 + \frac{x}{p^3 + p^2 + p + 1} \quad (18)$$

Substituting the value of p from equation (12) above into (18) we've

$$\sqrt[4]{1+x} = 1 + \frac{x}{1 + \sqrt[4]{1+x} + (\sqrt[4]{1+x})^2 + (\sqrt[4]{1+x})^3} \quad (19)$$

From equation (5), (11) and (19) it is seen that studying the series above we've the general formular for positive fractional index as

$$\sqrt[n]{1+x} = 1 + \frac{x}{1 + \sqrt[n]{1+x} + (\sqrt[n]{1+x})^2 + \dots + (\sqrt[n]{1+x})^{n-1}} \quad (20)$$

2.2. General Case of the Positive Fractional Binomial Expansion

$$(x+a)^{\frac{1}{n}} = \left[x \left(1 + \frac{a}{x} \right) \right]^{\frac{1}{n}} = x^{\frac{1}{n}} \cdot \left(1 + \frac{a}{x} \right)^{\frac{1}{n}} \quad (21)$$

∴ using the Analogy of equation (20) in (21) we've

$$\begin{aligned} (x+a)^{\frac{1}{n}} &= x^{\frac{1}{n}} \cdot \left(1 + \frac{a}{x} \right)^{\frac{1}{n}} = \\ &= x^{\frac{1}{n}} \cdot \left(1 + \frac{\left(\frac{a}{x} \right)}{1 + \sqrt[n]{1 + \frac{a}{x}} + \left(\sqrt[n]{1 + \frac{a}{x}} \right)^2 + \dots + \left(\sqrt[n]{1 + \frac{a}{x}} \right)^{n-1}} \right)^{\frac{1}{n}} \\ &= \left(x^{\frac{1}{n}} + \frac{x^{\frac{1}{n}} \cdot \left(\frac{a}{x} \right)}{1 + \sqrt[n]{1 + \frac{a}{x}} + \left(\sqrt[n]{1 + \frac{a}{x}} \right)^2 + \dots + \left(\sqrt[n]{1 + \frac{a}{x}} \right)^{n-1}} \right) \\ &\Rightarrow \\ &= \left(x^{\frac{1}{n}} + \frac{a \cdot x^{\left(\frac{1-n}{n} \right)}}{1 + \sqrt[n]{1 + \frac{a}{x}} + \left(\sqrt[n]{1 + \frac{a}{x}} \right)^2 + \dots + \left(\sqrt[n]{1 + \frac{a}{x}} \right)^{n-1}} \right) \end{aligned}$$

2.3. Negative Fractional Binomial Expansion

Given that

$$\sqrt[n]{1+x} = 1 + \frac{x}{1 + \sqrt[n]{1+x} + (\sqrt[n]{1+x})^2 + \dots + (\sqrt[n]{1+x})^{n-1}}$$

we've

$$\begin{aligned} (1+x)^{-\frac{1}{n}} &= \\ \frac{1}{(1+x)^{\frac{1}{n}}} &= \frac{1 + \sqrt[n]{1+x} + (\sqrt[n]{1+x})^2 + \dots + (\sqrt[n]{1+x})^{n-1}}{x + 1 + \sqrt[n]{1+x} + (\sqrt[n]{1+x})^2 + \dots + (\sqrt[n]{1+x})^{n-1}} \end{aligned}$$

The equation above is the general formula for negative fractional index of the given form.

2.4. General Case of the Negative Fractional Binomial Expansion

$$(x+a)^{-\frac{1}{n}} = \frac{1}{(x+a)^{\frac{1}{n}}} = \left(\frac{1 + n\sqrt{\frac{a}{x}} + \binom{n}{2}\left(\sqrt{\frac{a}{x}}\right)^2 + \dots + \binom{n}{n-1}\left(\sqrt{\frac{a}{x}}\right)^{n-1}}{x^{\frac{1}{n}} \left[1 + n\sqrt{\frac{a}{x}} + \binom{n}{2}\left(\sqrt{\frac{a}{x}}\right)^2 + \dots + \binom{n}{n-1}\left(\sqrt{\frac{a}{x}}\right)^{n-1} \right] + a \cdot x^{\frac{1-n}{n}}} \right)$$

2.5. Real Life Application of Binomial Theorem

The binomial theorem has a lot of Applications. [7-9] Some of the applications in real life Situations are:

★ Computing.

In computing areas, binomial theorem has been very Useful such as in distribution of IP addresses. With Binomial theorem, the automatic distribution of IP Addresses is not only possible but also the Distribution of virtual IP addresses.

★ Economy

Economists used binomial theorem to count probabilities that depend on numerous and very distributed variables to predict the way the economy will behave in the next few years. To be able to come up with realistic predictions, binomial theorem is used in this field.

★ Architecture

Architecture industry in design of infrastructure, allows engineers, to calculate the magnitudes of the projects and thus delivering accurate estimates of not only the costs but also time required to construct them. For contractors, it is a very important tool to help ensuring the costing projects is competent enough to deliver profits.

2.6. Applications of Binomial Expansion in Physics

The binomial Expansion has other Applications in physics amongst which we've its Applications in

- ★ Gravitational time dilation.
- ★ Kinetic energy
- ★ Electric quadrupole field.
- ★ Relativity factor gamma
- ★ Kinematic time dilation.

Other Applications of Binomial Expansions are in:

- ★ [10] Agriculture in Solving Problems in Genetics.

3. Conclusion

From the worked examples done above we've seen the correct solution to both positive fractional binomial expansion, Negative fractional binomial expansion and learnt how to solve both positive fractional binomial expansion and Negative fractional binomial expansion using the general formula above.

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